PRACTICE EXAM 55: ALGEBRA II REGENTS SIMULATION

PART I — Multiple Choice (24 questions × 2 credits = 48 credits)

1. What is the value of i^{62}?

A. i

B. 1

C. −1

D. −i

2. What is the value of 27^(2/3)?

A. 9

B. 18

C. 6

D. 81

3. What are the solutions to the equation $x^2 - 2x + 17 = 0$?

A. $x = -1 \pm 4i$

B. $x = 1 \pm 8i$

C. $x = 2 \pm 4i$

D. $x = 1 \pm 4i$

4. Factored completely, the expression $27x^3 + 8$ is equivalent to

A. $(3x + 2)(9x^2 + 6x + 4)$

B. $(3x + 2)(9x^2 - 6x + 4)$

C. $(3x - 2)(9x^2 + 6x + 4)$

D. $(3x + 2)(3x^2 + 4)$

5. What is the value of $\log_{10}(10000)$?

A. 1000

B. 10

C. 4

D. 5

6. What is the 5th term of a geometric sequence whose first term is 6 and whose common ratio is 2?

A. 96

B. 192

C. 48

D. 64

7. When $p(x) = x^3 + x^2 - 4$ is divided by $(x - 1)$, the remainder is

A. 2

B. −2

C. 0

D. −4

8. If $f(x) = 4x$ and $g(x) = x - 5$, what is the value of $g(f(3))$?

A. 12

B. −5

C. 28

D. 7

9. What is the solution to the equation $\sqrt{(3x + 4)} = 5$?

A. $x = 7$

B. $x = 25$

C. $x = 21$

D. $x = 9$

10. What is the average rate of change of $f(x) = x^2 + 2x$ over the interval [0, 4]?

A. 24

B. 16

C. 6

D. 8

11. A drug starts at 200 mg in the bloodstream and decreases by 20% each hour. Which function models the amount remaining after t hours?

A. $f(t) = 200(1.2)^t$

B. $f(t) = 200(0.2)^t$

C. $f(t) = 200 - 20t$

D. $f(t) = 200(0.8)^t$

12. What is the solution to the equation $\log_2(x - 4) = 3$?

A. $x = 7$

B. $x = 12$

C. $x = 6$

D. $x = 24$

13. Which statement best describes the nature of the solutions of $x^2 - 4x + 1 = 0$?

A. Two distinct rational solutions

B. One repeated rational solution

C. Two distinct irrational solutions

D. Two complex (non-real) solutions

14. What is the exact value of $\tan(\pi/6)$?

A. $\sqrt{3}/3$

B. $\sqrt{3}$

C. 1/2

D. 1

15. For the function $y = 5 \sin(2x)$, what is the period?

A. 2

B. π

C. 2π

D. 5

16. What is the sum of the first 8 terms of an arithmetic sequence whose first term is 5 and whose common difference is 4?

A. 140

B. 160

C. 144

D. 152

17. If f(x) = 2x + 9, what is $f^{-1}(x)$?

A. 2x − 9

B. (x + 9)/2

C. (x − 9)/2

D. 1 / (2x + 9)

18. A bag contains 6 blue chips and 4 yellow chips. If two chips are drawn at random without replacement, what is the probability that both are blue?

A. 1/3

B. 9/25

C. 3/5

D. 2/3

19. A data set is normally distributed with a mean of 60 and a standard deviation of 8. Approximately what percent of the data falls between 52 and 68?

A. 34%

B. 95%

C. 50%

D. 68%

20. Given that $\cos \theta = 5/13$ and θ is in Quadrant IV, what is the value of $\tan \theta$?

A. 12/5

B. −12/5

C. −5/12

D. 12/13

21. Written in vertex form, the expression $x^2 + 4x - 5$ is equivalent to

A. $(x + 2)^2 - 5$

B. $(x - 2)^2 - 9$

C. $(x + 2)^2 - 9$

D. $(x + 4)^2 - 21$

22. What is the sum of the infinite geometric series $30 + 10 + 10/3 + \dots$?

A. 40

B. 60

C. 36

D. 45

23. What is the solution to the equation $2^x = 30$, rounded to the nearest hundredth?

A. x = 4.91

B. x = 15.00

C. x = 3.40

D. x = 5.00

24. A study finds that ice cream sales and drowning deaths both rise during the same months of the year. What is the most likely explanation for this association?

A. Eating ice cream causes drowning

B. A third factor, warm weather, increases both

C. Drowning deaths increase ice cream sales

D. The two variables are completely unrelated by coincidence

PART II — Short Constructed Response (8 questions × 2 credits = 16 credits)

Show all work. A correct answer with no supporting work will receive only 1 credit.

25. Solve the equation $x^2 + 6x + 25 = 0$ algebraically. Express your solutions in a + bi form.

26. Solve algebraically for x: $3 / (x - 2) = 9 / (x + 4)$. State any restrictions on the variable and identify any extraneous solutions that must be rejected.

27. Given $f(x) = x^3 + 3x^2 - 4$, determine whether $(x - 1)$ is a factor of $f(x)$. Justify your answer using the Remainder Theorem or the Factor Theorem.

28. Write an equation, in the form $g(x) = a \cdot f(x - h) + k$, for the function obtained by vertically stretching $f(x) = |x|$ by a factor of 2 and then translating the result 3 units to the right.

29. Solve algebraically for x: 2^(3x) = 32.

30. Express $5 / (x - 3) - 2 / (x + 1)$ as a single rational expression in simplest form. State any restrictions on the variable.

31. A sequence is defined recursively by $a_1 = 4$ and $a_n = 2a_{n-1} - 3$ for $n \geq 2$. Find the value of a_4.

32. Given that $\sin \theta = 8/17$ and θ terminates in Quadrant II, find the exact value of $\tan \theta$. Show the algebraic work that justifies your answer.

PART III — Extended Constructed Response (3 questions × 4 credits = 12 credits)

Show all work. Partial credit is awarded according to the scoring rubric.

33. The value of an investment is modeled by the function $A(t) = 2500(1.04)^t$, where $A(t)$ is the value in dollars and t is the time in years.

(a) State the initial value of the investment and the annual percent rate of growth.

(b) Algebraically determine the number of years it will take for the investment to grow to \$4000. Round your answer to the nearest tenth of a year.

34. The table below shows the height h(x), in meters, of a thrown ball at horizontal distance x meters from the thrower.

x (meters)	0	1	2	3	4
h(x) (meters)	5	14	17	14	5

(a) Using regression, write a quadratic function $h(x) = ax^2 + bx + c$ that best models the data. Round each coefficient to the nearest tenth.

(b) Use the regression model from part (a) to determine the maximum height of the ball and the horizontal distance at which the maximum height occurs. Round each answer to the nearest tenth.

35. A circle is defined by the equation $x^2 + y^2 = 62$, and a line is defined by the equation $y = x + 6$.

(a) Algebraically determine all points of intersection between the circle and the line. Express coordinates in exact form (radicals permitted).

(b) Verify your answer by substituting one of the intersection points back into both original equations. Show the substitution clearly.

PART IV — Long Constructed Response (1 question × 6 credits = 6 credits)

Show all work. This problem requires multiple steps and integrates concepts from several chapters.

36. A fishing bobber floats on a lake and moves up and down as waves pass. Its height relative to the still-water level ranges from 0.3 meters above to 0.3 meters below that level, completing one full cycle every 4 seconds. At time $t = 0$, the bobber is at its highest point.

(a) Write a function $h(t)$ that models the height of the bobber in meters relative to the still-water level as a function of time t in seconds.

(b) State the amplitude, period, and midline of the function $h(t)$, and explain what each represents in the context of the bobbing float.

(c) Algebraically determine all times during the first 4 seconds at which the bobber is 0.15 meters above the still-water level. Round each answer to the nearest hundredth of a second. Show all algebraic work.

ANSWER KEY WITH EXPLANATIONS – PRACTICE EXAM 55

1. C — Powers of i cycle every four: 62 divided by 4 leaves remainder 2, so $i^{62} = i^2 = -1$. The remainder determines the equivalent power. The value is −1.

2. A — Apply the cube root, then square: $27^{(2/3)} = (27^{(1/3)})^2 = 3^2 = 9$. The denominator 3 takes the cube root and the numerator 2 squares it. The value is 9.

3. D — Quadratic formula: $x = [2 \pm \sqrt{(4 - 68)}]/2 = [2 \pm \sqrt{(-64)}]/2 = [2 \pm 8i]/2 = 1 \pm 4i$. The negative discriminant produces the imaginary part. Dividing both terms by 2 gives the simplified form.

4. B — This is a sum of cubes, $(3x)^3 + 2^3 = (3x + 2)(9x^2 - 6x + 4)$. The middle term of the quadratic factor is $-6x$, making it negative. The sign pattern distinguishes it from a difference of cubes.

5. C — A logarithm asks for the exponent on the base: $10^4 = 10000$, so $\log_{10}(10000) = 4$. Counting the zeros gives the power of 10. The value is 4.

6. A — The nth term is $a_n = a_1 \cdot r^{(n-1)}$, so $a_5 = 6(2^4) = 6(16) = 96$. The ratio applied four times scales the term. The fifth term is 96.

7. B — By the Remainder Theorem, the remainder is $p(1) = 1 + 1 - 4 = -2$. Substituting the zero of the divisor avoids long division. The remainder is −2.

8. D — Evaluate the inner function first: $f(3) = 4(3) = 12$, then $g(12) = 12 - 5 = 7$. Composition works from the inside out. Subtracting 5 from 12 gives 7.

9. A — Square both sides: $3x + 4 = 25$, so $3x = 21$ and $x = 7$. Squaring removes the radical. Checking, $\sqrt{3 \cdot 7 + 4} = \sqrt{25} = 5$ confirms the solution.

10. C — Average rate of change is $[f(4) - f(0)]/(4 - 0) = (24 - 0)/4 = 6$. This is the slope of the secant line over the interval. With $f(4) = 24$ and $f(0) = 0$, the result is 6.

11. D — A quantity that decreases by 20% uses base $1 - 0.20 = 0.8$, with 200 as the initial amount: $f(t) = 200(0.8)^t$. A base below 1 signals decay. This models the declining drug level correctly.

12. B — Rewrite in exponential form: $x - 4 = 2^3 = 8$, so $x = 12$. Converting the logarithmic equation isolates x. The base 2 raised to 3 equals 8.

13. C — The discriminant is $b^2 - 4ac = 16 - 4 = 12$, which is positive but not a perfect square. This produces two distinct irrational real solutions. The nature of the discriminant classifies the roots.

14. A — The tangent of $\pi/6$ (30°) is $1/\sqrt{3}$, which rationalizes to $\sqrt{3}/3$. The 30-60-90 reference triangle gives this ratio. The value is $\sqrt{3}/3$.

15. B — The period of $\sin(bx)$ is $2\pi/b$; with $b = 2$, the period is $2\pi/2 = \pi$. The coefficient of x sets the period. The period is π.

16. D — Using $S = n/2(2a_1 + (n - 1)d) = 8/2(10 + 7 \cdot 4) = 4(10 + 28) = 4(38) = 152$. The formula sums the arithmetic series directly. The sum of the first 8 terms is 152.

17. C — To invert, solve $y = 2x + 9$ for x: $x = (y - 9)/2$, so $f^{-1}(x) = (x - 9)/2$. The inverse undoes multiplication by 2 and addition of 9 in reverse order. The result is $(x - 9)/2$.

18. A — Without replacement, multiply the probabilities: $(6/10)(5/9) = 30/90 = 1/3$. The second draw has one fewer blue chip and one fewer total. The probability both are blue is 1/3.

19. D — The values 52 and 68 are one standard deviation below and above the mean of 60. By the empirical rule, about 68% of data lies within ±1 SD. The interval captures 68%.

20. B — In Quadrant IV cosine is positive while sine is negative, and the 5-12-13 triangle gives $\sin\theta = -12/13$. Then $\tan\theta = \sin/\cos = (-12/13)/(5/13) = -12/5$. The quadrant makes the tangent negative.

21. C — Complete the square: $x^2 + 4x - 5 = (x^2 + 4x + 4) - 4 - 5 = (x + 2)^2 - 9$. Half of 4 squared is 4, added and subtracted to preserve value. The vertex form reveals the vertex at (−2, −9).

22. D — The common ratio is 1/3, and an infinite geometric series with $|r| < 1$ sums to $a_1/(1 - r) = 30/(1 - 1/3) = 30/(2/3) = 45$. The ratio ensures convergence. The series sums to 45.

23. A — Take the log of both sides: $x = \log_2(30) = \ln(30)/\ln(2) \approx 4.91$. The change-of-base formula converts to natural logs. The result rounds to 4.91.

24. B — Warm summer weather independently raises both ice cream sales and the number of people swimming, which increases drownings. The two variables move together because of this shared cause, not because one causes the other. This is a classic confounding relationship.

Part II (Short Constructed Response)

25. $x = -3 \pm 4i$ — Quadratic formula: $x = [-6 \pm \sqrt{36 - 100}]/2 = [-6 \pm \sqrt{-64}]/2 = [-6 \pm 8i]/2 = -3 \pm 4i$. The negative discriminant produces complex conjugates. Dividing each term by 2 gives the simplified $a + bi$ form.

26. $x = 5$; no extraneous solutions — Cross-multiplying gives $3(x + 4) = 9(x - 2)$, which expands to $3x + 12 = 9x - 18$ and simplifies to $-6x = -30$, so $x = 5$. The restrictions are $x \neq 2$ and $x \neq -4$, and $x = 5$ violates neither, so it is valid.

27. Yes, $(x - 1)$ is a factor — By the Factor Theorem, $f(1) = 1 + 3 - 4 = 0$. A remainder of zero confirms that $(x - 1)$ divides $f(x)$ evenly. A nonzero value would have meant it is not a factor.

28. $g(x) = 2|x - 3|$ — A vertical stretch by 2 multiplies the function by 2, giving $2|x|$; translating 3 units right replaces x with $(x - 3)$. Combining these produces $2|x - 3|$. The horizontal shift moves opposite the sign inside.

29. $x = 5/3$ — Write the right side as a power of 2: $32 = 2^5$, so $2^{(3x)} = 2^5$ requires $3x = 5$. Solving gives $x = 5/3$. Equal bases allow the exponents to be set equal.

30. $(3x + 11) / [(x - 3)(x + 1)]$, $x \neq 3$, $x \neq -1$ — Over the common denominator $(x - 3)(x + 1)$: $5(x + 1) - 2(x - 3) = 5x + 5 - 2x + 6 = 3x + 11$. The numerator combines to $3x + 11$, which does not factor further. Restrictions come from the original denominators.

31. $a_4 = 11$ — Apply the recursion: $a_2 = 2(4) - 3 = 5$, $a_3 = 2(5) - 3 = 7$, $a_4 = 2(7) - 3 = 11$. Each term doubles the previous term and subtracts 3. Building up term by term reaches 11.

32. $\tan\theta = -8/15$ — With $\sin\theta = 8/17$, the 8-15-17 Pythagorean triple gives the adjacent leg of 15, and in Quadrant II cosine is negative, so $\cos\theta = -15/17$. Then $\tan\theta = \sin/\cos = (8/17)/(-15/17) = -8/15$. The 17s cancel in the ratio.

Part III (Extended Constructed Response)

33. (a) \$2,500, 4% growth; (b) ≈ 12.0 years — The coefficient 2500 is the initial value, and the base 1.04 = 1 + 0.04 indicates 4% annual growth. For part (b), $4000 = 2500(1.04)^t$ gives $1.04^t = 1.6$, so $t = \ln(1.6)/\ln(1.04) \approx 12.0$ years. Logarithms isolate the exponent.

34. (a) $h(x) = -3.0x^2 + 12.0x + 5.0$; (b) 17.0 m at x = 2.0 m — Quadratic regression on the data returns a = −3.0, b = 12.0, c = 5.0. The vertex occurs at $x = -b/(2a) = -12/(-6) = 2.0$ meters, and $h(2) = -12 + 24 + 5 = 17.0$ meters. The negative leading coefficient confirms a maximum.

35. (a) $(-3 + \sqrt{22}, 3 + \sqrt{22})$ and $(-3 - \sqrt{22}, 3 - \sqrt{22})$ — Substituting y = x + 6 into the circle gives $x^2 + (x + 6)^2 = 62$, which simplifies to $2x^2 + 12x - 26 = 0$, then $x^2 + 6x - 13 = 0$, so $x = -3 \pm \sqrt{22}$. The y-values follow from y = x + 6. **(b)** Checking $(-3 + \sqrt{22}, 3 + \sqrt{22})$: $x^2 + y^2 = (31 - 6\sqrt{22}) + (31 + 6\sqrt{22}) = 62$, and $y = x + 6 = (-3 + \sqrt{22}) + 6 = 3 + \sqrt{22}$, so both equations hold.

Part IV (Long Constructed Response)

36. (a) $h(t) = 0.3\cos(\pi t/2)$ — The amplitude is 0.3 meters and the midline is 0 since the motion is centered on the still-water level. The period of 4 seconds gives $b = 2\pi/4 = \pi/2$, and starting at the highest point at t = 0 calls for a positive cosine. This yields $h(t) = 0.3\cos(\pi t/2)$.

(b) Amplitude 0.3, period 4 s, midline h = 0 — The amplitude of 0.3 meters is the maximum rise or fall from the still-water level. The period of 4 seconds is the time for one complete up-and-down cycle. The midline h = 0 is the still-water level about which the bobber oscillates.

(c) t ≈ 0.67 and 3.33 seconds — Setting $0.15 = 0.3\cos(\pi t/2)$ gives $\cos(\pi t/2) = 0.5$. Over the first 4 seconds, $\pi t/2 = \pi/3$ and $5\pi/3$, so t = 2/3 and 10/3. These round to 0.67 and 3.33 seconds, the times the descending and rising bobber passes 0.15 meters above the still-water level.