# PRACTICE EXAM 6: ALGEBRA II REGENTS SIMULATION

**PART I — Multiple Choice (24 questions × 2 credits = 48 credits)**

1. What is the quotient when $6x^3 + x^2 - 4x + 1$ is divided by $(2x - 1)$?

A. $3x^2 - 2x - 1$

B. $3x^2 + x - 1$

C. $3x^2 + 2x - 1$

D. $6x^2 + 2x - 1$

2. The expression $x^{(3/4)} \cdot x^{(1/2)}$ is equivalent to

A. $x^{(3/8)}$

B. $x^{(5/4)}$

C. $x^{(3/4)}$

D. $x^{(2/3)}$

3. A polynomial equation with real coefficients has solutions $x = 3$ and $x = -2i$. The minimum possible degree of this polynomial equation is

A. 2

B. 4

C. 5

D. 3

4. The exact solutions to the equation $x^2 + 2x - 4 = 0$ are

A. $x = -1 \pm \sqrt{5}$

B. $x = 1 \pm \sqrt{5}$

C. $x = -1 \pm 2\sqrt{5}$

D. $x = -2 \pm \sqrt{5}$

5. A pipe can fill a tank in 6 hours, and a second pipe can fill the same tank in 9 hours. If both pipes are used simultaneously, which equation can be used to find the time t needed to fill the tank?

A. $6 + 9 = t$

B. $t/6 + t/9 = 1/2$

C. $1/6 + 1/9 = 1/t$

D. $6t + 9t = 1$

6. What is the solution to the equation $2\sqrt{(x - 3)} = x - 6$?

A. $x = 12$

B. $x = 4$

C. $x = 4$ and $x = 12$

D. no real solution

7. How many real solutions does the system $y = x^2$ and $y = 4 - x^2$ have?

A. 0

B. 1

C. 4

D. 2

8. A medication leaves the bloodstream at a rate of 25% per hour. If the initial dose is 400 mg, which equation models the amount A remaining after t hours?

A. $A = 400(1.25)^t$

B. $A = 400(0.75)^t$

C. $A = 400 - 25t$

D. $A = 400(0.25)^t$

9. What is the domain of $f(x) = 1 / \sqrt{(x - 5)}$?

A. $x \geq 5$

B. $x \leq 5$

C. $x > 5$

D. all real numbers

10. A sequence is defined by $a_1 = 3$ and $a_n = 2a_{n-1} - 1$ for $n \geq 2$. What is the value of $a_4$?

A. 17

B. 11

C. 13

D. 33

11. The first four terms of a sequence are 4, 12, 36, 108. The explicit formula for the nth term is

A. $a_n = 4 + 3(n - 1)$

B. $a_n = 4n + 3$

C. $a_n = 4 \cdot 3^n$

D. $a_n = 4 \cdot 3$^$(n - 1)$

12. The graph of $g(x)$ is obtained from the graph of $f(x)$ by reflecting $f(x)$ over the y-axis. The relationship between $g(x)$ and $f(x)$ is

A. $g(x) = -f(x)$

B. $g(x) = f(x + 1)$

C. $g(x) = -f(-x)$

D. $g(x) = f(-x)$

13. A population grows from 8,000 to 12,500 over a period of 5 years according to an exponential model. The annual growth factor, rounded to four decimal places, is

A. 1.0500

B. 1.0934

C. 1.5625

D. 1.1250

14. Solve algebraically: $(\log x)^2 - 5 \log x + 4 = 0$.

A. $x = 10$ and $x = 10{,}000$

B. $x = 1$ and $x = 4$

C. $x = 10$ and $x = 4$

D. $x = 1$ and $x = 10{,}000$

15. If $\cos \theta = 5/13$ and $\theta$ is in Quadrant I, then the value of $\sec \theta$ is

A. 5/13

B. 12/13

C. 13/5

D. 13/12

16. The angle $5\pi/12$ radians is equivalent to which of the following in degrees?

A. 45°

B. 60°

C. 105°

D. 75°

17. For the function $y = -2 \sin(3x) + 4$, the amplitude is

A. 2

B. −2

C. 3

D. 4

18. The expression $(1 - \sin^2\theta) / \cos\theta$, where $\cos\theta \neq 0$, is equivalent to

A. $\sin\theta$

B. 1

C. $\cos\theta$

D. $\tan\theta$

19. The expression $3 \log_2(x) + \log_2(y) - \log_2(z)$, written as a single logarithm, is

A. $\log_2(3x + y - z)$

B. $\log_2(3xy / z)$

C. $\log_2(x^3 + y - z)$

D. $\log_2(x^3y / z)$

20. A poll reports that 52% of voters support a policy, with a margin of error of ±3% at the 95% confidence level. The 95% confidence interval for the true proportion of voters who support the policy is

A. (47%, 55%)

B. (49%, 55%)

C. (50%, 54%)

D. (49%, 52%)

21. Five students received test scores of 70, 75, 80, 85, and 100. A sixth student is added to the group with a score of 60. As a result of adding this score, the mean and median both decrease. Which best describes the change?

A. The mean decreases more than the median

B. The median decreases more than the mean

C. Both decrease by the same amount

D. Only the mean decreases

22. A data set has a mean of 50 and a standard deviation of 5. Which of the following data sets has a smaller standard deviation than this data set?

A. 45, 50, 50, 50, 55

B. 30, 40, 50, 60, 70

C. 49, 50, 50, 50, 51

D. 40, 45, 50, 55, 60

23. A bag contains 5 red marbles and 3 blue marbles. Two marbles are drawn at random without replacement. What is the probability that the second marble is blue, given that the first marble drawn was red?

A. 3/8

B. 3/7

C. 5/8

D. 1/2

24. A spinner is divided into 5 equal sectors numbered 1 through 5. The spinner is spun twice. What is the probability of getting a 3 on the first spin and an even number on the second spin?

A. 1/5

B. 4/25

C. 1/25

D. 2/25

**PART II — Short Constructed Response (8 questions × 2 credits = 16 credits)**

**Show all work. A correct answer with no supporting work will receive only 1 credit.**

25. Solve the equation $x^2 - 4x + 13 = 0$ algebraically. Express the solutions in $a + bi$ form.

26. Solve algebraically: $2 / (x - 3) = 3 / (x + 2)$. State any restrictions on x and identify whether any extraneous solutions arise.

27. A polynomial f(x) with a leading coefficient of 1 has factors $(x - 4)$, $(x + 2)$, and $(x - 1)$. Write f(x) as a polynomial expression in expanded standard form.

28. Solve the exponential equation 4^(2x) = 8^(x + 1) algebraically by rewriting both sides with a common base. Show all algebraic steps.

29. Use the Pythagorean identity to find the exact value of sin θ given that cos θ = −7/25 and θ is in Quadrant III. Show all algebraic work.

30. A sequence is defined by $a_1 = 6$ and $a_n = a_{n-1} + 2n$ for $n \geq 2$. Find the value of $a_5$. Show each step in the recursion.

31. Solve the equation $2 \tan(x) = 2$ for all values of x in the interval $[0, 2\pi)$. Show how you determined each solution.

32. A coin is biased so that the probability of heads on any given flip is 0.6. The coin is flipped three times. Determine the probability of getting exactly two heads. Express your answer as a fraction in simplest form.

**PART III — Extended Constructed Response (3 questions × 4 credits = 12 credits)**

**Show all work. Partial credit is awarded according to the scoring rubric.**

33. A coffee shop estimates that the temperature T (in degrees Fahrenheit) of a freshly brewed cup of coffee t minutes after pouring is modeled by $T(t) = 70 + 110\ e^{(-0.05t)}$.

(a) Determine the initial temperature of the coffee and the temperature it approaches over time. Justify your reasoning.

(b) Algebraically determine the time at which the coffee reaches a temperature of 100°F. Round your answer to the nearest tenth of a minute.

34. The polynomial function $p(x) = 2x^3 + x^2 - 8x - 4$ has $x = -1/2$ as a known root.

(a) Use synthetic division (or another method) to find the quotient when p(x) is divided by the linear factor $(2x + 1)$. Show your work.

(b) Factor p(x) completely over the integers.

(c) State all real solutions to the equation $p(x) = 0$.

35. A scientist tracks the population of a fish species in a lake using the logistic model $P(t) = 5000 / (1 + 4e^{(-0.3t)})$, where t is measured in years.

(a) Determine the initial population (at $t = 0$).

(b) Determine the long-term population the model approaches as $t \to \infty$. Justify your reasoning.

(c) Algebraically determine the time at which the population reaches 4,000. Round your answer to the nearest tenth of a year.

**PART IV — Long Constructed Response (1 question × 6 credits = 6 credits)**

**Show all work. This problem integrates concepts from several chapters.**

36. A graduate student is comparing two investment options:

- Fund A: An initial investment of $5,000 growing at an annual rate of 4%, compounded annually.
- Fund B: An initial investment of $3,000 growing at an annual rate of 7%, compounded annually.

(a) Write a function A(t) for the value of Fund A after t years, and a function B(t) for the value of Fund B after t years.

(b) Determine the value of each fund after 10 years. Round each value to the nearest dollar.

(c) Algebraically determine the number of years after which Fund B will first exceed Fund A in value. Round your answer to the nearest tenth of a year. Show all algebraic work using logarithms.

# PRACTICE EXAM 6 – ANSWER KEY AND EXPLANATIONS

**1. C** — Perform polynomial long division: $6x^3 \div 2x = 3x^2$, and $(2x - 1)(3x^2) = 6x^3 - 3x^2$, leaving remainder $4x^2 - 4x + 1$. Next, $4x^2 \div 2x = 2x$, and $(2x - 1)(2x) = 4x^2 - 2x$, leaving $-2x + 1$. Finally, $-2x \div 2x = -1$, and $(2x - 1)(-1) = -2x + 1$, leaving 0. The quotient is $3x^2 + 2x - 1$ with zero remainder.

**2. B** — When multiplying powers with the same base, add the exponents: $3/4 + 1/2 = 3/4 + 2/4 = 5/4$. Therefore $x^{3/4} \cdot x^{1/2} = x^{5/4}$.

**3. D** — Complex roots of polynomials with real coefficients always occur in conjugate pairs. Since $-2i$ is a root, $2i$ must also be a root, and together with $x = 3$, the polynomial has at least three roots. The minimum degree of such a polynomial is therefore 3.

**4. A** — Apply the quadratic formula with $a = 1$, $b = 2$, $c = -4$: $x = [-2 \pm \sqrt{4 + 16}]/2 = [-2 \pm \sqrt{20}]/2 = [-2 \pm 2\sqrt{5}]/2 = -1 \pm \sqrt{5}$. Simplification of $\sqrt{20}$ to $2\sqrt{5}$ is essential for matching the answer form.

**5. C** — In work-rate problems, individual rates add to give the combined rate when both work simultaneously: Pipe 1 rate = 1/6 tank/hr, Pipe 2 rate = 1/9 tank/hr, and the combined rate is 1/t tank/hr. The equation $1/6 + 1/9 = 1/t$ encodes this relationship.

**6. A** — Square both sides: $4(x - 3) = (x - 6)^2$, giving $x^2 - 16x + 48 = 0$, which factors as $(x - 12)(x - 4) = 0$. The candidate $x = 12$ satisfies $2\sqrt{9} = 6 = 12 - 6$ ✓, but $x = 4$ gives $2\sqrt{1} = 2$ while $4 - 6 = -2$, so $x = 4$ is extraneous (the principal square root cannot equal a negative value). The only solution is $x = 12$.

**7. D** — Substitute to eliminate y: $x^2 = 4 - x^2$, so $2x^2 = 4$ and $x^2 = 2$, giving $x = \pm\sqrt{2}$. Each x-value produces one solution, so the system has 2 real solutions corresponding to $(\sqrt{2}, 2)$ and $(-\sqrt{2}, 2)$.

**8. B** — A loss rate of 25% per hour means each hour 75% of the previous amount remains, so the decay factor is $1 - 0.25 = 0.75$. The exponential decay model $A = 400(0.75)^t$ multiplies the initial amount by this factor t times.

**9. C** — The expression has two restrictions: the radicand must be non-negative ($x - 5 \geq 0$) and the denominator must be nonzero ($\sqrt{x - 5} \neq 0$). The combined condition is $x - 5 > 0$, or $x > 5$. Equality with 5 would cause division by zero.

**10. A** — Apply the recursion step by step: $a_1 = 3$, $a_2 = 2(3) - 1 = 5$, $a_3 = 2(5) - 1 = 9$, $a_4 = 2(9) - 1 = 17$. Each term doubles the previous and subtracts 1.

**11. D** — The successive ratios $12/4 = 3$, $36/12 = 3$, $108/36 = 3$ confirm a geometric sequence with first term 4 and common ratio 3. The explicit formula for a geometric sequence is $a_n = a_1 \cdot r^{(n - 1)} = 4 \cdot 3^{(n - 1)}$. Verification: $a_4 = 4 \cdot 3^3 = 108$ ✓.

**12. D** — Reflecting a graph across the y-axis replaces every x with −x while leaving the function's outputs unchanged in sign. Therefore $g(x) = f(-x)$. Option A reflects across the x-axis, option B is a horizontal shift, and option C is a reflection through the origin.

**13. B** — Set up 12,500 = 8,000 · $r^5$, so $r^5$ = 12,500/8,000 = 1.5625. Take the fifth root: r = 1.5625^(1/5). Using logarithms: log(r) = log(1.5625)/5 ≈ 0.19382/5 ≈ 0.03876, so r ≈ 10^(0.03876) ≈ 1.0934.

**14. A** — Substitute u = log x to get $u^2 - 5u + 4 = 0$, which factors as (u − 1)(u − 4) = 0, giving u = 1 or u = 4. Back-substitute: log x = 1 means x = $10^1$ = 10, and log x = 4 means x = $10^4$ = 10,000. Both satisfy the domain x > 0.

**15. C** — The secant is the reciprocal of cosine: sec θ = 1/cos θ = 1/(5/13) = 13/5. Reciprocal trig identities convert between the primary and reciprocal trig functions directly.

**16. D** — Convert from radians to degrees by multiplying by 180/π: (5π/12)(180/π) = (5 · 180)/12 = 900/12 = 75°. The π factors cancel, leaving a simple arithmetic calculation.

**17. A** — For y = A sin(Bx) + D, the amplitude is |A|, regardless of sign. Here |−2| = 2, so the amplitude is 2. The negative sign reflects the curve across the midline but does not affect amplitude.

**18. C** — Apply the Pythagorean identity $\sin^2\theta + \cos^2\theta = 1$, which rearranges to $1 - \sin^2\theta = \cos^2\theta$. Substituting: $(1 - \sin^2\theta)/\cos\theta = \cos^2\theta/\cos\theta = \cos\theta$.

**19. D** — Apply log properties in order: 3 $\log_2(x) = \log_2(x^3)$. Then $\log_2(x^3) + \log_2(y) = \log_2(x^3y)$, and $\log_2(x^3y) - \log_2(z) = \log_2(x^3y / z)$. The power rule applies first, then the product and quotient rules combine the remaining logs.

**20. B** — The margin of error gives the half-width of the confidence interval: lower bound = 52% − 3% = 49%, upper bound = 52% + 3% = 55%. The interval is (49%, 55%).

**21. A** — Original: mean = 410/5 = 82 and median = 80. After adding 60: new mean = 470/6 ≈ 78.33 (decrease of about 3.67), and new median = (75 + 80)/2 = 77.5 (decrease of 2.5). The mean is more sensitive to extreme values than the median, so it decreased more.

**22. C** — Standard deviation measures spread around the mean. Set C has values 49, 50, 50, 50, 51 with mean 50 and deviations of magnitude at most 1, producing σ ≈ 0.63 — by far the smallest spread of the options and well below the original σ = 5. Sets B (range 40) and D (range 20) have larger spreads, so their standard deviations are larger than 5.

**23. B** — After the first marble drawn is red, 7 marbles remain in the bag: 4 red and 3 blue. The conditional probability P(2nd blue | 1st red) = 3/7. Without replacement reduces the total available and updates the conditional probability.

**24. D** — Independent events allow multiplying probabilities: P(3 on first) = 1/5 since 3 is one of five equally likely outcomes; P(even on second) = 2/5 since 2 and 4 are the even numbers in {1, 2, 3, 4, 5}. Multiply: (1/5)(2/5) = 2/25.

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# PART II — Short Constructed-Response Explanations

**25.** [2 credits] Apply the quadratic formula with $a = 1$, $b = -4$, $c = 13$: $x = [4 \pm \sqrt{(16 - 52)}]/2 = [4 \pm \sqrt{(-36)}]/2 = [4 \pm 6i]/2 = \mathbf{2 \pm 3i}$. Solutions in a + bi form: $x = 2 + 3i$ and $x = 2 - 3i$.

**26.** [2 credits] Restrictions: $\mathbf{x \neq 3}$ **and** $\mathbf{x \neq -2}$. Cross-multiply: $2(x + 2) = 3(x - 3)$, giving $2x + 4 = 3x - 9$, so $\mathbf{x = 13}$. Since 13 violates neither restriction, the solution is valid and **no extraneous solutions arise**.

**27.** [2 credits] Multiply two factors first: $(x - 4)(x + 2) = x^2 - 2x - 8$. Then multiply by the third factor: $(x^2 - 2x - 8)(x - 1) = x^3 - x^2 - 2x^2 + 2x - 8x + 8 = \mathbf{x^3 - 3x^2 - 6x + 8}$.

**28.** [2 credits] Rewrite both sides with base 2: $4 = 2^2$ and $8 = 2^3$, so $(2^2)^{(2x)} = (2^3)^{(x + 1)}$, giving $2^{(4x)} = 2^{(3x + 3)}$. Equate exponents: $4x = 3x + 3$, so $\mathbf{x = 3}$.

**29.** [2 credits] Apply the Pythagorean identity: $\sin^2\theta = 1 - \cos^2\theta = 1 - 49/625 = 576/625$, so $|\sin \theta| = 24/25$. In Quadrant III sine is negative, so $\mathbf{\sin \theta = -24/25}$. Verification: $(-24/25)^2 + (-7/25)^2 = 576/625 + 49/625 = 625/625 = 1$ ✓.

**30.** [2 credits] Apply the recursion: $a_1 = 6$, $a_2 = 6 + 2(2) = 10$, $a_3 = 10 + 2(3) = 16$, $a_4 = 16 + 2(4) = 24$, $\mathbf{a_5 = 24 + 2(5) = 34}$.

**31.** [2 credits] Divide both sides by 2: $\tan(x) = 1$. Tangent equals 1 at reference angle $\pi/4$, in Quadrants I and III where tangent is positive: $\mathbf{x = \pi/4}$ **and** $\mathbf{x = 5\pi/4}$ (since $5\pi/4 = \pi/4 + \pi$). Both values lie within $[0, 2\pi)$.

**32.** [2 credits] Apply the binomial probability formula $P(X = k) = C(n, k) \cdot p^k \cdot (1 - p)^{(n - k)}$ with $n = 3$, $k = 2$, $p = 0.6$: $P(X = 2) = C(3, 2) \cdot (0.6)^2 \cdot (0.4)^1 = 3 \cdot 0.36 \cdot 0.4 = 0.432$. As a fraction: $3 \cdot (36/100) \cdot (4/10) = 432/1000 = \mathbf{54/125}$ (in simplest form, since $432 \div 8 = 54$ and $1000 \div 8 = 125$).

# PART III — Extended Constructed-Response Explanations

**33.**

**(a)** [2 credits] At $t = 0$: $T(0) = 70 + 110e^0 = 70 + 110 = \mathbf{180°F}$ (the brewing temperature). As $t \to \infty$, $e^{(-0.05t)} \to 0$, so $T(t) \to \mathbf{70°F}$, representing the ambient room temperature the coffee approaches but never reaches as it cools.

**(b)** [2 credits] Set $T(t) = 100$: $70 + 110e^{(-0.05t)} = 100$, so $110e^{(-0.05t)} = 30$ and $e^{(-0.05t)} = 3/11$. Take the natural log: $-0.05t = \ln(3/11) \approx -1.2993$, giving $t \approx -1.2993/(-0.05) \approx$ **26.0 minutes**.

**34.**

**(a)** [2 credits] Use synthetic division with $x = -1/2$ (the root of $2x + 1 = 0$):

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-1/2 |  2   1  -8  -4
     |     -1   0   4
     |__________________________
        2   0  -8   0
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Dividing by $(x + 1/2)$ gives quotient $2x^2 - 8$. Since $(2x + 1) = 2(x + 1/2)$, divide the quotient by 2 to get the quotient when $p(x)$ is divided by $(2x + 1)$: $\mathbf{x^2 - 4}$.

**(b)** [1 credit] $x^2 - 4 = (x - 2)(x + 2)$, so $p(x) = \mathbf{(2x + 1)(x - 2)(x + 2)}$.

**(c)** [1 credit] Setting each factor equal to zero: **$x = -1/2$, $x = 2$, and $x = -2$**.

**35.**

**(a)** [1 credit] At $t = 0$: $P(0) = 5000/(1 + 4\cdot1) = 5000/5 =$ **1,000 fish**.

**(b)** [1 credit] As $t \to \infty$, $e^{-0.3t} \to 0$, so the denominator approaches $1 + 0 = 1$, and $P(t) \to$ **5,000 fish**. This value represents the carrying capacity of the lake.

**(c)** [2 credits] Set $P(t) = 4{,}000$: $5000/(1 + 4e^{-0.3t}) = 4000$. Cross-multiply: $5000 = 4000 + 16{,}000e^{-0.3t}$, so $16{,}000e^{-0.3t} = 1000$ and $e^{-0.3t} = 1/16$. Take the natural log: $-0.3t = \ln(1/16) = -\ln(16) \approx -2.7726$, giving $t \approx -2.7726/(-0.3) \approx$ **9.2 years**.

# PART IV — Long Constructed-Response Explanation

**36.**

**(a)** [1 credit] Compound interest applies the annual growth factor t times: $\mathbf{A(t) = 5{,}000(1.04)^t}$ and $\mathbf{B(t) = 3{,}000(1.07)^t}$.

**(b)** [2 credits] $A(10) = 5{,}000(1.04)^{10} \approx 5{,}000 \cdot 1.48024 \approx$ **\$7,401**. $B(10) = 3{,}000(1.07)^{10} \approx 3{,}000 \cdot 1.96715 \approx$ **\$5,901**. After 10 years, Fund A still has more value because of its larger initial investment, even though Fund B grows at a higher rate.

**(c)** [3 credits] Set $B(t) = A(t)$: $3{,}000(1.07)^t = 5{,}000(1.04)^t$. Divide both sides by $3{,}000(1.04)^t$: $(1.07/1.04)^t = 5/3$. Take the natural log: $t \cdot \ln(1.07/1.04) = \ln(5/3)$, so $t = \ln(5/3) / \ln(1.07/1.04)$. Compute: $\ln(5/3) \approx 0.5108$, $\ln(1.07/1.04) = \ln(1.07) - \ln(1.04) \approx 0.06766 - 0.03922 = 0.02844$, giving $t \approx 0.5108/0.02844 \approx$ **18.0 years**. Fund B first exceeds Fund A at approximately $t = 18.0$ years.