PRACTICE EXAM 9: ALGEBRA II REGENTS SIMULATION

PART I — Multiple Choice (24 questions × 2 credits = 48 credits)

1. The expression $(5 - 3i) - (2 - 7i)$, written in $a + bi$ form, is

A. $7 - 10i$

B. $3 - 10i$

C. $7 + 4i$

D. $3 + 4i$

2. The expression $27^{(4/3)}$ is equivalent to

A. 12

B. 81

C. 27^4

D. 36

3. For the equation $2x^2 - 10x + 8 = 0$, the sum and product of the roots are

A. sum = −5, product = 4

B. sum = 5, product = −4

C. sum = 5, product = 4

D. sum = −5, product = −4

4. The polynomial $x^3 - 2x^2 - 9x + 18$, factored completely over the integers, is

A. $(x - 2)(x - 3)(x + 3)$

B. $(x + 2)(x - 3)(x + 3)$

C. $(x - 2)(x^2 + 9)$

D. $(x - 2)(x - 3)^2$

5. The solutions to the equation $x^2 - 4x + 13 = 0$ are

A. $4 \pm 3i$

B. $-2 \pm 3i$

C. $2 \pm 4i$

D. $2 \pm 3i$

6. What is the solution to the equation $5/(x + 1) - 2/(x - 1) = 4/(x^2 - 1)$?

A. $x = -1$

B. $x = 1$

C. no solution

D. $x = 11/3$

7. One zero of the polynomial $p(x) = x^3 - 2x^2 - 5x + 6$ is $x = 1$. The other two zeros are

A. $x = 3$ and $x = 2$

B. $x = 3$ and $x = -2$

C. $x = -3$ and $x = 2$

D. $x = -3$ and $x = -2$

8. What is the solution to the equation $\sqrt[3]{(x + 5)} = 3$?

A. $x = 22$

B. $x = 8$

C. $x = 4$

D. $x = 32$

9. How many real solutions does the system $y = x^2 + 1$ and $y = -x + 3$ have?

A. 0

B. 1

C. 2

D. 3

10. A drug dosage in the bloodstream decreases by 30% every hour. If the initial dosage is 200 mg, which function models the amount A remaining after t hours?

A. $A(t) = 200(0.30)^t$

B. $A(t) = 200(0.70)^t$

C. $A(t) = 200 - 30t$

D. $A(t) = 200(1.30)^t$

11. If $f(x) = (x - 1) / (x + 3)$, then the value of $f(5)$ is

A. 1/2

B. 2

C. 2/3

D. 5/8

12. The inverse of the function $f(x) = 3x - 7$ is $f^{-1}(x)$ equal to

A. $7 - 3x$

B. $(x - 7) / 3$

C. $-(3x - 7)$

D. $(x + 7) / 3$

13. The sum of the infinite geometric series $8 + 4 + 2 + 1 + \cdots$ is

A. 8

B. 12

C. 16

D. infinite (does not converge)

14. Solve algebraically for x: $\log(3x) = 2$.

A. $x = 50$

B. $x = 100/3$

C. $x = 20$

D. $x = 2/3$

15. The exact value of $\tan(5\pi/4)$ is

A. 1

B. -1

C. $\sqrt{3}$

D. $-\sqrt{3}$

16. What is the period of the function $y = 5\sin(4\pi x) + 2$?

A. 4π

B. 2

C. 1/2

D. 4

17. The expression $\log_8(x)$, rewritten using the change of base formula with base 2, is equal to

A. $\log_2(x) - 3$

B. $3 \log_2(x)$

C. $(\log_2 x) / 3$

D. $\log_2(x / 3)$

18. What is the sum of the first 30 terms of the arithmetic sequence 3, 7, 11, 15, …?

A. 1,530

B. 1,620

C. 1,740

D. 1,830

19. The displacement of a particle from equilibrium is modeled by $y = 4 \sin((\pi/3)t)$, where y is measured in meters and t is measured in seconds. The maximum displacement of the particle from equilibrium is

A. 4 m

B. 8 m

C. 3 m

D. $\pi/3$ m

20. If $\sin \theta = -3/5$ and θ is in Quadrant IV, then the value of $\cos \theta$ is

A. $-4/5$

B. $4/5$

C. $-3/4$

D. $3/4$

21. A simple random sample of $n = 100$ students yields a sample mean weight of 150 pounds with a sample standard deviation of 20 pounds. The standard error of the sample mean is

A. 20

B. 100

C. 10

D. 2

22. A statistician runs 10,000 simulations of rolling a fair six-sided die 60 times. The mean number of 6's observed per simulation will be approximately

A. 10

B. 60

C. 6

D. 1/6

23. A box contains 6 red marbles and 4 blue marbles. Two marbles are drawn at random without replacement. What is the probability that the first marble is red and the second marble is blue?

A. 6/25

B. 4/15

C. 2/5

D. 1/5

24. The probability of at least one event A occurring is 0.85. What is the probability that A does not occur?

A. 0.85

B. 0.5

C. 0.15

D. 0.7

PART II — Short Constructed Response (8 questions × 2 credits = 16 credits)

Show all work. A correct answer with no supporting work will receive only 1 credit.

25. Solve the equation $x^2 - 2x + 10 = 0$ algebraically. Express the solutions in $a + bi$ form.

26. Simplify the rational expression $(x^2 - 8x + 16) / (x^2 - 16)$ for $x \neq \pm 4$. Show all factoring steps.

27. Use synthetic division to determine whether $(x + 3)$ is a factor of $2x^3 + 5x^2 - x - 6$. State your conclusion clearly and justify it with the value of the remainder.

28. Solve algebraically: $\log_2(x) + \log_2(x - 6) = 4$. Show all algebraic steps and verify your solution.

29. Find the explicit formula for the arithmetic sequence whose third term is $a_3 = 11$ and whose seventh term is $a_7 = 27$. Show all algebraic work.

30. Verify algebraically that $(\sin^2\theta) / (1 - \cos\theta) = 1 + \cos\theta$ for $\cos\theta \neq 1$. Show every algebraic step using a Pythagorean identity.

31. Solve the equation $\sin(2x) = \sin(x)$ for all values of x in the interval $[0, \pi]$. Show all algebraic work using a double-angle identity.

32. A test consists of 5 multiple-choice questions, each with 4 possible answer options of which exactly one is correct. A student guesses on every question. Determine the probability that the student gets exactly 3 questions correct. Express your answer as a fraction in simplest form.

PART III — Extended Constructed Response (3 questions × 4 credits = 12 credits)

Show all work. Partial credit is awarded according to the scoring rubric.

33. A car loan of $24,000 is repaid over 5 years at an annual interest rate of 6%, compounded monthly. The monthly payment M is given by the formula $M = P \cdot [r(1 + r)^n] / [(1 + r)^n - 1]$, where P is the loan amount, r is the monthly interest rate, and n is the total number of monthly payments.

(a) State the values of P, r, and n that apply to this loan.

(b) Algebraically determine the monthly payment M. Round your final answer to the nearest cent.

34. The polynomial function $p(x) = x^3 - 5x^2 + x + 3$ has $x = 1$ as a known real root.

(a) Use synthetic division to find the quotient when $p(x)$ is divided by $(x - 1)$. Show your synthetic division work.

(b) Use the quadratic formula on the quotient to find the remaining roots. Express your answers in simplest radical form.

(c) State all real solutions to the equation $p(x) = 0$.

35. A city's daily high temperatures over a 30-day period are approximately normally distributed with a mean of 75°F and a standard deviation of 6°F.

(a) Determine the approximate percentage of days during which the high temperature was between 69°F and 87°F. Justify your answer using the Empirical Rule.

(b) Determine the z-score for a day with a high temperature of 84°F. Explain what this z-score represents in context.

(c) The city's climate office defines an "unusually hot day" as a day with a z-score greater than 2 in temperature. Determine the minimum temperature (in °F) at which a day qualifies as unusually hot.

PART IV — Long Constructed Response (1 question × 6 credits = 6 credits)

Show all work. This problem integrates concepts from several chapters.

36. A spring is set in motion, with displacement d (in centimeters) above its rest position at time t (in seconds) modeled by $d(t) = -8\cos((\pi/3)t)$. A positive value of d means the spring is above its rest position; a negative value means the spring is below.

(a) Determine the period of oscillation. Justify your reasoning based on the model.

(b) Determine the displacement of the spring at t = 2 seconds. Round your answer to the nearest tenth of a centimeter.

(c) Algebraically determine all values of t in the interval [0, 6] seconds at which the spring is exactly 5 centimeters above its rest position. Round each value of t to the nearest hundredth of a second. Show all algebraic work.

PRACTICE EXAM 9 — ANSWER KEY AND EXPLANATIONS

1. D — Subtract complex numbers component-wise: $(5 - 3i) - (2 - 7i) = (5 - 2) + (-3i + 7i) = 3 + 4i$. The negative sign distributes through both the real and imaginary parts of the second number.

2. B — Apply the rational exponent rule: $27^{(4/3)} = (27^{(1/3)})^4$. The cube root of 27 is 3, and $3^4 = 81$. Taking the cube root first keeps the intermediate calculations small.

3. C — Apply Vieta's formulas for $ax^2 + bx + c = 0$: sum of roots $= -b/a = -(-10)/2 = 5$, and product of roots $= c/a = 8/2 = 4$. These relationships hold for any quadratic, including those with complex roots.

4. A — Group the four terms in pairs: $x^2(x - 2) - 9(x - 2) = (x - 2)(x^2 - 9)$. The quadratic factor is a difference of squares: $x^2 - 9 = (x - 3)(x + 3)$, giving $(x - 2)(x - 3)(x + 3)$.

5. D — Apply the quadratic formula with $a = 1$, $b = -4$, $c = 13$: $x = [4 \pm \sqrt{(16 - 52)}]/2 = [4 \pm \sqrt{(-36)}]/2 = [4 \pm 6i]/2 = 2 \pm 3i$. The negative discriminant produces a complex conjugate pair.

6. D — The LCD is $x^2 - 1 = (x + 1)(x - 1)$ with restrictions $x \neq \pm 1$. Multiply through: $5(x - 1) - 2(x + 1) = 4$, giving $3x - 7 = 4$ and $x = 11/3$. Since $11/3 \neq \pm 1$, the solution is valid.

7. B — Apply synthetic division by $(x - 1)$ to obtain quotient $x^2 - x - 6$, which factors as $(x - 3)(x + 2)$. Setting each factor to zero gives the other zeros $x = 3$ and $x = -2$.

8. A — Cube both sides to eliminate the radical: $x + 5 = 27$, giving $x = 22$. The cube root function is one-to-one, so no extraneous solutions can arise.

9. C — Substitute to eliminate y: $x^2 + 1 = -x + 3$, giving $x^2 + x - 2 = 0$. The discriminant $1 - 4(1)(-2) = 9$ is positive, so there are two distinct real solutions where the line crosses the parabola.

10. B — A 30% decrease per hour means 70% remains, so the hourly multiplier is 0.70. The exponential decay model is $A(t) = 200(0.70)^t$. Verification at $t = 1$: $200 \cdot 0.7 = 140$, which is 70% of 200 ✓.

11. A — Substitute $x = 5$: $f(5) = (5 - 1)/(5 + 3) = 4/8 = 1/2$. The fraction simplifies by dividing numerator and denominator by 4.

12. D — Swap x and y in $y = 3x - 7$ to get $x = 3y - 7$. Solve for y: $3y = x + 7$, so $y = (x + 7)/3$. Verification: $f(f^{-1}(x)) = 3 \cdot (x + 7)/3 - 7 = x + 7 - 7 = x$ ✓.

13. C — Since $|r| = 1/2 < 1$, the series converges. Apply the formula $S = a_1/(1 - r) = 8/(1 - 1/2) = 8/(1/2) = 16$. The terms shrink toward zero quickly enough for the infinite sum to exist.

14. B — Convert the logarithmic equation to exponential form: $\log(3x) = 2$ means $10^2 = 3x$, so $100 = 3x$ and $x = 100/3$. The base of common log is 10.

15. A — The angle $5\pi/4$ lies in Quadrant III with reference angle $\pi/4$. In Quadrant III both sine and cosine are negative, so their ratio (tangent) is positive: $\tan(5\pi/4) = \tan(\pi/4) = 1$.

16. C — For $y = A \sin(Bx) + D$, the period is $2\pi/B$. With $B = 4\pi$: period $= 2\pi/(4\pi) = 1/2$. The function completes one cycle every 1/2 unit on the x-axis.

17. C — The change of base formula gives $\log_8(x) = \log_2(x)/\log_2(8)$. Since $8 = 2^3$, $\log_2(8) = 3$, so $\log_8(x) = \log_2(x)/3$.

18. D — Apply the arithmetic series formula $S_n = (n/2)(2a_1 + (n - 1)d)$ with $a_1 = 3$, $d = 4$, $n = 30$: $S_{30} = 15(6 + 116) = 15(122) = 1{,}830$. The 30th term itself is $3 + 29(4) = 119$.

19. A — The maximum displacement of a sinusoidal function from its midline equals the amplitude $|A|$. With $A = 4$, the maximum displacement is 4 m. The period and phase are not relevant to amplitude.

20. B — Apply the Pythagorean identity: $\cos^2\theta = 1 - \sin^2\theta = 1 - 9/25 = 16/25$, so $|\cos \theta| = 4/5$. In Quadrant IV cosine is positive (x-coordinates are positive there), so $\cos \theta = 4/5$.

21. D — The standard error of the sample mean is $SE = s/\sqrt{n}$, which estimates how much the sample mean varies from the population mean. With $s = 20$ and $n = 100$: $SE = 20/\sqrt{100} = 20/10 = 2$.

22. A — For a binomial random variable with n trials and probability p of success per trial, the expected value is $\mu = np$. Here $n = 60$ and $p = 1/6$, so $\mu = 60(1/6) = 10$ — averaging across many simulations, the mean count converges to this expected value.

23. B — Without replacement, the events are dependent. P(red first) = 6/10 = 3/5; after one red is removed, P(blue second | red first) = 4/9 (4 blue out of 9 remaining). Multiply: (3/5)(4/9) = 12/45 = 4/15.

24. C — The complement rule states P(A does not occur) = 1 − P(A occurs). With P(A) = 0.85: P(not A) = 1 − 0.85 = 0.15. Probabilities of complementary events sum to 1.

PART II — Short Constructed-Response Explanations

25. [2 credits] Apply the quadratic formula with a = 1, b = −2, c = 10: x = [2 ± √(4 − 40)]/2 = [2 ± √(−36)]/2 = [2 ± 6i]/2 = **1 ± 3i**. Solutions in a + bi form: x = 1 + 3i and x = 1 − 3i.

26. [2 credits] Factor: $x^2 - 8x + 16 = (x - 4)^2$ and $x^2 - 16 = (x - 4)(x + 4)$. Cancel the common factor: $(x - 4)^2/[(x - 4)(x + 4)]$ = **(x − 4)/(x + 4)** for x ≠ ±4.

27. [2 credits] Apply synthetic division with k = −3 on coefficients 2, 5, −1, −6:

```
-3 |  2     5    -1    -6
   |       -6     3    -6
   |___________________________
      2    -1     2   -12
```

The remainder is **−12 ≠ 0**, so by the Factor Theorem **(x + 3) is NOT a factor** of $2x^3 + 5x^2 - x - 6$.

28. [2 credits] Apply the product rule: $\log_2(x(x - 6)) = 4$. Convert to exponential form: $x(x - 6) = 2^4 = 16$, giving $x^2 - 6x - 16 = 0$ and (x − 8)(x + 2) = 0. Candidate solutions: x = 8 and x = −2. Domain requires x > 6 (so both log arguments are positive), so x = −2 is rejected and **x = 8**. Verification: $\log_2(8) + \log_2(2)$ = 3 + 1 = 4 ✓.

29. [2 credits] Find d: $a_7 - a_3 = 4d$, so 27 − 11 = 4d, giving d = 4. Find a_1: $a_3 = a_1 + 2d$, so $11 = a_1 + 8$, giving $a_1 = 3$. The explicit formula is $a_n = a_1 + (n - 1)d = 3 + 4(n - 1)$ = **4n − 1**.

30. [2 credits] Apply the Pythagorean identity $\sin^2\theta + \cos^2\theta = 1$ to rewrite $\sin^2\theta = 1 - \cos^2\theta$. Recognize that $1 - \cos^2\theta$ is a difference of squares: $1 - \cos^2\theta$ = **(1 − cos θ)(1 + cos θ)**. Substitute into the left side: $(\sin^2\theta)/(1 - \cos\theta) = [(1 - \cos\theta)(1 + \cos\theta)]/(1 - \cos\theta)$ = **1 + cos θ** for cos θ ≠ 1 ✓. Both sides are equal, identity verified.

31. [2 credits] Apply the double-angle identity sin(2x) = 2 sin x cos x to get 2 sin x cos x = sin x. Subtract: 2 sin x cos x − sin x = 0, then factor: sin x (2 cos x − 1) = 0. Solving sin x = 0 in [0, π] gives **x = 0 and x = π**. Solving 2 cos x − 1 = 0 gives cos x = 1/2, so **x = π/3**. All three solutions: x = 0, π/3, and π.

32. [2 credits] Apply the binomial probability formula $P(X = k) = C(n, k) \cdot p^k \cdot (1 - p)$^(n − k) with n = 5, k = 3, p = 1/4: $P(X = 3) = C(5, 3) \cdot (1/4)^3 \cdot (3/4)^2 = 10 \cdot (1/64) \cdot (9/16)$ = 90/1,024 = **45/512** (in simplest form, since GCD(90, 1024) = 2).

PART III — Extended Constructed-Response Explanations

33.

(a) [1 credit] **$P = 24{,}000$** (loan amount in dollars), **$r = 0.06/12 = 0.005$** (monthly interest rate), and **$n = 5 \times 12 = 60$** (total monthly payments over 5 years).

(b) [3 credits] Compute $(1.005)^{60} \approx 1.34885$. Substitute into the formula: $M = 24{,}000 \cdot [0.005 \cdot 1.34885]/[1.34885 - 1] = 24{,}000 \cdot [0.006744]/[0.34885] = 24{,}000 \cdot 0.01933 \approx$ **\$463.99**. The borrower pays approximately \$463.99 each month for 60 months.

34.

(a) [2 credits] Apply synthetic division with $k = 1$ on coefficients 1, −5, 1, 3:

```
1 |  1   -5    1    3
  |        1   -4   -3
  |_____________________________
     1   -4   -3    0
```

The quotient is **$x^2 - 4x - 3$** with remainder 0, confirming $x = 1$ as a root.

(b) [1 credit] Apply the quadratic formula to $x^2 - 4x - 3 = 0$ with $a = 1$, $b = -4$, $c = -3$: $x = [4 \pm \sqrt{(16 + 12)}]/2 = [4 \pm \sqrt{28}]/2 = [4 \pm 2\sqrt{7}]/2 =$ **$2 \pm \sqrt{7}$**.

(c) [1 credit] All real solutions: **$x = 1$, $x = 2 + \sqrt{7}$, and $x = 2 - \sqrt{7}$**.

35.

(a) [1 credit] 69°F = 75 − 6 = mean − 1σ, and 87°F = 75 + 12 = mean + 2σ. By the Empirical Rule, ~34% of data lies between the mean and 1σ below, and ~47.5% lies between the mean and 2σ above. Total: 34% + 47.5% ≈ **81.5%** of days had high temperatures between 69°F and 87°F.

(b) [2 credits] $z = (84 - 75)/6 = 9/6 =$ **1.5**. This z-score means the temperature of 84°F lies **1.5 standard deviations above the mean** — it is warmer than average but not yet at the extreme tails of the distribution.

(c) [1 credit] $z > 2$ corresponds to temperature > mean + 2σ = 75 + 2(6) = **87°F**. Any day with a high temperature greater than 87°F qualifies as unusually hot under the office's criterion.

PART IV — Long Constructed-Response Explanation

36.

(a) [2 credits] In the model $d(t) = -8\cos((\pi/3)t)$, the coefficient of t inside cosine is $B = \pi/3$. The period of a sinusoidal function is $2\pi/B = 2\pi/(\pi/3) =$ **6 seconds**. This means the spring completes one full oscillation every 6 seconds.

(b) [1 credit] Substitute $t = 2$: $d(2) = -8\cos((\pi/3)(2)) = -8\cos(2\pi/3)$. Since $\cos(2\pi/3) = -1/2$: $d(2) = -8(-1/2) =$ **4.0 cm**. The spring is 4 cm above its rest position at t = 2 seconds.

(c) [3 credits] Set $d(t) = 5$: $-8\cos((\pi/3)t) = 5$, giving $\cos((\pi/3)t) = -5/8$. Let $u = (\pi/3)t$; as t ranges over [0, 6], u ranges over $[0, 2\pi]$. Solve $\cos(u) = -5/8$: $u_1 = \arccos(-5/8) \approx 2.2459$ rad, and by symmetry $u_2 = 2\pi - \arccos(-5/8) \approx 4.0373$ rad. Convert back using $t = (3/\pi)u$: $t_1 \approx (3/\pi)(2.2459) \approx$ **2.14 seconds**, and $t_2 \approx (3/\pi)(4.0373) \approx$ **3.86 seconds**. The spring is 5 cm above its rest position at two times during one full oscillation, symmetrically placed around the time of maximum displacement (t = 3 s).