PRACTICE EXAM 7: ALGEBRA II REGENTS SIMULATION

PART I — Multiple Choice (24 questions × 2 credits = 48 credits)

1. The expression $i^{17} - i^{10}$ simplifies to

A. $-1 - i$

B. $-1 + i$

C. $1 - i$

D. $1 + i$

2. The complex conjugate of $-3 + 5i$ is

A. $-3 - 5i$

B. $3 + 5i$

C. $3 - 5i$

D. $-5 + 3i$

3. A polynomial function f(x) of degree 4 has a positive leading coefficient and zeros at $x = -3$ (multiplicity 1), $x = 0$ (multiplicity 2), and $x = 2$ (multiplicity 1). Which expression could represent f(x)?

A. $f(x) = x(x + 3)(x - 2)$

B. $f(x) = (x + 3)^2(x - 2)$

C. $f(x) = x^2(x + 3)(x - 2)$

D. $f(x) = x(x + 3)^2(x - 2)$

4. When the polynomial $p(x) = x^4 + 2x^3 - 5x + 3$ is divided by $(x + 1)$, the remainder is

A. 0

B. 7

C. 1

D. −7

5. The equation $x^2 + 2x + 5 = 0$ has how many real solutions?

A. 0

B. 1

C. 2

D. 4

6. What is the solution to the equation $x / (x - 4) = 4 / (x - 4) + 3$?

A. no solution

B. $x = 4$

C. $x = -4$

D. $x = 0$

7. The solution set of the equation $\sqrt{x + 3} = x - 3$ is

A. {1}

B. {1, 6}

C. {6}

D. { }

8. For what value of k does the system $y = x^2 + 4x + k$ and $y = 4$ have exactly one real solution?

A. $k = 4$

B. $k = 0$

C. $k = -4$

D. $k = 8$

9. A culture of yeast cells triples every 4 hours. If the initial culture has 80 cells, which function models the cell count P(t) after t hours?

A. $P(t) = 80 \cdot 3^{4t}$

B. $P(t) = 80 \cdot 3^{t/4}$

C. $P(t) = 80 \cdot 3^t$

D. $P(t) = 80 \cdot 4^{t/3}$

10. The expression $8x^3 - 27$ factored completely is

A. $(2x + 3)(4x^2 - 6x + 9)$

B. $(2x - 3)(2x + 3)^2$

C. $(2x - 3)(4x^2 + 6x + 9)$

D. $(2x - 3)(4x^2 - 6x + 9)$

11. The domain of the function $f(x) = \log_2(x - 4)$ is

A. $x > 4$

B. $x \geq 4$

C. $x < 4$

D. all real numbers

12. The 4th term of a geometric sequence is 54, and the common ratio is 3. What is the first term?

A. 18

B. 2

C. 6

D. 9

13. If $f(x) = 2x - 1$ and $g(x) = x^2 + 3$, then $g(f(2))$ is equal to

A. 7

B. 49

C. 5

D. 12

14. The function $f(x) = (x - 5)^2 - 2$, defined for $x \geq 5$, has an inverse $f^{-1}(x)$ equal to

A. $(x + 2)^2 - 5$

B. $5 - \sqrt{(x + 2)}$

C. $5 + \sqrt{(x + 2)}$

D. $\sqrt{(x + 2)} - 5$

15. The exact value of $\cos(5\pi/6)$ is

A. $\sqrt{3} / 2$

B. $-\sqrt{3} / 2$

C. $1 / 2$

D. $-1 / 2$

16. What is the solution to the equation $\log_3(2x + 1) = \log_3(x + 7)$?

A. $x = 7$

B. $x = 6$

C. $x = 1$

D. $x = 8$

17. The function $T(d) = 15 \sin((2\pi/365)(d - 80)) + 60$ models the average daily temperature in degrees Fahrenheit on day d of the year. According to this model, what is the average annual temperature?

A. 15°F

B. 80°F

C. 75°F

D. 60°F

18. The expression $\log((x^2y) / z)$, expanded as a sum or difference of logarithms, is

A. $2 \log(x) + \log(y) - \log(z)$

B. $2 \log(x) - \log(y) + \log(z)$

C. $2(\log(x) + \log(y) - \log(z))$

D. $\log(2x + y - z)$

19. $1,000 is invested at 6% interest compounded annually. Which equation correctly solves for the number of years t needed for the investment to double?

A. $t = \log(2000 / 1000)$

B. $t = \log(2) / 0.06$

C. $t = \log(2) / \log(1.06)$

D. t = log(0.06) / log(2)

20. If sin θ = 4/5 and θ is in Quadrant II, then the value of sec θ is

A. 5/3

B. 3/5

C. −3/5

D. −5/3

21. A normally distributed data set has a mean of 100 and a standard deviation of 15. Approximately what percent of the data falls between 70 and 130?

A. 68%

B. 50%

C. 95%

D. 99.7%

22. A company wants to survey customers about their satisfaction with a recent product launch. Which sampling method is most likely to produce biased results?

A. Sending an email survey only to customers who recently filed complaints

B. Randomly selecting customers from the entire customer database

C. Surveying every 10th customer who visited the store last month

D. Using a random number generator to select 200 customer accounts

23. A committee of 4 students is to be selected from a group of 10 students. In how many distinct ways can this committee be formed, assuming the order of selection does not matter?

A. 40

B. 5,040

C. 24

D. 210

24. Events A and B are independent, with $P(A) = 0.7$ and $P(A \cap B) = 0.21$. What is $P(B)$?

A. 0.21

B. 0.3

C. 0.7

D. 0.49

PART II — Short Constructed Response (8 questions × 2 credits = 16 credits)

Show all work. A correct answer with no supporting work will receive only 1 credit.

25. Solve the equation $x^2 + 8x + 25 = 0$ algebraically. Express the solutions in $a + bi$ form.

26. Solve algebraically and check for extraneous solutions: $(2x) / (x - 1) = 3 / (x - 1) + 4$. State any restrictions on x.

27. Use synthetic division to determine whether $(x - 3)$ is a factor of $x^3 - 5x^2 + 8x - 6$. State your conclusion and identify the quotient.

28. Solve algebraically for x: 9^(x + 1) = 27^(2x − 1). Show all steps using a common base.

29. A sequence is defined recursively by $a_1 = 100$ and $a_n = a_{n-1} - 7$ for $n \geq 2$. Write an explicit formula for a_n, and use it to find the value of a_{12}.

30. Find the average rate of change of the function $f(x) = 2^x$ on the interval [1, 4]. Express your answer as a fraction in simplest form.

31. Solve the equation $\sin(x) = -1/2$ for all values of x in the interval $[0, 2\pi)$. Show how you determined each solution.

32. Bag X contains 4 white marbles and 6 black marbles. Bag Y contains 3 white marbles and 5 black marbles. One marble is drawn at random from each bag. Determine the probability that both marbles drawn are white. Express your answer as a fraction in simplest form.

PART III — Extended Constructed Response (3 questions × 4 credits = 12 credits)

Show all work. Partial credit is awarded according to the scoring rubric.

33. A radioactive isotope decays according to the formula $M(t) = M_0 \cdot$ e^(−kt), where M_0 is the initial mass, M(t) is the mass remaining after t years, and k is the decay constant for the isotope. The isotope has a half-life of 25 years.

(a) Algebraically determine the decay constant k. Round your answer to four decimal places.

(b) If the initial mass is 200 grams, algebraically determine the time at which 50 grams of the isotope remains. Round your answer to the nearest tenth of a year.

34. The polynomial function $p(x) = x^3 - 4x^2 - 11x + 30$ has $x = 2$ as one of its real roots.

(a) Use synthetic division to find the quotient when $p(x)$ is divided by $(x - 2)$. Show your synthetic division work.

(b) Factor $p(x)$ completely over the integers.

(c) State all real solutions to the equation $p(x) = 0$.

35. A survey of 500 randomly selected high school students found that 280 of them preferred online classes over in-person classes.

(a) Determine the sample proportion $\hat{p}$ of students who prefer online classes.

(b) Construct a 95% confidence interval for the true population proportion of students who prefer online classes. Use the margin of error formula $ME = 1.96 \cdot \sqrt{(\hat{p}(1 - \hat{p})/n)}$. Round bounds to the nearest thousandth.

(c) Interpret the confidence interval in the context of the problem.

PART IV — Long Constructed Response (1 question × 6 credits = 6 credits)

Show all work. This problem integrates concepts from several chapters.

36. The depth d (in meters) of water at a coastal pier is modeled by $d(t) = 2.5 \cos((\pi/6)\, t) + 4.5$, where t is the number of hours after midnight on a particular day.

(a) Determine the maximum and minimum depth of the water and the times in the 24-hour period ($0 \le t \le 24$) at which each occurs. Justify your reasoning.

(b) Determine the depth of the water at $t = 9$ hours after midnight (i.e., 9:00 AM). Round your answer to the nearest hundredth of a meter.

(c) Algebraically determine all values of t in the interval [0, 24] at which the water depth equals exactly 5.5 meters. Round each value to the nearest tenth of an hour. Show all algebraic work.

PRACTICE EXAM 7 – ANSWER KEY AND EXPLANATIONS

1. D — Powers of i cycle through $\{i, -1, -i, 1\}$ with period 4. Since $17 = 4(4) + 1$, $i^{17} = i$; since $10 = 4(2) + 2$, $i^{10} = i^2 = -1$. Therefore $i^{17} - i^{10} = i - (-1) = 1 + i$ in standard $a + bi$ form.

2. A — The complex conjugate of $a + bi$ is $a - bi$, formed by negating only the imaginary part. For $-3 + 5i$, the conjugate is $-3 - 5i$. Complex conjugates are used to rationalize denominators and to find magnitudes.

3. C — Each zero contributes a factor (x − zero) raised to its multiplicity. Zeros at $x = -3, 0, 2$ with multiplicities 1, 2, 1 produce factors $(x + 3)$, x^2, and $(x - 2)$. The product $x^2(x + 3)(x - 2)$ is degree 4 with positive leading coefficient when expanded.

4. B — Apply the Remainder Theorem: dividing by $(x + 1)$ means evaluating at $x = -1$. Compute $p(-1) = (-1)^4 + 2(-1)^3 - 5(-1) + 3 = 1 - 2 + 5 + 3 = 7$. The polynomial has no x^2 term, but the formula still applies.

5. A — The discriminant is $b^2 - 4ac = 4 - 4(1)(5) = 4 - 20 = -16$, which is negative. A negative discriminant means no real solutions — the two roots are a complex conjugate pair.

6. A — Restriction: $x \neq 4$. Multiply both sides by $(x - 4)$: $x = 4 + 3(x - 4)$, giving $x = 3x - 8$ and $x = 4$. Since $x = 4$ violates the restriction (creating division by zero), it is extraneous. The equation has no solution.

7. C — Square both sides: $x + 3 = (x - 3)^2$, giving $x^2 - 7x + 6 = 0$ and roots $x = 1$ or $x = 6$. Check $x = 1$: $\sqrt{4} = 2$ but $1 - 3 = -2$ (extraneous since principal square root is non-negative). Check $x = 6$: $\sqrt{9} = 3 = 6 - 3$ ✓. Solution set: {6}.

8. D — Setting $y = 4$ yields $x^2 + 4x + (k - 4) = 0$. For exactly one real solution, the discriminant must equal zero: $16 - 4(k - 4) = 0$, giving $32 = 4k$ and $k = 8$. At $k = 8$, the line is tangent to the parabola.

9. B — Tripling every 4 hours means the exponent on 3 must equal 1 when $t = 4$. The form 3^(t/4) achieves this: at $t = 4$, $P = 80 \cdot 3^1 = 240$; at $t = 8$, $P = 80 \cdot 3^2 = 720$. The structure base^(t/period) is standard for tripling-time models.

10. C — Apply the difference of cubes identity $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$ with $a = 2x$ and $b = 3$: $(2x - 3)((2x)^2 + (2x)(3) + 3^2) = (2x - 3)(4x^2 + 6x + 9)$. The middle term of the quadratic factor takes a positive sign for differences of cubes.

11. A — The logarithm $\log_2(x - 4)$ is defined only when the argument is strictly positive: $x - 4 > 0$, so $x > 4$. Equality with 4 is excluded because log of zero is undefined.

12. B — Apply $a_n = a_1 \cdot r$^(n − 1) with $a_4 = 54$ and $r = 3$: $54 = a_1 \cdot 3^3 = 27\ a_1$, so $a_1 = 2$. The 4th term is the first term multiplied by the common ratio three times.

13. D — Composition is evaluated inside-out. First compute $f(2) = 2(2) - 1 = 3$, then $g(3) = 3^2 + 3 = 12$. The output of f becomes the input of g.

14. C — Swap x and y in $y = (x - 5)^2 - 2$ to get $x = (y - 5)^2 - 2$, so $y - 5 = \pm\sqrt{(x + 2)}$. The original domain $x \geq 5$ means $x - 5 \geq 0$, so the inverse's output must also satisfy $y - 5 \geq 0$; take the positive square root: $y = 5 + \sqrt{(x + 2)}$. This restriction is necessary because the unrestricted parabola is not one-to-one.

15. B — The angle $5\pi/6$ lies in Quadrant II with reference angle $\pi - 5\pi/6 = \pi/6$. Cosine is negative in Quadrant II, and $\cos(\pi/6) = \sqrt{3}/2$, so $\cos(5\pi/6) = -\sqrt{3}/2$.

16. B — When two logarithms with the same base are equal, their arguments must be equal: $2x + 1 = x + 7$, giving $x = 6$. Verification: both arguments equal 13, and $\log_3(13)$ is defined since $13 > 0$.

17. D — In $y = A \sin(Bx) + D$, the midline (vertical shift D) represents the average value. Here $D = 60$, so the average annual temperature is 60°F. The amplitude 15 represents the seasonal range above and below this average.

18. A — Apply log properties in order: $\log(x^2y/z) = \log(x^2y) - \log(z)$ by the quotient rule, then $\log(x^2y) = \log(x^2) + \log(y)$ by the product rule, and $\log(x^2) = 2 \log(x)$ by the power rule. Combine: $2 \log(x) + \log(y) - \log(z)$.

19. C — Set $2000 = 1000(1.06)^t$ to get $2 = (1.06)^t$. Take the logarithm of both sides: $\log(2) = t \cdot \log(1.06)$, so $t = \log(2)/\log(1.06)$. The change-of-base formula avoids needing log base 1.06 directly.

20. D — Apply the Pythagorean identity: $\cos^2\theta = 1 - \sin^2\theta = 1 - 16/25 = 9/25$, so $|\cos\theta| = 3/5$. In Quadrant II cosine is negative: $\cos\theta = -3/5$. Therefore $\sec\theta = 1/\cos\theta = -5/3$.

21. C — The range 70 to 130 corresponds to $100 \pm 2(15)$ = mean ± 2 standard deviations. By the Empirical Rule, approximately 95% of normally distributed data falls within 2 standard deviations of the mean.

22. A — Sampling only customers who filed complaints systematically excludes satisfied customers, producing a sample that is not representative of the full customer base. This is a clear case of selection bias. The other three methods use random or systematic procedures that capture a balanced view.

23. D — Since order of selection does not matter, use combinations: $C(10, 4) = 10!/(4! \cdot 6!) = (10 \cdot 9 \cdot 8 \cdot 7)/(4 \cdot 3 \cdot 2 \cdot 1) = 5{,}040/24 = 210$. Option B is the permutation count, which would apply only if order mattered.

24. B — For independent events, $P(A \cap B) = P(A) \cdot P(B)$. Substitute and solve: $0.21 = 0.7 \cdot P(B)$, giving $P(B) = 0.21/0.7 = 0.3$.

PART II — Short Constructed-Response Explanations

25. [2 credits] Apply the quadratic formula with $a = 1$, $b = 8$, $c = 25$: $x = [-8 \pm \sqrt{(64 - 100)}]/2 = [-8 \pm \sqrt{(-36)}]/2 = [-8 \pm 6i]/2 = \mathbf{-4 \pm 3i}$. Solutions in $a + bi$ form: $x = -4 + 3i$ and $x = -4 - 3i$.

26. [2 credits] Restriction: $\mathbf{x \neq 1}$. Multiply both sides by $(x - 1)$: $2x = 3 + 4(x - 1)$, so $2x = 4x - 1$, giving $2x = 1$ and $\mathbf{x = 1/2}$. Verification: $2(1/2)/(1/2 - 1) = 1/(-1/2) = -2$, and $3/(-1/2) + 4 = -6 + 4 = -2$ ✓. Since $1/2 \neq 1$, **no extraneous solutions arise**.

27. [2 credits] Apply synthetic division with $k = 3$ on coefficients $1, -5, 8, -6$:

```
3 |  1   -5    8   -6
  |        3   -6    6
  |________________________________
     1   -2    2    0
```

The remainder is **0**, so **(x − 3) IS a factor** of $x^3 - 5x^2 + 8x - 6$. The quotient is $\mathbf{x^2 - 2x + 2}$.

28. [2 credits] Rewrite both sides with base 3: $9 = 3^2$ and $27 = 3^3$, so $(3^2)^{(x + 1)} = (3^3)^{(2x - 1)}$, giving $3^{(2x + 2)} = 3^{(6x - 3)}$. Equate exponents: $2x + 2 = 6x - 3$, so $4x = 5$ and $\mathbf{x = 5/4}$.

29. [2 credits] The sequence is arithmetic with first term $a_1 = 100$ and common difference $d = -7$. The explicit formula is $a_n = a_1 + (n - 1)d = 100 - 7(n - 1) = \mathbf{107 - 7n}$. Apply for $n = 12$: $\mathbf{a_{12} = 107 - 84 = 23}$.

30. [2 credits] Compute $f(1) = 2^1 = 2$ and $f(4) = 2^4 = 16$. Apply the average rate of change formula: $[f(4) - f(1)]/(4 - 1) = (16 - 2)/3 = \mathbf{14/3}$.

31. [2 credits] The reference angle for sin = 1/2 is $\pi/6$. Sine is negative in Quadrants III and IV. Quadrant III: $x = \pi + \pi/6 = \mathbf{7\pi/6}$. Quadrant IV: $x = 2\pi - \pi/6 = \mathbf{11\pi/6}$. Both values lie within $[0, 2\pi)$.

32. [2 credits] The draws from the two bags are independent. P(white from X) = 4/(4 + 6) = 4/10 = 2/5. P(white from Y) = 3/(3 + 5) = 3/8. Multiply: P(both white) = (2/5)(3/8) = 6/40 = **3/20**.

PART III — Extended Constructed-Response Explanations

33.

(a) [2 credits] At t = 25, the mass is half the initial: $M_0/2 = M_0 \cdot e^{-25k}$, so $1/2 = e^{-25k}$. Take the natural log: $-25k = \ln(1/2) = -\ln(2)$, giving $k = \ln(2)/25 \approx 0.6931/25 \approx$ **0.0277** (rounded to four decimal places).

(b) [2 credits] Set $200 e^{-0.0277t} = 50$, so $e^{-0.0277t} = 0.25 = 1/4$. Take the natural log: $-0.0277t = \ln(0.25) \approx -1.3863$, giving $t \approx 1.3863/0.0277 \approx$ **50.0 years**. This matches expectations: going from 200 g to 50 g requires two half-life periods (200 → 100 → 50), and 2 × 25 = 50 years.

34.

(a) [2 credits] Apply synthetic division with k = 2 on coefficients 1, −4, −11, 30:

```
2 |  1    -4   -11    30
  |        2    -4   -30
  |______________________________
     1    -2   -15     0
```

The quotient is $\mathbf{x^2 - 2x - 15}$ with remainder 0, confirming x = 2 as a root.

(b) [1 credit] Factor the quadratic: $x^2 - 2x - 15 = (x - 5)(x + 3)$, so $p(x) = \mathbf{(x - 2)(x - 5)(x + 3)}$.

(c) [1 credit] Setting each factor to zero: **x = 2, x = 5, and x = −3**.

35.

(a) [1 credit] Sample proportion: $\hat{p} = 280/500 =$ **0.56** (or 56%).

(b) [2 credits] Compute the margin of error: $ME = 1.96 \cdot \sqrt{0.56 \cdot 0.44/500} = 1.96 \cdot \sqrt{0.2464/500} = 1.96 \cdot \sqrt{0.0004928} \approx 1.96 \cdot 0.02220 \approx 0.04351$. The confidence interval is 0.56 ± 0.04351: lower bound ≈ 0.56 − 0.04351 ≈ **0.516**, upper bound ≈ 0.56 + 0.04351 ≈ **0.604**. CI ≈ **(0.516, 0.604)**.

(c) [1 credit] We are 95% confident that the true proportion of all high school students who prefer online classes lies between 0.516 (51.6%) and 0.604 (60.4%). This range expresses the uncertainty in estimating the population proportion from the sample of 500 students.

PART IV — Long Constructed-Response Explanation

36.

(a) [2 credits] **Maximum depth = 7 meters**, occurring when $\cos((\pi/6)t) = 1$, i.e., $(\pi/6)t = 0, 2\pi, 4\pi$, giving **t = 0, 12, and 24 hours** (midnight, noon, and the following midnight). **Minimum depth = 2 meters**, occurring when $\cos((\pi/6)t) = -1$, i.e., $(\pi/6)t = \pi, 3\pi$, giving **t = 6 and 18 hours** (6:00 AM and 6:00 PM). The wheel of the cosine produces this regular alternation across the 12-hour tidal cycle.

(b) [1 credit] Substitute $t = 9$: $d(9) = 2.5 \cos((\pi/6)(9)) + 4.5 = 2.5 \cos(3\pi/2) + 4.5$. Since $\cos(3\pi/2) = 0$: $d(9) = 2.5(0) + 4.5 =$ **4.50 meters**.

(c) [3 credits] Set $d(t) = 5.5$: $2.5 \cos((\pi/6)t) + 4.5 = 5.5$, so $2.5 \cos((\pi/6)t) = 1$ and $\cos((\pi/6)t) = 0.4$. Let $u = (\pi/6)t$. For t in [0, 24], u ranges over $[0, 4\pi]$. In $[0, 2\pi)$, $\cos(u) = 0.4$ at $u_1 = \arccos(0.4) \approx 1.1593$ and $u_2 = 2\pi - 1.1593 \approx 5.1239$. The full period 2π then repeats: $u_3 \approx 7.4424$ and $u_4 \approx 11.4071$. Convert each to t using $t = (6/\pi)u$: $t_1 \approx (6/\pi)(1.1593) \approx$ **2.2 hours**, $t_2 \approx (6/\pi)(5.1239) \approx$ **9.8 hours**, $t_3 \approx (6/\pi)(7.4424) \approx$ **14.2 hours**, $t_4 \approx (6/\pi)(11.4071) \approx$ **21.8 hours**. The water depth equals 5.5 m four times during the 24-hour period.