PRACTICE EXAM 54: ALGEBRA II REGENTS SIMULATION

PART I — Multiple Choice (24 questions × 2 credits = 48 credits)

1. Express the product (1 − 2i)(3 + i) in a + bi form.

A. 5 − 5i

B. 5 + 5i

C. 1 − 5i

D. 3 − 2i

2. What is the value of 81^(1/2)?

A. 40.5

B. 27

C. 9

D. 18

3. What are the solutions to the equation $x^2 + 4x + 8 = 0$?

A. $x = 2 \pm 2i$

B. $x = -2 \pm 4i$

C. $x = -4 \pm 2i$

D. $x = -2 \pm 2i$

4. Factored completely, the expression $x^3 - 64$ is equivalent to

A. $(x - 4)(x^2 - 4x + 16)$

B. $(x - 4)(x^2 + 4x + 16)$

C. $(x + 4)(x^2 - 4x + 16)$

D. $(x - 4)(x^2 + 16)$

5. What is the value of $\log_3(1/9)$?

A. -2

B. 2

C. $-1/2$

D. 9

6. What is the sum of the finite geometric series $2 + 8 + 32 + 128$?

A. 128

B. 256

C. 160

D. 170

7. When $p(x) = 2x^3 - 3x^2 + x - 5$ is divided by $(x - 1)$, the remainder is

A. 1

B. −3

C. −5

D. 0

8. If $f(x) = x + 7$ and $g(x) = 2x - 1$, what is the value of $g(f(0))$?

A. 6

B. 13

C. −1

D. 7

9. What is the solution to the equation $\sqrt{(x + 5)} = x - 1$?

A. $x = 4$ and $x = -1$

B. $x = -1$

C. $x = 9$

D. $x = 4$

10. What is the average rate of change of $f(x) = x^2 + x$ over the interval $[-2, 2]$?

A. 1

B. 4

C. 2

D. 0

11. A quantity is modeled by $f(t) = 1500(1.045)^t$, where t is the time in years. What is the annual percent rate of growth?

A. 45%

B. 4.5%

C. 1.045%

D. 104.5%

12. What is the solution to the equation $\log_3(2x - 1) = 3$?

A. $x = 5$

B. $x = 13$

C. $x = 14$

D. $x = 41$

13. Which statement best describes the nature of the solutions of $4x^2 - 12x + 9 = 0$?

A. One repeated rational solution

B. Two distinct rational solutions

C. Two distinct irrational solutions

D. Two complex (non-real) solutions

14. What is the exact value of $\cos(5\pi/3)$?

A. $-1/2$

B. $\sqrt{3}/2$

C. $-\sqrt{3}/2$

D. $1/2$

15. For the function $y = -3\cos(5x) + 4$, what is the amplitude?

A. 5

B. 4

C. 3

D. -3

16. In an arithmetic sequence, the second term is -1 and the eighth term is 23. What is the common difference?

A. 3

B. 4

C. 6

D. 5

17. If $f(x) = (x - 4)/2$, what is $f^{-1}(x)$?

A. $2x + 4$

B. $(x + 4)/2$

C. $2x - 4$

D. $(2 - x)/4$

18. A standard die is rolled twice. What is the probability that both rolls show a 6?

A. 1/3

B. 1/12

C. 1/36

D. 2/6

19. A data set is normally distributed with a mean of 500 and a standard deviation of 50. Approximately what percent of the data is less than 450?

A. 34%

B. 68%

C. 84%

D. 16

20. Given that $\tan \theta = 40/9$ and θ is in Quadrant I, what is the value of $\sin \theta$?

A. 9/41

B. 40/41

C. 41/40

D. 9/40

21. Written in vertex form, the expression $2x^2 + 12x + 7$ is equivalent to

A. $2(x + 3)^2 + 7$

B. $(x + 6)^2 - 29$

C. $2(x + 3)^2 - 11$

D. $2(x + 6)^2 - 65$

22. What is the sum of the infinite geometric series $18 - 6 + 2 - \ldots$?

A. 13.5

B. 27

C. 12

D. 9

23. What is the solution to the equation $3^x = 120$, rounded to the nearest hundredth?

A. x = 40.00

B. x = 4.36

C. x = 2.41

D. x = 5.00

24. A pollster wants to estimate the average income of a town's residents. Why is it important to use a random sample?

A. To make the survey cheaper to administer

B. To guarantee a specific result

C. To ensure only high earners are surveyed

D. So every resident has an equal chance of selection, reducing bias

PART II — Short Constructed Response (8 questions × 2 credits = 16 credits)

Show all work. A correct answer with no supporting work will receive only 1 credit.

25. Solve the equation $x^2 + 10x + 34 = 0$ algebraically. Express your solutions in a + bi form.

26. Solve algebraically for x: $6 / (x - 4) = 2 / x$. State any restrictions on the variable and identify any extraneous solutions that must be rejected.

27. Given $f(x) = x^3 + 5x^2 + 4x - 4$, determine whether $(x + 2)$ is a factor of $f(x)$. Justify your answer using the Remainder Theorem or the Factor Theorem.

28. Write an equation, in the form $g(x) = a \cdot f(x) + k$, for the function obtained by reflecting $f(x) = \sqrt{x}$ across the x-axis and then translating the result 2 units up.

29. Solve algebraically for x: 4^(x − 1) = 16.

30. Express 2 / (x + 3) + 5 / (x − 2) as a single rational expression in simplest form. State any restrictions on the variable.

31. A sequence is defined recursively by $a_1 = 1$ and $a_n = 2a_{n-1} + 3$ for $n \geq 2$. Find the value of a_4.

32. Given that $\sin \theta = -24/25$ and θ terminates in Quadrant IV, find the exact value of $\cos \theta$. Show the algebraic work that justifies your answer.

PART III — Extended Constructed Response (3 questions × 4 credits = 12 credits)

Show all work. Partial credit is awarded according to the scoring rubric.

33. The value of an investment is modeled by the function $A(t) = 6000(1.07)^t$, where A(t) is the value in dollars and t is the time in years.

(a) State the initial value of the investment and the annual percent rate of growth.

(b) Algebraically determine the number of years it will take for the investment to grow to \$9000. Round your answer to the nearest tenth of a year.

34. The table below shows the height h(x), in meters, of a basketball shot at horizontal distance x meters from the shooter.

x (meters)	0	1	2	3	4
h(x) (meters)	3	11	15	15	11

(a) Using regression, write a quadratic function $h(x) = ax^2 + bx + c$ that best models the data. Round each coefficient to the nearest tenth.

(b) Use the regression model from part (a) to determine the maximum height of the shot and the horizontal distance at which the maximum height occurs. Round each answer to the nearest tenth.

35. A circle is defined by the equation $x^2 + y^2 = 54$, and a line is defined by the equation $y = x + 6$.

(a) Algebraically determine all points of intersection between the circle and the line. Express coordinates in exact form (radicals permitted).

(b) Verify your answer by substituting one of the intersection points back into both original equations. Show the substitution clearly.

PART IV — Long Constructed Response (1 question × 6 credits = 6 credits)

Show all work. This problem requires multiple steps and integrates concepts from several chapters.

36. A pendulum swings back and forth. Its horizontal displacement from the center rest position ranges from 20 cm to the right (+20) to 20 cm to the left (−20), completing one full swing every 2 seconds. At time t = 0, the pendulum is at the center and moving to the right (the positive direction).

(a) Write a function d(t) that models the horizontal displacement of the pendulum in centimeters as a function of time t in seconds.

(b) State the amplitude, period, and midline of the function d(t), and explain what each represents in the context of the swinging pendulum.

(c) Algebraically determine all times during the first 2 seconds at which the pendulum is 12 cm to the right of center (displacement +12). Round each answer to the nearest hundredth of a second. Show all algebraic work.

ANSWER KEY WITH EXPLANATIONS — PRACTICE EXAM 54

1. A — Distribute and apply $i^2 = -1$: $(1 - 2i)(3 + i) = 3 + i - 6i - 2i^2 = 3 - 5i + 2 = 5 - 5i$. The $-2i^2$ term becomes +2, raising the real part to 5. The middle terms combine to $-5i$.

2. C — A one-half power is a square root: $81^{(1/2)} = \sqrt{81} = 9$. The exponent 1/2 takes the square root. The value is 9.

3. D — Quadratic formula: $x = [-4 \pm \sqrt{(16 - 32)}]/2 = [-4 \pm \sqrt{(-16)}]/2 = [-4 \pm 4i]/2 = -2 \pm 2i$. The negative discriminant produces the imaginary part. Dividing both terms by 2 gives the simplified form.

4. B — This is a difference of cubes, $x^3 - 4^3 = (x - 4)(x^2 + 4x + 16)$. The middle term of the quadratic factor is +4x, making it positive. The sign pattern distinguishes it from a sum of cubes.

5. A — Rewrite the argument as a power of 3: $1/9 = 3^{-2}$, so $\log_3(1/9) = -2$. The negative exponent reflects a value below 1. The result is −2.

6. D — Using $S = a_1(r^n - 1)/(r - 1) = 2(4^4 - 1)/(4 - 1) = 2(255)/3 = 170$. The common ratio is 4 and there are four terms. Direct addition of the terms also gives 170.

7. C — By the Remainder Theorem, the remainder is $p(1) = 2 - 3 + 1 - 5 = -5$. Substituting the zero of the divisor avoids long division. The remainder is −5.

8. B — Evaluate the inner function first: $f(0) = 0 + 7 = 7$, then $g(7) = 2(7) - 1 = 13$. Composition works from the inside out. Multiplying 7 by 2 and subtracting 1 gives 13.

9. D — Squaring gives $x + 5 = (x - 1)^2$, which becomes $x^2 - 3x - 4 = 0$ and $(x - 4)(x + 1) = 0$, yielding $x = 4$ or $x = -1$. Checking, $x = -1$ fails because $\sqrt{4} = 2 \neq -2$, leaving $x = 4$ as the only valid solution. Squaring can introduce extraneous roots that must be tested.

10. A — Average rate of change is $[f(2) - f(-2)]/(2 - (-2)) = (6 - 2)/4 = 4/4 = 1$. This is the slope of the secant line over the interval. With $f(2) = 6$ and $f(-2) = 2$, the result is 1.

11. B — In $a(1 + r)^t$, the base 1.045 equals $1 + 0.045$, so the growth rate is 4.5%. A base above 1 signals growth, and the base minus 1 gives the rate. The decimal 0.045 converts to 4.5%.

12. C — Rewrite in exponential form: $2x - 1 = 3^3 = 27$, so $2x = 28$ and $x = 14$. Converting the logarithmic equation isolates x. The base 3 raised to 3 equals 27.

13. A — The discriminant is $b^2 - 4ac = 144 - 144 = 0$, indicating one repeated real solution. Since $4x^2 - 12x + 9 = (2x - 3)^2$, the repeated root 3/2 is rational. A zero discriminant always yields a single repeated solution.

14. D — The angle $5\pi/3$ lies in Quadrant IV with reference angle $\pi/3$, where cosine is positive. Since $\cos(\pi/3) = 1/2$, $\cos(5\pi/3) = 1/2$. The quadrant keeps the cosine positive.

15. C — In $y = a\cos(bx) + d$, the amplitude is $|a|$, which is $|-3| = 3$. The amplitude is the absolute value of the cosine coefficient. The negative sign reflects the graph but does not change the amplitude.

16. B — The common difference is $d = (23 - (-1))/(8 - 2) = 24/6 = 4$. Dividing the change in terms by the change in position gives d. The common difference is 4.

17. A — To invert, solve $y = (x - 4)/2$ for x: $2y = x - 4$, so $x = 2y + 4$ and $f^{-1}(x) = 2x + 4$. The inverse undoes subtraction of 4 and division by 2 in reverse order. The result is $2x + 4$.

18. C — The two rolls are independent, so multiply the probabilities: $(1/6)(1/6) = 1/36$. Each roll has a 1/6 chance of a 6. The probability of two sixes is 1/36.

19. D — A value of 450 is one standard deviation below the mean of 500. Since 50% lies below the mean and 34% lies between the mean and −1 SD, about 16% lies below −1 SD. The empirical rule gives the tail percentage.

20. B — In Quadrant I all ratios are positive, and the 9-40-41 triangle gives the sine ratio. Therefore $\sin\theta = 40/41$. The Pythagorean triple supplies the hypotenuse of 41.

21. C — Factor 2 from the variable terms, then complete the square: $2(x^2 + 6x) + 7 = 2(x^2 + 6x + 9) - 18 + 7 = 2(x + 3)^2 - 11$. The subtracted 18 accounts for the factor of 2 times the added 9. The vertex form reveals the vertex at $(-3, -11)$.

22. A — The common ratio is $-1/3$, and an infinite geometric series with $|r| < 1$ sums to $a_1/(1 - r) = 18/(1 - (-1/3)) = 18/(4/3) = 13.5$. The alternating signs come from the negative ratio. The series sums to 13.5.

23. B — Take the log of both sides: $x = \log_3(120) = \ln(120)/\ln(3) \approx 4.36$. The change-of-base formula converts to natural logs. The result rounds to 4.36.

24. D — Random sampling gives every resident an equal chance of selection, producing a representative sample and minimizing bias. Without randomization, certain income groups could be over- or under-represented. This is what makes the estimate trustworthy.

Part II (Short Constructed Response)

25. $x = -5 \pm 3i$ — Quadratic formula: $x = [-10 \pm \sqrt{(100 - 136)}]/2 = [-10 \pm \sqrt{(-36)}]/2 = [-10 \pm 6i]/2 = -5 \pm 3i$. The negative discriminant produces complex conjugates. Dividing each term by 2 gives the simplified a + bi form.

26. $x = -2$; no extraneous solutions — Cross-multiplying gives $6x = 2(x - 4)$, which expands to $6x = 2x - 8$ and simplifies to $4x = -8$, so $x = -2$. The restrictions are $x \neq 4$ and $x \neq 0$, and $x = -2$ violates neither, so it is valid.

27. Yes, (x + 2) is a factor — By the Factor Theorem, $f(-2) = -8 + 20 - 8 - 4 = 0$. A remainder of zero confirms that (x + 2) divides f(x) evenly. A nonzero value would have meant it is not a factor.

28. $g(x) = -\sqrt{x} + 2$ — Reflecting across the x-axis negates the function to $-\sqrt{x}$; translating 2 units up adds 2. Combining these produces $-\sqrt{x} + 2$. The reflection flips the curve below the x-axis before the shift.

29. $x = 3$ — Write the right side as a power of 4: $16 = 4^2$, so $4^{(x - 1)} = 4^2$ requires $x - 1 = 2$. Solving gives $x = 3$. Equal bases allow the exponents to be set equal.

30. $(7x + 11) / [(x + 3)(x - 2)]$, $x \neq -3$, $x \neq 2$ — Over the common denominator $(x + 3)(x - 2)$: $2(x - 2) + 5(x + 3) = 2x - 4 + 5x + 15 = 7x + 11$. The numerator combines to $7x + 11$, which does not factor further. Restrictions come from the original denominators.

31. $a_4 = 29$ — Apply the recursion: $a_2 = 2(1) + 3 = 5$, $a_3 = 2(5) + 3 = 13$, $a_4 = 2(13) + 3 = 29$. Each term doubles the previous term and adds 3. Building up term by term reaches 29.

32. $\cos \theta = 7/25$ — With $\sin \theta = -24/25$, the 7-24-25 Pythagorean triple gives the adjacent leg of 7. In Quadrant IV cosine is positive, so $\cos \theta = 7/25$. The quadrant fixes the positive sign.

Part III (Extended Constructed Response)

33. (a) $6,000, 7% growth; (b) ≈ 6.0 years — The coefficient 6000 is the initial value, and the base 1.07 = 1 + 0.07 indicates 7% annual growth. For part (b), $9000 = 6000(1.07)^t$ gives $1.07^t = 1.5$, so $t = \ln(1.5)/\ln(1.07) \approx 6.0$ years. Logarithms isolate the exponent.

34. (a) $h(x) = -2.0x^2 + 10.0x + 3.0$; (b) 15.5 m at x = 2.5 m — Quadratic regression on the data returns $a = -2.0$, $b = 10.0$, $c = 3.0$. The vertex occurs at $x = -b/(2a) = -10/(-4) = 2.5$ meters, and $h(2.5) = -12.5 + 25 + 3 = 15.5$ meters. The negative leading coefficient confirms a maximum.

35. (a) $(-3 + 3\sqrt{2}, 3 + 3\sqrt{2})$ and $(-3 - 3\sqrt{2}, 3 - 3\sqrt{2})$ — Substituting $y = x + 6$ into the circle gives $x^2 + (x + 6)^2 = 54$, which simplifies to $2x^2 + 12x - 18 = 0$, then $x^2 + 6x - 9 = 0$, so $x = -3 \pm 3\sqrt{2}$. The y-values follow from $y = x + 6$. **(b)** Checking $(-3 + 3\sqrt{2}, 3 + 3\sqrt{2})$: $x^2 + y^2 = (27 - 18\sqrt{2}) + (27 + 18\sqrt{2}) = 54$, and $y = x + 6 = (-3 + 3\sqrt{2}) + 6 = 3 + 3\sqrt{2}$, so both equations hold.

Part IV (Long Constructed Response)

36. (a) $d(t) = 20 \sin(\pi t)$ — The amplitude is 20 cm and the midline is 0 since the motion is centered on the rest position. The period of 2 seconds gives $b = 2\pi/2 = \pi$, and starting at the center moving in the positive direction at $t = 0$ calls for a positive sine. This yields $d(t) = 20 \sin(\pi t)$.

(b) Amplitude 20, period 2 s, midline d = 0 — The amplitude of 20 cm is the maximum displacement to either side of the rest position. The period of 2 seconds is the time for one complete back-and-forth swing. The midline $d = 0$ is the rest position about which the pendulum oscillates.

(c) $t \approx 0.20$ and 0.80 seconds — Setting $12 = 20 \sin(\pi t)$ gives $\sin(\pi t) = 0.6$. Over the first 2 seconds, $\pi t = 0.6435$ and 2.4981 radians, so $t = 0.20$ and 0.80 seconds. These are the two times the pendulum passes 12 cm to the right, once moving outward and once returning.