PRACTICE EXAM 57: ALGEBRA II REGENTS SIMULATION

PART I — Multiple Choice (24 questions × 2 credits = 48 credits)

1. Express the product $(2 + 5i)(2 - 5i)$ in $a + bi$ form.

A. 29

B. $4 - 25i$

C. -21

D. $4 + 10i$

2. What is the value of 9^(5/2)?

A. 45

B. 243

C. 81

D. 22.5

3. What are the solutions to the equation $x^2 + 8x + 25 = 0$?

A. $x = 4 \pm 3i$

B. $x = -4 \pm 6i$

C. $x = -8 \pm 3i$

D. $x = -4 \pm 3i$

4. Factored completely, the expression $8x^3 - 125$ is equivalent to

A. $(2x - 5)(4x^2 - 10x + 25)$

B. $(2x + 5)(4x^2 - 10x + 25)$

C. $(2x - 5)(4x^2 + 10x + 25)$

D. $(2x - 5)(4x^2 + 25)$

5. What is the value of $\log_3(243)$?

A. 5

B. 81

C. 3

D. 27

6. What is the 5th term of a geometric sequence whose first term is 7 and whose common ratio is 2?

A. 224

B. 56

C. 64

D. 112

7. When $p(x) = x^3 - 4x^2 + 6$ is divided by $(x - 2)$, the remainder is

A. 2

B. −2

C. 6

D. 0

8. If $f(x) = x - 6$ and $g(x) = 3x + 1$, what is the value of $g(f(4))$?

A. 5

B. 7

C. −5

D. −2

9. What is the solution to the equation $\sqrt{(x - 3)} = 6$?

A. $x = 6$

B. $x = 33$

C. $x = 9$

D. $x = 39$

10. What is the average rate of change of $f(x) = x^2 - 5$ over the interval [2, 6]?

A. 8

B. 32

C. 4

D. 16

11. A quantity is modeled by $f(t) = 950(1.08)^t$, where t is the time in years. What is the annual percent rate of growth?

A. 80%

B. 108%

C. 8%

D. 1.08%

12. What is the solution to the equation $\log_3(x) - \log_3(4) = 1$?

A. x = 7

B. x = 12

C. x = 9

D. x = 4

13. Which statement best describes the nature of the solutions of $3x^2 + 2x + 5 = 0$?

A. Two complex (non-real) solutions

B. Two distinct rational solutions

C. Two distinct irrational solutions

D. One repeated rational solution

14. What is the exact value of $\cos(\pi/3)$?

A. $\sqrt{3}/2$

B. $\sqrt{2}/2$

C. $-1/2$

D. $1/2$

15. For the function $y = 8 \sin(x) + 2$, what is the amplitude?

A. 2

B. 10

C. 8

D. 4

16. In an arithmetic sequence, the fifth term is 17 and the twelfth term is 45. What is the first term?

A. 3

B. 1

C. 5

D. 2

17. If $f(x) = (x + 5)/2$, what is $f^{-1}(x)$?

A. $(x - 5)/2$

B. $2x + 5$

C. $(2 - x)/5$

D. $2x - 5$

18. A spinner is divided into 4 equal sections numbered 1 through 4. If the spinner is spun twice, what is the probability that it lands on 3 both times?

A. 1/16

B. 1/8

C. 1/4

D. 2/4

19. A data set is normally distributed with a mean of 40 and a standard deviation of 5. Approximately what percent of the data is greater than 50?

A. 16%

B. 5%

C. 2.5%

D. 34%

20. Given that $\cos \theta = -24/25$ and θ is in Quadrant II, what is the value of $\sin \theta$?

A. $-7/25$

B. $7/25$

C. $24/7$

D. $-24/25$

21. Written in vertex form, the expression $x^2 + 6x + 1$ is equivalent to

A. $(x + 3)^2 - 8$

B. $(x + 3)^2 + 1$

C. $(x - 3)^2 - 8$

D. $(x + 6)^2 - 35$

22. What is the sum of the infinite geometric series $16 + 12 + 9 + \ldots$?

A. 48

B. 32

C. 64

D. 80

23. What is the solution to the equation $6^x = 250$, rounded to the nearest hundredth?

A. x = 41.67

B. x = 3.08

C. x = 2.40

D. x = 3.50

24. A school surveys students about the cafeteria food by handing out forms only in the lunchroom during lunch. Why might this bias the results?

A. The sample is too large to interpret

B. Lunchroom surveys are always inaccurate

C. The forms cannot be counted accurately

D. Students who avoid the cafeteria are excluded, so the sample is not representative

PART II — Short Constructed Response (8 questions × 2 credits = 16 credits)

Show all work. A correct answer with no supporting work will receive only 1 credit.

25. Solve the equation $x^2 + 2x + 5 = 0$ algebraically. Express your solutions in a + bi form.

26. Solve algebraically for x: $7 / (x + 1) = 5 / (x - 1)$. State any restrictions on the variable and identify any extraneous solutions that must be rejected.

27. Given $f(x) = x^3 - 7x - 6$, determine whether $(x + 2)$ is a factor of $f(x)$. Justify your answer using the Remainder Theorem or the Factor Theorem.

28. Write an equation, in the form $g(x) = a \cdot f(x - h) + k$, for the function obtained by translating $f(x) = \sqrt[3]{x}$ to the left 4 units and up 1 unit.

29. Solve algebraically for x: 7^(2x) = 49.

30. Express 2 / (x − 4) + 6 / (x + 2) as a single rational expression in simplest form. State any restrictions on the variable.

31. A sequence is defined recursively by $a_1 = 5$ and $a_n = 2a_{n-1} - 4$ for $n \geq 2$. Find the value of a_4.

32. Given that $\sin \theta = -7/25$ and θ terminates in Quadrant III, find the exact value of $\cos \theta$. Show the algebraic work that justifies your answer.

PART III — Extended Constructed Response (3 questions × 4 credits = 12 credits)

Show all work. Partial credit is awarded according to the scoring rubric.

33. The value of an investment is modeled by the function $A(t) = 7500(1.03)^t$, where $A(t)$ is the value in dollars and t is the time in years.

(a) State the initial value of the investment and the annual percent rate of growth.

(b) Algebraically determine the number of years it will take for the investment to grow to \$10000. Round your answer to the nearest tenth of a year.

34. The table below shows the height $h(x)$, in meters, of a long jumper's trajectory at horizontal distance x meters from takeoff.

x (meters)	0	1	2	3	4
h(x) (meters)	3	15	23	27	27

(a) Using regression, write a quadratic function $h(x) = ax^2 + bx + c$ that best models the data. Round each coefficient to the nearest tenth.

(b) Use the regression model from part (a) to determine the maximum height of the trajectory and the horizontal distance at which the maximum height occurs. Round each answer to the nearest tenth.

35. A circle is defined by the equation $x^2 + y^2 = 66$, and a line is defined by the equation $y = x + 6$.

(a) Algebraically determine all points of intersection between the circle and the line. Express coordinates in exact form (radicals permitted).

(b) Verify your answer by substituting one of the intersection points back into both original equations. Show the substitution clearly.

PART IV — Long Constructed Response (1 question × 6 credits = 6 credits)

Show all work. This problem requires multiple steps and integrates concepts from several chapters.

36. A water wheel at a mill rotates steadily. A marked bucket on the rim has a height that ranges from a minimum of 1 meter below the water surface (−1 m) to a maximum of 5 meters above the surface, completing one full rotation every 10 seconds. At time t = 0, the marked bucket is at its lowest point.

(a) Write a function h(t) that models the height of the bucket in meters relative to the water surface as a function of time t in seconds.

(b) State the amplitude, period, and midline of the function h(t), and explain what each represents in the context of the water wheel.

(c) Algebraically determine all times during the first 10 seconds at which the bucket is exactly at the water surface (height 0 meters). Round each answer to the nearest hundredth of a second. Show all algebraic work.

ANSWER KEY WITH EXPLANATIONS – PRACTICE EXAM 57

1. A — Multiply the conjugates: $(2 + 5i)(2 - 5i) = 4 - 25i^2 = 4 + 25 = 29$. The product of conjugates eliminates the imaginary part. The result is the real number 29.

2. B — Apply the square root, then raise to the fifth: $9^{(5/2)} = (9^{(1/2)})^5 = 3^5 = 243$. The denominator 2 takes the square root and the numerator 5 raises it to the fifth power. The value is 243.

3. D — Quadratic formula: $x = [-8 \pm \sqrt{(64 - 100)}]/2 = [-8 \pm \sqrt{(-36)}]/2 = [-8 \pm 6i]/2 = -4 \pm 3i$. The negative discriminant produces the imaginary part. Dividing both terms by 2 gives the simplified form.

4. C — This is a difference of cubes, $(2x)^3 - 5^3 = (2x - 5)(4x^2 + 10x + 25)$. The middle term of the quadratic factor is $+ab = +10x$, making it positive. The sign pattern distinguishes it from a sum of cubes.

5. A — A logarithm asks for the exponent on the base: $3^5 = 243$, so $\log_3(243) = 5$. Rewriting in exponential form makes the value clear. The base 3 raised to 5 equals 243.

6. D — The nth term is $a_n = a_1 \cdot r^{(n-1)}$, so $a_5 = 7(2^4) = 7(16) = 112$. The ratio applied four times scales the term. The fifth term is 112.

7. B — By the Remainder Theorem, the remainder is $p(2) = 8 - 16 + 6 = -2$. Substituting the zero of the divisor avoids long division. The remainder is −2.

8. C — Evaluate the inner function first: $f(4) = 4 - 6 = -2$, then $g(-2) = 3(-2) + 1 = -5$. Composition works from the inside out. Multiplying −2 by 3 and adding 1 gives −5.

9. D — Square both sides: $x - 3 = 36$, so $x = 39$. Squaring removes the radical. Checking, $\sqrt{(39 - 3)} = \sqrt{36} = 6$ confirms the solution.

10. A — Average rate of change is $[f(6) - f(2)]/(6 - 2) = (31 - (-1))/4 = 32/4 = 8$. This is the slope of the secant line over the interval. With $f(6) = 31$ and $f(2) = -1$, the result is 8.

11. C — In $a(1 + r)^t$, the base 1.08 equals 1 + 0.08, so the growth rate is 8%. A base above 1 signals growth, and the base minus 1 gives the rate. The decimal 0.08 converts to 8%.

12. B — Combine using the quotient rule: $\log_3(x/4) = 1$, so $x/4 = 3^1 = 3$ and $x = 12$. The difference of logs becomes a quotient. Solving the exponential form gives 12.

13. A — The discriminant is $b^2 - 4ac = 4 - 60 = -56$, which is negative. A negative discriminant produces two complex conjugate solutions. No real solutions exist.

14. D — The cosine of $\pi/3$ (60°) is a standard unit-circle value equal to 1/2. The 30-60-90 reference triangle gives this ratio. The value is 1/2.

15. C — In $y = a \sin(x) + d$, the amplitude is $|a|$, which is 8. The amplitude is the coefficient of the sine term. The vertical shift of 2 does not affect it.

16. B — The common difference is $d = (45 - 17)/(12 - 5) = 28/7 = 4$. Working back, $a_1 = a_5 - 4d = 17 - 16 = 1$. Subtracting four common differences recovers the first term.

17. D — To invert, solve $y = (x + 5)/2$ for x: $2y = x + 5$, so $x = 2y - 5$ and $f^{-1}(x) = 2x - 5$. The inverse undoes the operations in reverse order. The result is $2x - 5$.

18. A — The two spins are independent, so multiply the probabilities: (1/4)(1/4) = 1/16. Each spin has a 1/4 chance of landing on 3. The probability of two 3's is 1/16.

19. C — A value of 50 is two standard deviations above the mean of 40. By the empirical rule, about 2.5% of data lies above +2 SD. The upper tail captures 2.5%.

20. B — In Quadrant II sine is positive while cosine is negative, and the 7-24-25 triangle gives the sine ratio. Therefore $\sin \theta = 7/25$. The quadrant makes the sine positive.

21. A — Complete the square: $x^2 + 6x + 1 = (x^2 + 6x + 9) - 9 + 1 = (x + 3)^2 - 8$. Half of 6 squared is 9, added and subtracted to preserve value. The vertex form reveals the vertex at $(-3, -8)$.

22. C — The common ratio is 3/4, and an infinite geometric series with $|r| < 1$ sums to $a_1/(1 - r) = 16/(1 - 3/4) = 16/(1/4) = 64$. The ratio ensures convergence. The series sums to 64.

23. B — Take the log of both sides: $x = \log_6(250) = \ln(250)/\ln(6) \approx 3.08$. The change-of-base formula converts to natural logs. The result rounds to 3.08.

24. D — Distributing forms only inside the cafeteria during lunch excludes students who bring their own food or skip the cafeteria, whose opinions likely differ. The sample is therefore not representative of all students. This selection bias skews the findings.

Part II (Short Constructed Response)

25. $x = -1 \pm 2i$ — Quadratic formula: $x = [-2 \pm \sqrt{(4 - 20)}]/2 = [-2 \pm \sqrt{(-16)}]/2 = [-2 \pm 4i]/2 = -1 \pm 2i$. The negative discriminant produces complex conjugates. Dividing each term by 2 gives the simplified $a + bi$ form.

26. $x = 6$; no extraneous solutions — Cross-multiplying gives $7(x - 1) = 5(x + 1)$, which expands to $7x - 7 = 5x + 5$ and simplifies to $2x = 12$, so $x = 6$. The restrictions are $x \neq -1$ and $x \neq 1$, and $x = 6$ violates neither, so it is valid.

27. Yes, $(x + 2)$ is a factor — By the Factor Theorem, $f(-2) = -8 + 14 - 6 = 0$. A remainder of zero confirms that $(x + 2)$ divides $f(x)$ evenly. A nonzero value would have meant it is not a factor.

28. $g(x) = \sqrt[3]{(x + 4)} + 1$ — Translating left 4 units replaces x with $(x + 4)$; translating up 1 unit adds 1. Combining these gives $\sqrt[3]{(x + 4)} + 1$. The horizontal shift moves opposite the sign inside.

29. $x = 1$ — Write the right side as a power of 7: $49 = 7^2$, so $7^{(2x)} = 7^2$ requires $2x = 2$. Solving gives $x = 1$. Equal bases allow the exponents to be set equal.

30. $(8x - 20) / [(x - 4)(x + 2)]$, $x \neq 4$, $x \neq -2$ — Over the common denominator $(x - 4)(x + 2)$: $2(x + 2) + 6(x - 4) = 2x + 4 + 6x - 24 = 8x - 20$. The numerator combines to $8x - 20$, which factors as $4(2x - 5)$. Restrictions come from the original denominators.

31. $a_4 = 12$ — Apply the recursion: $a_2 = 2(5) - 4 = 6$, $a_3 = 2(6) - 4 = 8$, $a_4 = 2(8) - 4 = 12$. Each term doubles the previous term and subtracts 4. Building up term by term reaches 12.

32. $\cos \theta = -24/25$ — With $\sin \theta = -7/25$, the 7-24-25 Pythagorean triple gives the adjacent leg of 24. In Quadrant III cosine is negative, so $\cos \theta = -24/25$. The quadrant fixes the negative sign.

Part III (Extended Constructed Response)

33. (a) $7,500, 3% growth; (b) ≈ 9.7 years — The coefficient 7500 is the initial value, and the base 1.03 = 1 + 0.03 indicates 3% annual growth. For part (b), $10000 = 7500(1.03)^t$ gives $1.03^t = 1.3333$, so $t = \ln(1.3333)/\ln(1.03) \approx 9.7$ years. Logarithms isolate the exponent.

34. (a) $h(x) = -2.0x^2 + 14.0x + 3.0$; (b) 27.5 m at x = 3.5 m — Quadratic regression on the data returns $a = -2.0$, $b = 14.0$, $c = 3.0$. The vertex occurs at $x = -b/(2a) = -14/(-4) = 3.5$ meters, and $h(3.5) = -24.5 + 49 + 3 = 27.5$ meters. The negative leading coefficient confirms a maximum.

35. (a) $(-3 + 2\sqrt{6}, 3 + 2\sqrt{6})$ and $(-3 - 2\sqrt{6}, 3 - 2\sqrt{6})$ — Substituting $y = x + 6$ into the circle gives $x^2 + (x + 6)^2 = 66$, which simplifies to $2x^2 + 12x - 30 = 0$, then $x^2 + 6x - 15 = 0$, so $x = -3 \pm 2\sqrt{6}$. The y-values follow from $y = x + 6$. **(b)** Checking $(-3 + 2\sqrt{6}, 3 + 2\sqrt{6})$: $x^2 + y^2 = (33 - 12\sqrt{6}) + (33 + 12\sqrt{6}) = 66$, and $y = x + 6 = (-3 + 2\sqrt{6}) + 6 = 3 + 2\sqrt{6}$, so both equations hold.

Part IV (Long Constructed Response)

36. (a) $h(t) = -3\cos(\pi t/5) + 2$ — The amplitude is $(5 - (-1))/2 = 3$ and the midline is $(5 + (-1))/2 = 2$. The period of 10 seconds gives $b = 2\pi/10 = \pi/5$, and starting at the lowest point at $t = 0$ calls for a negative cosine. This yields $h(t) = -3\cos(\pi t/5) + 2$.

(b) Amplitude 3, period 10 s, midline h = 2 — The amplitude of 3 meters equals the wheel's radius, the distance from the center to the rim. The period of 10 seconds is the time for one full rotation. The midline h = 2 meters is the height of the wheel's center relative to the water surface, about which the bucket's height oscillates.

(c) t ≈ 1.34 and 8.66 seconds — Setting $0 = -3\cos(\pi t/5) + 2$ gives $\cos(\pi t/5) = 2/3 \approx 0.6667$. Over the first 10 seconds, $\pi t/5 = 0.8411$ and 5.4421 radians, so $t = 1.34$ and 8.66 seconds. These are the two times the rising and falling bucket passes through the water surface.