PRACTICE EXAM 56: ALGEBRA II REGENTS SIMULATION

PART I — Multiple Choice (24 questions × 2 credits = 48 credits)

1. Express the sum $(7 - 3i) + (-2 + 8i)$ in $a + bi$ form.

A. $5 - 5i$

B. $5 + 5i$

C. $9 + 5i$

D. $-5 + 5i$

2. What is the value of 16^(3/2)?

A. 24

B. 96

C. 64

D. 48

3. What are the solutions to the equation $x^2 - 10x + 34 = 0$?

A. $x = 5 \pm 3i$

B. $x = -5 \pm 3i$

C. $x = 5 \pm 6i$

D. $x = 10 \pm 3i$

4. Factored completely, the expression $x^3 + 2x^2 - 16x - 32$ is equivalent to

A. $(x + 2)(x^2 - 16)$

B. $(x - 2)(x - 4)(x + 4)$

C. $(x + 2)(x - 16)(x + 2)$

D. $(x + 2)(x - 4)(x + 4)$

5. What is the value of $\log_2(1/16)$?

A. 4

B. −4

C. −1/4

D. 8

6. What is the sum of the finite geometric series 1 + 3 + 9 + 27 + 81 + 243?

A. 364

B. 729

C. 243

D. 486

7. When $p(x) = x^3 - 3x + 5$ is divided by $(x + 2)$, the remainder is

A. 7

B. −3

C. 3

D. 1

8. If $f(x) = 2x + 3$ and $g(x) = x^2$, what is the value of $f(g(-2))$?

A. 1

B. −1

C. 19

D. 11

9. What is the solution to the equation $\sqrt{(6x - 2)} = 4$?

A. $x = 4$

B. $x = 3$

C. $x = 16$

D. $x = 18$

10. What is the average rate of change of $f(x) = x^3$ over the interval [1, 2]?

A. 7

B. 8

C. 9

D. 4

11. \$4000 is invested at an annual interest rate of 5% compounded annually. What is the value of the investment after 2 years?

A. \$4400.00

B. \$4200.00

C. \$4410.00

D. \$4500.00

12. What is the solution to the equation $\log_4(3x) = 2$?

A. x = 8

B. x = 5

C. x = 4

D. x = 16/3

13. Which statement best describes the nature of the solutions of $x^2 + 8x + 16 = 0$?

A. Two distinct rational solutions

B. One repeated rational solution

C. Two distinct irrational solutions

D. Two complex (non-real) solutions

14. What is the exact value of $\sin(2\pi/3)$?

A. 1/2

B. $-\sqrt{3}/2$

C. $\sqrt{3}/2$

D. $-1/2$

15. For the function $y = 2\cos(\pi x/3)$, what is the period?

A. 6

B. 3

C. 2π

D. $\pi/3$

16. What is the 9th term of an arithmetic sequence whose first term is 12 and whose common difference is 5?

A. 45

B. 57

C. 48

D. 52

17. If $f(x) = 3^x$, what is $f^{-1}(x)$?

A. $(1/3)^x$

B. $\log_3(x)$

C. x^3

D. $\sqrt{x}$

18. In how many ways can 3 people be chosen from a group of 12 people?

A. 1320

B. 36

C. 220

D. 1728

19. A data set is normally distributed with a mean of 90 and a standard deviation of 6. What value corresponds to a z-score of 2?

A. 102

B. 78

C. 96

D. 12

20. Given that $\sin \theta = -12/13$ and θ is in Quadrant III, what is the value of $\cos \theta$?

A. 5/13

B. 12/5

C. −12/13

D. −5/13

21. Written in vertex form, the expression $x^2 - 8x + 20$ is equivalent to

A. $(x - 4)^2 + 20$

B. $(x + 4)^2 + 4$

C. $(x - 4)^2 + 4$

D. $(x - 8)^2 - 44$

22. What is the sum of the infinite geometric series 48 + 12 + 3 + …?

A. 60

B. 64

C. 72

D. 96

23. What is the solution to the equation $5^x = 80$, rounded to the nearest hundredth?

A. x = 16.00

B. x = 3.50

C. $x = 2.30$

D. $x = 2.72$

24. In a drug trial, why is a placebo group used?

A. To provide a baseline so the drug's true effect can be separated from the placebo effect

B. To increase the number of participants who recover

C. To ensure the drug always appears effective

D. To reduce the total cost of the study

PART II — Short Constructed Response (8 questions × 2 credits = 16 credits)

Show all work. A correct answer with no supporting work will receive only 1 credit.

25. Solve the equation $x^2 - 4x + 20 = 0$ algebraically. Express your solutions in $a + bi$ form.

26. Solve algebraically for x: $4 / (x + 2) = 2 / (x - 1)$. State any restrictions on the variable and identify any extraneous solutions that must be rejected.

27. Given $f(x) = x^3 - 2x^2 - 5x + 6$, determine whether $(x - 3)$ is a factor of $f(x)$. Justify your answer using the Remainder Theorem or the Factor Theorem.

28. Write an equation, in the form $g(x) = a \cdot f(x - h) + k$, for the function obtained by reflecting $f(x) = x^2$ across the x-axis and then translating the result 1 unit to the right.

29. Solve algebraically for x: $3^{(x + 2)} = 27$.

30. Express $3 / (x^2 - 9) + 2 / (x - 3)$ as a single rational expression in simplest form. State any restrictions on the variable.

31. A sequence is defined recursively by $a_1 = 3$ and $a_n = (a_{n-1})^2 - 5$ for $n \geq 2$. Find the value of a_4.

32. Given that $\tan \theta = -3/4$ and θ terminates in Quadrant II, find the exact value of $\sin \theta$. Show the algebraic work that justifies your answer.

PART III — Extended Constructed Response (3 questions × 4 credits = 12 credits)

Show all work. Partial credit is awarded according to the scoring rubric.

33. The value of a machine is modeled by the function $V(t) = 18000(0.9)^t$, where $V(t)$ is the value in dollars and t is the time in years.

(a) State the initial value of the machine and the annual percent rate of depreciation.

(b) Algebraically determine the number of years it will take for the value to fall to \$9000. Round your answer to the nearest tenth of a year.

34. The table below shows the height $h(x)$, in meters, of a fountain's water arc at horizontal distance x meters from the spout.

x (meters)	0	1	2	3	4

h(x) (meters)	4	19	24	19	4

(a) Using regression, write a quadratic function $h(x) = ax^2 + bx + c$ that best models the data. Round each coefficient to the nearest tenth.

(b) Use the regression model from part (a) to determine the maximum height of the water and the horizontal distance at which the maximum height occurs. Round each answer to the nearest tenth.

35. A circle is defined by the equation $x^2 + y^2 = 70$, and a line is defined by the equation $y = x + 6$.

(a) Algebraically determine all points of intersection between the circle and the line. Express coordinates in exact form (radicals permitted).

(b) Verify your answer by substituting one of the intersection points back into both original equations. Show the substitution clearly.

PART IV — Long Constructed Response (1 question × 6 credits = 6 credits)

Show all work. This problem requires multiple steps and integrates concepts from several chapters.

36. The water depth at a harbor varies with the tides. The maximum depth is 9 meters and the minimum depth is 3 meters. One complete tidal cycle takes 12 hours. At time $t = 0$ (noon), the water is at its maximum depth.

(a) Write a function $d(t)$ that models the water depth in meters as a function of time t in hours.

(b) State the amplitude, period, and midline of the function d(t), and explain what each represents in the context of the tides.

(c) Algebraically determine all times during the first 12 hours at which the water depth is 7 meters. Round each answer to the nearest hundredth of an hour. Show all algebraic work.

ANSWER KEY WITH EXPLANATIONS – PRACTICE EXAM 56

1. B — Combine real and imaginary parts: $(7 - 2) + (-3 + 8)i = 5 + 5i$. The real parts sum to 5 and the imaginary parts to 5i. The result is $5 + 5i$.

2. C — Apply the square root, then cube: $16^{3/2} = (16^{1/2})^3 = 4^3 = 64$. The denominator 2 takes the square root and the numerator 3 cubes it. The value is 64.

3. A — Quadratic formula: $x = [10 \pm \sqrt{100 - 136}]/2 = [10 \pm \sqrt{-36}]/2 = [10 \pm 6i]/2 = 5 \pm 3i$. The negative discriminant produces the imaginary part. Dividing both terms by 2 gives the simplified form.

4. D — Group and factor: $x^2(x + 2) - 16(x + 2) = (x + 2)(x^2 - 16)$, then the difference of squares gives $(x + 2)(x - 4)(x + 4)$. Complete factoring requires breaking down $x^2 - 16$. The fully factored form has three factors.

5. B — Rewrite the argument as a power of 2: $1/16 = 2^{-4}$, so $\log_2(1/16) = -4$. The negative exponent reflects a value below 1. The result is -4.

6. A — Using $S = a_1(r^n - 1)/(r - 1) = (3^6 - 1)/(3 - 1) = 728/2 = 364$. The common ratio is 3 and there are six terms. Direct addition of the terms also gives 364.

7. C — By the Remainder Theorem, the remainder is $p(-2) = -8 + 6 + 5 = 3$. Substituting the zero of the divisor avoids long division. The remainder is 3.

8. D — Evaluate the inner function first: $g(-2) = (-2)^2 = 4$, then $f(4) = 2(4) + 3 = 11$. Composition works from the inside out. Multiplying 4 by 2 and adding 3 gives 11.

9. B — Square both sides: $6x - 2 = 16$, so $6x = 18$ and $x = 3$. Squaring removes the radical. Checking, $\sqrt{6 \cdot 3 - 2} = \sqrt{16} = 4$ confirms the solution.

10. A — Average rate of change is $[f(2) - f(1)]/(2 - 1) = (8 - 1)/1 = 7$. This is the slope of the secant line over the interval. The cubes 8 and 1 drive the result.

11. C — Apply $A = P(1 + r)^t = 4000(1.05)^2 = 4000(1.1025) = 4410$. Compounding multiplies by 1.05 each year for two years. The value grows to \$4410.

12. D — Rewrite in exponential form: $3x = 4^2 = 16$, so $x = 16/3$. Converting the logarithmic equation isolates x. The base 4 raised to 2 equals 16.

13. B — The discriminant is $b^2 - 4ac = 64 - 64 = 0$, indicating one repeated real solution. Since $x^2 + 8x + 16 = (x + 4)^2$, the repeated root -4 is rational. A zero discriminant always yields a single repeated solution.

14. C — The angle $2\pi/3$ lies in Quadrant II with reference angle $\pi/3$, where sine is positive. Since $\sin(\pi/3) = \sqrt{3}/2$, $\sin(2\pi/3) = \sqrt{3}/2$. The quadrant keeps the sine positive.

15. A — The period of $\cos(bx)$ is $2\pi/b$; with $b = \pi/3$, the period is $2\pi/(\pi/3) = 6$. Dividing by the fractional coefficient enlarges the period. The period is 6.

16. D — The nth term is $a_n = a_1 + (n - 1)d$, so $a_9 = 12 + 8(5) = 12 + 40 = 52$. The common difference applied eight times raises the term. The ninth term is 52.

17. B — The inverse of an exponential with base 3 is the logarithm with base 3: $f^{-1}(x) = \log_3(x)$. Exponentials and logarithms are inverse operations. The inverse is $\log_3(x)$.

18. C — Order does not matter, so use combinations: $C(12, 3) = (12 \cdot 11 \cdot 10)/(3 \cdot 2 \cdot 1) = 220$. The combination formula counts unordered selections. There are 220 ways.

19. A — Rearrange the z-score formula: value = mean + z · SD = 90 + 2(6) = 102. A z-score of 2 sits two standard deviations above the mean. The value is 102.

20. D — In Quadrant III both sine and cosine are negative, and the 5-12-13 triangle gives the cosine ratio. Therefore $\cos \theta = -5/13$. The quadrant makes the cosine negative.

21. C — Complete the square: $x^2 - 8x + 20 = (x^2 - 8x + 16) - 16 + 20 = (x - 4)^2 + 4$. Half of -8 squared is 16, added and subtracted to preserve value. The vertex form reveals the vertex at (4, 4).

22. B — The common ratio is 1/4, and an infinite geometric series with $|r| < 1$ sums to $a_1/(1 - r) = 48/(1 - 1/4) = 48/(3/4) = 64$. The ratio ensures convergence. The series sums to 64.

23. D — Take the log of both sides: $x = \log_5(80) = \ln(80)/\ln(5) \approx 2.72$. The change-of-base formula converts to natural logs. The result rounds to 2.72.

24. A — The placebo group provides a baseline that controls for the placebo effect, so improvement from the drug itself can be isolated from improvement people experience simply from being treated. Comparing the two groups reveals the drug's true effect. Without it, the placebo response could be mistaken for efficacy.

Part II (Short Constructed Response)

25. $x = 2 \pm 4i$ — Quadratic formula: $x = [4 \pm \sqrt{(16 - 80)}]/2 = [4 \pm \sqrt{(-64)}]/2 = [4 \pm 8i]/2 = 2 \pm 4i$. The negative discriminant produces complex conjugates. Dividing each term by 2 gives the simplified a + bi form.

26. $x = 4$; no extraneous solutions — Cross-multiplying gives $4(x - 1) = 2(x + 2)$, which expands to $4x - 4 = 2x + 4$ and simplifies to $2x = 8$, so $x = 4$. The restrictions are $x \neq -2$ and $x \neq 1$, and $x = 4$ violates neither, so it is valid.

27. Yes, $(x - 3)$ is a factor — By the Factor Theorem, $f(3) = 27 - 18 - 15 + 6 = 0$. A remainder of zero confirms that $(x - 3)$ divides $f(x)$ evenly. A nonzero value would have meant it is not a factor.

28. $g(x) = -(x - 1)^2$ — Reflecting across the x-axis negates the function to $-x^2$; translating 1 unit right replaces x with $(x - 1)$. Combining these produces $-(x - 1)^2$. The reflection flips the parabola to open downward.

29. $x = 1$ — Write the right side as a power of 3: $27 = 3^3$, so $3^{(x + 2)} = 3^3$ requires $x + 2 = 3$. Solving gives $x = 1$. Equal bases allow the exponents to be set equal.

30. $(2x + 9) / [(x - 3)(x + 3)]$, $x \neq 3$, $x \neq -3$ — Factor the first denominator as $(x - 3)(x + 3)$, then use it as the common denominator: $3/[(x - 3)(x + 3)] + 2(x + 3)/[(x - 3)(x + 3)] = (3 + 2x + 6)/[(x - 3)(x + 3)] = (2x + 9)/[(x - 3)(x + 3)]$. The numerator combines to $2x + 9$.

31. $a_4 = 116$ — Apply the recursion: $a_2 = (3)^2 - 5 = 4$, $a_3 = (4)^2 - 5 = 11$, $a_4 = (11)^2 - 5 = 116$. Each term squares the previous term and subtracts 5. Building up term by term reaches 116.

32. $\sin \theta = 3/5$ — With $\tan \theta = -3/4$, the 3-4-5 reference triangle has legs 3 and 4 with hypotenuse 5. In Quadrant II sine is positive, so $\sin \theta = 3/5$. The quadrant fixes the positive sign.

Part III (Extended Constructed Response)

33. (a) \$18,000, 10% depreciation; (b) ≈ 6.6 years — The coefficient 18000 is the initial value, and the base $0.9 = 1 - 0.10$ indicates 10% annual depreciation. For part (b), $9000 = 18000(0.9)^t$ gives $0.9^t = 0.5$, so $t = \ln(0.5)/\ln(0.9) \approx 6.6$ years. Logarithms isolate the exponent.

34. (a) $h(x) = -5.0x^2 + 20.0x + 4.0$; (b) 24.0 m at $x = 2.0$ m — Quadratic regression on the data returns $a = -5.0$, $b = 20.0$, $c = 4.0$. The vertex occurs at $x = -b/(2a) = -20/(-10) = 2.0$ meters, and $h(2) = -20 + 40 + 4 = 24.0$ meters. The negative leading coefficient confirms a maximum.

35. (a) $(-3 + \sqrt{26}, 3 + \sqrt{26})$ and $(-3 - \sqrt{26}, 3 - \sqrt{26})$ — Substituting $y = x + 6$ into the circle gives $x^2 + (x + 6)^2 = 70$, which simplifies to $2x^2 + 12x - 34 = 0$, then $x^2 + 6x - 17 = 0$, so $x = -3 \pm \sqrt{26}$. The y-values follow from $y = x + 6$. **(b)** Checking $(-3 + \sqrt{26}, 3 + \sqrt{26})$: $x^2 + y^2 = (35 - 6\sqrt{26}) + (35 + 6\sqrt{26}) = 70$, and $y = x + 6 = (-3 + \sqrt{26}) + 6 = 3 + \sqrt{26}$, so both equations hold.

Part IV (Long Constructed Response)

36. (a) $d(t) = 3\cos(\pi t/6) + 6$ — The amplitude is $(9 - 3)/2 = 3$ and the midline is $(9 + 3)/2 = 6$. The period of 12 hours gives $b = 2\pi/12 = \pi/6$, and starting at the maximum depth at $t = 0$ calls for a positive cosine. This yields $d(t) = 3\cos(\pi t/6) + 6$.

(b) Amplitude 3, period 12 h, midline $d = 6$ — The amplitude of 3 meters is the distance from the midline to the maximum or minimum depth. The period of 12 hours is the time for one complete tidal cycle. The midline $d = 6$ meters is the average water depth about which the level oscillates.

(c) $t \approx 2.35$ and 9.65 hours — Setting $7 = 3\cos(\pi t/6) + 6$ gives $\cos(\pi t/6) = 1/3 \approx 0.3333$. Over the first 12 hours, $\pi t/6 = 1.2310$ and 5.0522 radians, so $t = 2.35$ and 9.65 hours. These are the two times the falling and rising tide passes a depth of 7 meters.