PRACTICE EXAM 8: ALGEBRA II REGENTS SIMULATION

PART I — Multiple Choice (24 questions × 2 credits = 48 credits)

1. The expression $(8x^6)$^(2/3) is equivalent to

A. $4x^4$

B. $4x^9$

C. $16x^4$

D. $8x^4$

2. Expressed in a + bi form, the quotient (4 + 3i) / (2 − i) equals

A. 5 + 10i

B. 1 − 2i

C. 1 + 2i

D. 2 + i

3. A cubic polynomial has factors (x + 1), (x − 2), and (x + 4), and a leading coefficient of 1. In standard form, the polynomial is

A. $x^3 + 3x^2 + 6x - 8$

B. $x^3 - 3x^2 - 6x - 8$

C. $x^3 + 3x^2 - 6x + 8$

D. $x^3 + 3x^2 - 6x - 8$

4. When the polynomial $2x^3 - x^2 + 5x + 4$ is divided by $(x - 1)$, the remainder is

A. 6

B. 10

C. 4

D. 12

5. When the equation $x^2 + 6x - 2 = 0$ is solved by completing the square, it can be rewritten as

A. $(x + 3)^2 = 11$

B. $(x + 3)^2 = -2$

C. $(x - 3)^2 = 11$

D. $(x + 6)^2 = 38$

6. What is the solution to the equation $4 / (x - 3) + 2 = 6 / (x - 3)$?

A. $x = 3$

B. no solution

C. $x = 4$

D. $x = 6$

7. The solution set of the equation $\sqrt{(x - 1)} + 3 = x$ is

A. {2}

B. {2, 5}

C. {5}

D. { }

8. For which value of c will the equation $x^2 - 6x + c = 0$ have a repeated real root?

A. $c = 4$

B. $c = 9$

C. $c = 6$

D. $c = 36$

9. A rectangle has a perimeter of 30 meters and an area of 50 square meters. Which system of equations can be used to find its length L and width W?

A. $2L + 2W = 30$; $LW = 50$

B. $L + W = 30$; $LW = 50$

C. $2L + 2W = 50$; $LW = 30$

D. $L \cdot W = 30$; $L + W = 50$

10. A football is kicked from ground level, and its height h (in feet) above the ground after t seconds is modeled by $h(t) = -16t^2 + 64t$. What is the maximum height the football reaches?

A. 32 ft

B. 48 ft

C. 56 ft

D. 64 ft

11. If $f(x) = 3x + 2$ and $g(x) = x^2 - 1$, then the value of $f(g(-1))$ is

A. −1

B. 0

C. 2

D. 8

12. The sum of the first five terms of the geometric sequence 4, 12, 36, … is

A. 324

B. 484

C. 1,212

D. 240

13. The graph of $f(x) = x^2$ is shifted 3 units to the right and reflected across the x-axis to produce g(x). The equation for g(x) is

A. $g(x) = -(x - 3)^2$

B. $g(x) = (x - 3)^2 + 3$

C. $g(x) = -(x + 3)^2$

D. $g(x) = (x + 3)^2 - 3$

14. As $x \to \infty$, the function $f(x) = -2(0.7)^x + 5$ approaches the value

A. 0

B. −2

C. $-\infty$

D. 5

15. The equation $\log_5(125) = y$ is equivalent to which exponential equation?

A. $5^y = 125$

B. $y^5 = 125$

C. $125^5 = y$

D. $y = 5^{125}$

16. What is the period of the function $y = 4 \cos((\pi/2)x) - 3$?

A. 2

B. 4

C. π

D. 2π

17. The exact value of $\csc(7\pi/6)$ is

A. 2

B. 1/2

C. −2

D. −1/2

18. Solve algebraically: $\ln(x + 5) - \ln(x - 1) = \ln(3)$.

A. $x = 1$

B. $x = 8$

C. $x = 5$

D. $x = 4$

19. A sequence is defined recursively by $a_1 = 5$ and $a_n = a_{n-1} + 3$ for $n \geq 2$. Which explicit formula represents this sequence?

A. $a_n = 3n + 2$

B. $a_n = 5n - 3$

C. $a_n = 3n - 2$

D. $a_n = 5n + 3$

20. The maximum value of the function $y = 6 \sin(2x - \pi/4) + 1$ is

A. 6

B. 1

C. 7

D. 8

21. Which statement about the standard deviation of a data set is true?

A. A larger standard deviation always indicates a larger mean

B. Standard deviation cannot be negative or equal to zero

C. Standard deviation is measured in the squared units of the data

D. A standard deviation of 0 indicates all values in the set are identical

22. In a survey, 240 of 400 randomly selected voters indicated support for a candidate. The sample proportion $\hat{p}$ of supporters is

A. 60%

B. 24%

C. 40%

D. 16.67%

23. In a class of 30 students, 18 students are in band and 12 students are in chorus. If 8 students are in both band and chorus, what is the probability that a randomly selected student is in chorus given that they are in band?

A. 8/30

B. 12/30

C. 8/12

D. 8/18

24. A standard deck of 52 playing cards contains 4 aces. Two cards are drawn at random without replacement. What is the probability that both cards drawn are aces?

A. 1/169

B. 1/221

C. 8/52

D. 16/52

PART II — Short Constructed Response (8 questions × 2 credits = 16 credits)

Show all work. A correct answer with no supporting work will receive only 1 credit.

25. Solve the equation $2x^2 - 6x + 5 = 0$ algebraically. Express the solutions in a + bi form.

26. Simplify the rational expression $(x^2 + 5x + 6) / (x^2 - 4)$ for $x \neq \pm 2$. Show all factoring steps.

27. Use the Remainder Theorem to determine whether $(x + 2)$ is a factor of $x^4 - 3x^2 - 2x - 8$. State your conclusion and justify it with the value of the polynomial at the appropriate x.

28. Solve algebraically: $2^{(3x - 1)} = 16^{(x + 1)}$. Show all algebraic steps using a common base.

29. A geometric sequence has first term $a_1 = 64$ and fourth term $a_4 = -8$. Algebraically determine the common ratio r. Show all work.

30. Using the Pythagorean identity, find the exact value of $\sin \theta$ given that $\cos \theta = 3/5$ and θ is in Quadrant IV. Show all algebraic work.

31. Solve the equation $2 \cos(x) + 1 = 0$ for all values of x in the interval $[0, 2\pi)$. Show how you determined each solution.

32. A standard six-sided die is rolled three times. Determine the probability that the number 4 appears at least once during the three rolls. Express your answer as a fraction in simplest form.

PART III — Extended Constructed Response (3 questions × 4 credits = 12 credits)

Show all work. Partial credit is awarded according to the scoring rubric.

33. A bank offers a savings account paying 4% annual interest compounded quarterly.

(a) Write a function A(t) for the value of an initial investment of $1,500 after t years.

(b) Algebraically determine the number of years required for the investment to double in value. Round your answer to the nearest tenth of a year.

34. The polynomial function $f(x) = x^3 - 7x + 6$ has $x = 1$ as a known real root.

(a) Use synthetic division to find the quotient when $f(x)$ is divided by $(x - 1)$. Show your synthetic division work.

(b) Factor $f(x)$ completely over the integers.

(c) State all real solutions to the equation $f(x) = 0$.

35. A survey of 600 randomly selected adults found that 252 preferred streaming services over traditional television.

(a) Determine the sample proportion $\hat{p}$ of adults who prefer streaming services.

(b) Construct a 95% confidence interval for the true population proportion who prefer streaming services. Use the margin of error formula $ME = 1.96 \cdot \sqrt{\hat{p}(1 - \hat{p})/n}$. Round each bound to three decimal places.

(c) A streaming company claims that 50% of adults prefer streaming services. Does your confidence interval support this claim? Justify your answer using the interval.

PART IV — Long Constructed Response (1 question × 6 credits = 6 credits)

Show all work. This problem integrates concepts from several chapters.

36. A pendulum's height above its lowest point, h (in centimeters), at time t (in seconds) after release is modeled by $h(t) = 6 - 6\cos((\pi/2)t)$.

(a) Determine the maximum height of the pendulum, the minimum height of the pendulum, and the period of motion. Show how each value is derived from the function.

(b) Determine the height of the pendulum at $t = 1.5$ seconds. Round your answer to the nearest tenth of a centimeter.

(c) Algebraically determine all values of t in the interval [0, 4] at which the pendulum reaches a height of exactly 8 centimeters. Round each value of t to the nearest hundredth of a second. Show all algebraic work.

PRACTICE EXAM 8 – ANSWER KEY AND EXPLANATIONS

1. A — Apply the rational exponent rule: $(8x^6)^{(2/3)} = 8^{(2/3)} \cdot x^{(6 \cdot 2/3)} = (8^{(1/3)})^2 \cdot x^4 = 2^2 \cdot x^4 = 4x^4$. The cube root of 8 is 2, and squaring it gives 4; the exponent on x is multiplied through.

2. C — Multiply numerator and denominator by the conjugate $(2 + i)$: the numerator becomes $(4 + 3i)(2 + i) = 8 + 4i + 6i + 3i^2 = 5 + 10i$, and the denominator becomes $(2 - i)(2 + i) = 4 + 1 = 5$. The result $(5 + 10i)/5$ simplifies to $1 + 2i$ in standard form.

3. D — Multiply factor pairs systematically: $(x + 1)(x - 2) = x^2 - x - 2$, then $(x^2 - x - 2)(x + 4) = x^3 + 4x^2 - x^2 - 4x - 2x - 8 = x^3 + 3x^2 - 6x - 8$. Each step combines like terms carefully.

4. B — By the Remainder Theorem, evaluate the polynomial at $x = 1$: $p(1) = 2(1)^3 - (1)^2 + 5(1) + 4 = 2 - 1 + 5 + 4 = 10$. The remainder of division by $(x - 1)$ equals $p(1)$.

5. A — Move the constant: $x^2 + 6x = 2$. Add $(6/2)^2 = 9$ to both sides: $x^2 + 6x + 9 = 11$. The left side is a perfect square: $(x + 3)^2 = 11$.

6. C — Restriction: $x \neq 3$. Multiply both sides by $(x - 3)$: $4 + 2(x - 3) = 6$, so $4 + 2x - 6 = 6$, giving $2x = 8$ and $x = 4$. Since $4 \neq 3$, the solution is valid; verification: $4/1 + 2 = 6$ ✓.

7. C — Isolate the radical: $\sqrt{(x - 1)} = x - 3$. Square both sides: $x - 1 = x^2 - 6x + 9$, giving $x^2 - 7x + 10 = 0$, which factors as $(x - 2)(x - 5) = 0$. Check $x = 2$: $\sqrt{1} + 3 = 4 \neq 2$ (extraneous); check $x = 5$: $\sqrt{4} + 3 = 5$ ✓. Solution set: {5}.

8. B — A repeated real root occurs when the discriminant equals zero: $b^2 - 4ac = 36 - 4c = 0$, so $c = 9$. At $c = 9$, the equation becomes $(x - 3)^2 = 0$ with the double root $x = 3$.

9. A — The perimeter of a rectangle is $P = 2L + 2W$ (not $L + W$, which is half the perimeter). The area is LW. With $P = 30$ and area 50, the system is $2L + 2W = 30$ and $LW = 50$.

10. D — The vertex of a parabola occurs at $t = -b/(2a) = -64/(2 \cdot -16) = 2$. Substitute: $h(2) = -16(4) + 64(2) = -64 + 128 = 64$ ft. This is the maximum since the leading coefficient is negative (parabola opens downward).

11. C — Composition is inside-out: $g(-1) = (-1)^2 - 1 = 0$, then $f(0) = 3(0) + 2 = 2$. The output of g becomes the input of f.

12. B — Apply the geometric series formula $S_n = a_1(1 - r^n)/(1 - r)$ with $a_1 = 4$, $r = 3$ (each term ×3), and $n = 5$: $S_5 = 4(1 - 243)/(1 - 3) = 4(-242)/(-2) = 484$. Verification by summing: $4 + 12 + 36 + 108 + 324 = 484$ ✓.

13. A — Shifting $f(x) = x^2$ three units right produces $(x - 3)^2$; reflecting that across the x-axis negates the output, giving $g(x) = -(x - 3)^2$. Horizontal shifts subtract inside the function; reflections across the x-axis place a negative outside.

14. D — As $x \to \infty$, $(0.7)^x \to 0$ because the base is between 0 and 1 (exponential decay). The function approaches $-2(0) + 5 = 5$, which is the horizontal asymptote.

15. A — A logarithm is the inverse of an exponential: $\log_b(N) = y$ is equivalent to $b^y = N$. Applying this with $b = 5$ and $N = 125$ gives $5^y = 125$, and the solution is $y = 3$.

16. B — For $y = A \cos(Bx) + D$, the period is $2\pi/B$. With $B = \pi/2$: period $= 2\pi/(\pi/2) = 2\pi \cdot 2/\pi = 4$. The vertical shift −3 and amplitude 4 do not affect the period.

17. C — The angle $7\pi/6$ lies in Quadrant III with reference angle $\pi/6$. Sine is negative in Quadrant III: $\sin(7\pi/6) = -\sin(\pi/6) = -1/2$. The cosecant is the reciprocal: $\csc(7\pi/6) = 1/(-1/2) = -2$.

18. D — Apply the quotient property: $\ln((x + 5)/(x - 1)) = \ln(3)$. Since the logarithms are equal, the arguments are equal: $(x + 5)/(x - 1) = 3$, so $x + 5 = 3x - 3$, giving $2x = 8$ and $x = 4$. Both arguments remain positive at $x = 4$, so the solution is valid.

19. A — Arithmetic sequence with $a_1 = 5$ and common difference $d = 3$: $a_n = a_1 + (n - 1)d = 5 + 3(n - 1) = 3n + 2$. Verification: $a_1 = 3(1) + 2 = 5$ ✓, $a_2 = 3(2) + 2 = 8 = 5 + 3$ ✓.

20. C — For $y = A \sin(Bx + C) + D$, the maximum equals midline + amplitude $= D + |A|$. Here $D = 1$ and $|A| = 6$, so max $= 1 + 6 = 7$. The phase shift and B do not affect maximum or minimum values.

21. D — A standard deviation of 0 means there is no spread — every value equals the mean, so all values are identical. Option A confuses spread with center; option B is false because $\sigma = 0$ is possible; option C describes variance, not standard deviation, which is in original units.

22. A — Sample proportion $\hat{p}$ = number of successes / sample size = 240/400 = 0.60 = 60%. The proportion is computed from the count of those who support divided by the total sampled.

23. D — Conditional probability restricts to the condition's sample space. Given the student is in band (18 students), the probability of also being in chorus is 8/18 (those in both, out of the band members). Without simplification, 8/18 directly matches the option.

24. B — Without replacement, multiply sequential probabilities: P(1st ace) = 4/52, and P(2nd ace | 1st ace) = 3/51 (one ace and one card removed). Product: (4/52)(3/51) = 12/2,652 = 1/221.

PART II — Short Constructed-Response Explanations

25. [2 credits] Apply the quadratic formula with a = 2, b = −6, c = 5: $x = [6 \pm \sqrt{36 - 40}]/4 = [6 \pm \sqrt{-4}]/4 = [6 \pm 2i]/4 = (3 \pm i)/2$. Solutions in a + bi form: **x = 3/2 + (1/2)i and x = 3/2 − (1/2)i**.

26. [2 credits] Factor: $x^2 + 5x + 6 = (x + 2)(x + 3)$, and $x^2 - 4 = (x - 2)(x + 2)$. The common factor (x + 2) cancels, leaving **(x + 3)/(x − 2)** for x ≠ ±2.

27. [2 credits] By the Factor Theorem, (x + 2) is a factor if and only if p(−2) = 0. Compute $p(-2) = (-2)^4 - 3(-2)^2 - 2(-2) - 8 = 16 - 12 + 4 - 8 =$ **0**. Since the remainder is 0, **(x + 2) IS a factor** of $x^4 - 3x^2 - 2x - 8$.

28. [2 credits] Rewrite 16 as 2^4: $2^{(3x - 1)} = (2^4)^{(x + 1)} = 2^{(4x + 4)}$. Equate exponents: 3x − 1 = 4x + 4, so −x = 5 and **x = −5**.

29. [2 credits] Apply $a_n = a_1 \cdot r^{(n - 1)}$ with $a_4 = a_1 \cdot r^3$: $-8 = 64r^3$, so $r^3 = -1/8$ and **r = −1/2**. The cube root of −1/8 is taken since cube roots of negatives are uniquely real.

30. [2 credits] Apply the Pythagorean identity: $\sin^2\theta = 1 - \cos^2\theta = 1 - 9/25 = 16/25$, so $|\sin \theta| = 4/5$. In Quadrant IV sine is negative, so **sin θ = −4/5**. Verification: $(-4/5)^2 + (3/5)^2 = 16/25 + 9/25 = 1$ ✓.

31. [2 credits] Solve cos(x) = −1/2. Reference angle is π/3 (since cos(π/3) = 1/2). Cosine is negative in Quadrants II and III. Quadrant II: x = π − π/3 = **2π/3**. Quadrant III: x = π + π/3 = **4π/3**. Both lie within [0, 2π).

32. [2 credits] Use the complement rule: P(at least one 4) = 1 − P(no 4 in 3 rolls). For each roll, P(not 4) = 5/6, so P(no 4 in 3 rolls) = $(5/6)^3$ = 125/216. Therefore P(at least one 4) = 1 − 125/216 = **91/216**.

PART III — Extended Constructed-Response Explanations

33.

(a) [1 credit] Compound interest formula: $A(t) = P(1 + r/n)^{(nt)}$ with P = 1500, r = 0.04, n = 4: **A(t) = 1,500(1.01)^(4t)**.

(b) [3 credits] Set A(t) = 3,000 (double the initial): $1{,}500(1.01)^{(4t)} = 3{,}000$, so $(1.01)^{(4t)} = 2$. Take the natural log: 4t · ln(1.01) = ln(2), giving 4t = ln(2)/ln(1.01) ≈ 0.6931/0.00995 ≈ 69.66, so t ≈ 69.66/4 ≈ **17.4 years**.

34.

(a) [2 credits] Apply synthetic division with k = 1 on coefficients 1, 0, −7, 6:

```
1 |  1   0   -7    6
  |      1    1   -6
  |___________________________
     1   1   -6    0
```

The quotient is $\mathbf{x^2 + x - 6}$ with remainder 0, confirming x = 1 as a root.

(b) [1 credit] Factor the quadratic: $x^2 + x - 6 = (x + 3)(x - 2)$, so $f(x) = \mathbf{(x - 1)(x + 3)(x - 2)}$.

(c) [1 credit] Setting each factor to zero: **x = 1, x = −3, and x = 2**.

35.

(a) [1 credit] Sample proportion: $\hat{p} = 252/600 =$ **0.42** (or 42%).

(b) [2 credits] Compute $ME = 1.96 \cdot \sqrt{(0.42 \cdot 0.58/600)} = 1.96 \cdot \sqrt{(0.2436/600)} = 1.96 \cdot \sqrt{0.000406} \approx 1.96 \cdot 0.02015 \approx 0.0395$. Confidence interval: 0.42 ± 0.0395, giving lower bound 0.42 − 0.0395 = **0.381** and upper bound 0.42 + 0.0395 = **0.459**. CI ≈ **(0.381, 0.459)**.

(c) [1 credit] The interval (0.381, 0.459) does **NOT contain 0.50**, so the data do not support the streaming company's claim that 50% of adults prefer streaming. Since the entire 95% interval lies well below 0.50, the sample evidence suggests the true proportion is significantly lower than 50%.

PART IV — Long Constructed-Response Explanation

36.

(a) [2 credits] **Maximum height = 12 cm**: this occurs when $\cos((\pi/2)t) = -1$ (which makes −6 cos = +6, giving h = 6 + 6 = 12). **Minimum height = 0 cm**: this occurs when $\cos((\pi/2)t) = 1$ (which makes −6 cos = −6, giving h = 6 − 6 = 0). **Period = 4 seconds**: from the formula period = $2\pi/B$ with $B = \pi/2$, giving $2\pi/(\pi/2) = 4$. The pendulum completes one full swing every 4 seconds.

(b) [1 credit] Substitute t = 1.5: $h(1.5) = 6 - 6\cos((\pi/2)(1.5)) = 6 - 6\cos(3\pi/4)$. Since $\cos(3\pi/4) = -\sqrt{2}/2 \approx -0.7071$: $h(1.5) = 6 - 6(-0.7071) \approx 6 + 4.243 \approx$ **10.2 cm**.

(c) [3 credits] Set h(t) = 8: $6 - 6\cos((\pi/2)t) = 8$, so $-6\cos((\pi/2)t) = 2$ and $\cos((\pi/2)t) = -1/3$. Let $u = (\pi/2)t$; as t ranges over [0, 4], u ranges over $[0, 2\pi]$. The equation $\cos(u) = -1/3$ has solutions $u_1 = \arccos(-1/3) \approx 1.9106$ rad and $u_2 = 2\pi - 1.9106 \approx 4.3726$ rad in $[0, 2\pi]$. Convert back: $t = (2/\pi)u$. $t_1 \approx (2/\pi)(1.9106) \approx$ **1.22 seconds**, and $t_2 \approx (2/\pi)(4.3726) \approx$ **2.78 seconds**. Both values lie within [0, 4]. The pendulum reaches a height of exactly 8 cm twice during the first full period, symmetrically about t = 2 (the time of maximum height).