PRACTICE EXAM 53: ALGEBRA II REGENTS SIMULATION

PART I — Multiple Choice (24 questions × 2 credits = 48 credits)

1. Express the sum $(-3 + 5i) + (8 - 2i)$ in $a + bi$ form.

A. $5 - 3i$

B. $5 + 3i$

C. $11 + 3i$

D. $-11 + 7i$

2. What is the value of 100^(3/2)?

A. 150

B. 300

C. 30

D. 1000

3. What are the solutions to the equation $x^2 - 4x + 13 = 0$?

A. $x = -2 \pm 3i$

B. $x = 2 \pm 6i$

C. $x = 2 \pm 3i$

D. $x = 4 \pm 3i$

4. Factored completely, the expression $x^3 - 3x^2 + 4x - 12$ is equivalent to

A. $(x - 3)(x^2 + 4)$

B. $(x + 3)(x^2 + 4)$

C. $(x - 3)(x^2 - 4)$

D. $(x - 3)(x + 4)$

5. What is the value of $\log_8(64)$?

A. 8

B. 32

C. 2

D. 1/2

6. What is the sum of the finite geometric series 5 + 10 + 20 + 40 + 80?

A. 80

B. 160

C. 75

D. 155

7. When $p(x) = x^3 - 2x^2 + x - 3$ is divided by $(x - 3)$, the remainder is

A. 9

B. −3

C. 0

D. 6

8. If $f(x) = 3x - 2$ and $g(x) = x^2$, what is the value of $g(f(2))$?

A. 10

B. 16

C. 4

D. 8

9. What is the solution to the equation $\sqrt{(5x - 1)} = 7$?

A. $x = 7$

B. $x = 49$

C. $x = 8$

D. $x = 10$

10. What is the average rate of change of $f(x) = x^2 - 4$ over the interval [1, 4]?

A. 5

B. 15

C. 3

D. 9

11. \$5000 is invested at an annual interest rate of 2% compounded annually. What is the value of the investment after 5 years?

A. \$5500.00

B. \$5100.00

C. \$5520.40

D. \$5612.00

12. What is the solution to the equation $\log_4(x) + \log_4(2) = 3$?

A. x = 16

B. x = 32

C. x = 30

D. x = 64

13. Which statement best describes the nature of the solutions of $x^2 + 5x + 7 = 0$?

A. Two complex (non-real) solutions

B. Two distinct rational solutions

C. Two distinct irrational solutions

D. One repeated rational solution

14. What is the exact value of $\sin(\pi/4)$?

A. 1/2

B. $\sqrt{3}/2$

C. 1

D. $\sqrt{2}/2$

15. For the function $y = 3\cos(\pi x/4)$, what is the period?

A. 4

B. 8

C. 2π

D. $\pi/4$

16. What is the 8th term of an arithmetic sequence whose first term is 100 and whose common difference is -7?

A. 44

B. 58

C. 51

D. 49

17. If $f(x) = 4^x$, what is $f^{-1}(x)$?

A. $(1/4)^x$

B. x^4

C. $\sqrt{x}$

D. $\log_4(x)$

18. In how many ways can 4 books be chosen from a collection of 9 distinct books?

A. 126

B. 3024

C. 36

D. 6561

19. A data set is normally distributed with a mean of 65 and a standard deviation of 4. What is the z-score for a value of 57?

A. 2

B. −8

C. −2

D. −0.5

20. Given that $\sin \theta = -15/17$ and θ is in Quadrant III, what is the value of $\cos \theta$?

A. 8/17

B. −8/17

C. 15/8

D. −15/8

21. Written in vertex form, the expression $x^2 - 10x + 30$ is equivalent to

A. $(x - 5)^2 + 5$

B. $(x - 5)^2 - 5$

C. $(x + 5)^2 + 5$

D. $(x - 10)^2 - 70$

22. What is the sum of the infinite geometric series 40 + 10 + 2.5 + …?

A. 50

B. 60

C. 48

D. 53.33

23. What is the solution to the equation $4^x = 250$, rounded to the nearest hundredth?

A. $x = 62.50$

B. $x = 3.98$

C. $x = 2.40$

D. $x = 4.50$

24. A researcher records that students who eat breakfast tend to score higher on tests, without assigning who eats breakfast. What type of study is this, and what can it conclude?

A. An experiment that proves breakfast causes higher scores

B. An experiment that disproves any link

C. An observational study, so it can show association but not causation

D. A randomized trial that establishes cause and effect

PART II — Short Constructed Response (8 questions × 2 credits = 16 credits)

Show all work. A correct answer with no supporting work will receive only 1 credit.

25. Solve the equation $x^2 - 6x + 18 = 0$ algebraically. Express your solutions in $a + bi$ form.

26. Solve algebraically for x: $5 / (x + 3) = 2 / (x - 3)$. State any restrictions on the variable and identify any extraneous solutions that must be rejected.

27. Given $f(x) = x^3 - x^2 - 4x + 4$, determine whether $(x - 2)$ is a factor of $f(x)$. Justify your answer using the Remainder Theorem or the Factor Theorem.

28. Write an equation, in the form $g(x) = a \cdot f(x - h) + k$, for the function obtained by translating $f(x) = x^3$ to the right 2 units and up 5 units.

29. Solve algebraically for x: $2^{(4x)} = 64$.

30. Express $4 / (x - 1) - 3 / (x + 2)$ as a single rational expression in simplest form. State any restrictions on the variable.

31. A sequence is defined recursively by $a_1 = 1$ and $a_n = 2a_{n-1} + n^2$ for $n \geq 2$. Find the value of a_4.

32. Given that $\cos \theta = -7/25$ and θ terminates in Quadrant II, find the exact value of $\sin \theta$. Show the algebraic work that justifies your answer.

PART III — Extended Constructed Response (3 questions × 4 credits = 12 credits)

Show all work. Partial credit is awarded according to the scoring rubric.

33. The amount of a radioactive substance is modeled by the function $A(t) = 240(1/2)^{(t/6)}$, where $A(t)$ is the amount in milligrams and t is the time in hours.

(a) State the initial amount of the substance and the half-life in hours.

(b) Algebraically determine the number of hours it will take for the amount to fall to 50 milligrams. Round your answer to the nearest tenth of an hour.

34. The table below shows the height $h(x)$, in meters, of a stream of water from a hose at horizontal distance x meters from the nozzle.

x (meters)	0	1	2	3	4
h(x) (meters)	1	36	57	64	57

(a) Using regression, write a quadratic function $h(x) = ax^2 + bx + c$ that best models the data. Round each coefficient to the nearest tenth.

(b) Use the regression model from part (a) to determine the maximum height of the water and the horizontal distance at which the maximum height occurs. Round each answer to the nearest tenth.

35. A circle is defined by the equation $x^2 + y^2 = 42$, and a line is defined by the equation $y = x + 2$.

(a) Algebraically determine all points of intersection between the circle and the line. Express coordinates in exact form (radicals permitted).

(b) Verify your answer by substituting one of the intersection points back into both original equations. Show the substitution clearly.

PART IV — Long Constructed Response (1 question × 6 credits = 6 credits)

Show all work. This problem requires multiple steps and integrates concepts from several chapters.

36. A piston in an engine moves up and down inside its cylinder. Its height above the bottom of the cylinder ranges from a maximum of 12 cm to a minimum of 2 cm, completing one full cycle every 0.5 seconds. At time t = 0, the piston is at its highest point.

(a) Write a function h(t) that models the height of the piston in centimeters above the bottom of the cylinder as a function of time t in seconds.

(b) State the amplitude, period, and midline of the function h(t), and explain what each represents in the context of the moving piston.

(c) Algebraically determine all times during the first 0.5 seconds at which the piston is at a height of 9 cm. Round each answer to the nearest hundredth of a second. Show all algebraic work.

ANSWER KEY WITH EXPLANATIONS – PRACTICE EXAM 53

1. B — Combine real and imaginary parts: (−3 + 8) + (5 − 2)i = 5 + 3i. The real parts sum to 5 and the imaginary parts to 3i. The result is 5 + 3i.

2. D — Apply the square root, then cube: $100^{(3/2)} = (100^{(1/2)})^3 = 10^3 = 1000$. The denominator 2 takes the square root and the numerator 3 cubes it. The value is 1000.

3. C — Quadratic formula: x = [4 ± √(16 − 52)]/2 = [4 ± √(−36)]/2 = [4 ± 6i]/2 = 2 ± 3i. The negative discriminant produces the imaginary part. Dividing both terms by 2 gives the simplified form.

4. A — Group and factor: $x^2(x - 3) + 4(x - 3) = (x - 3)(x^2 + 4)$. The common binomial (x − 3) factors out. The quadratic factor $x^2 + 4$ does not factor over the reals.

5. C — A logarithm asks for the exponent on the base: $8^2 = 64$, so $\log_8(64) = 2$. Rewriting in exponential form makes the value clear. The base 8 raised to 2 equals 64.

6. D — Using $S = a_1(r^n - 1)/(r - 1) = 5(2^5 - 1)/(2 - 1) = 5(31) = 155$. The common ratio is 2 and there are five terms. Direct addition of the terms also gives 155.

7. A — By the Remainder Theorem, the remainder is p(3) = 27 − 18 + 3 − 3 = 9. Substituting the zero of the divisor avoids long division. The remainder is 9.

8. B — Evaluate the inner function first: f(2) = 3(2) − 2 = 4, then $g(4) = 4^2 = 16$. Composition works from the inside out. Squaring 4 gives 16.

9. D — Square both sides: $5x - 1 = 49$, so $5x = 50$ and $x = 10$. Squaring removes the radical. Checking, $\sqrt{(5 \cdot 10 - 1)} = \sqrt{49} = 7$ confirms the solution.

10. A — Average rate of change is $[f(4) - f(1)]/(4 - 1) = (12 - (-3))/3 = 15/3 = 5$. This is the slope of the secant line over the interval. With $f(4) = 12$ and $f(1) = -3$, the result is 5.

11. C — Apply $A = P(1 + r)^t = 5000(1.02)^5 = 5000(1.10408) \approx 5520.40$. Compounding multiplies by 1.02 each year for five years. The value grows to about \$5520.40.

12. B — Combine the logs: $\log_4(2x) = 3$, so $2x = 4^3 = 64$ and $x = 32$. The product rule for logarithms merges the terms. Solving the exponential form gives 32.

13. A — The discriminant is $b^2 - 4ac = 25 - 28 = -3$, which is negative. A negative discriminant produces two complex conjugate solutions. No real solutions exist.

14. D — The sine of $\pi/4$ (45°) is a standard unit-circle value equal to $\sqrt{2}/2$. In a 45-45-90 triangle the legs are equal, giving this ratio. The value is $\sqrt{2}/2$.

15. B — The period of $\cos(bx)$ is $2\pi/b$; with $b = \pi/4$, the period is $2\pi/(\pi/4) = 8$. Dividing by the fractional coefficient enlarges the period. The period is 8.

16. C — The nth term is $a_n = a_1 + (n - 1)d$, so $a_8 = 100 + 7(-7) = 100 - 49 = 51$. The common difference applied seven times lowers the term. The eighth term is 51.

17. D — The inverse of an exponential with base 4 is the logarithm with base 4: $f^{-1}(x) = \log_4(x)$. Exponentials and logarithms are inverse operations. The inverse is $\log_4(x)$.

18. A — Order does not matter, so use combinations: $C(9, 4) = (9 \cdot 8 \cdot 7 \cdot 6)/(4 \cdot 3 \cdot 2 \cdot 1) = 126$. The combination formula counts unordered selections. There are 126 ways.

19. C — The z-score is (value − mean)/SD = $(57 - 65)/4 = -2$. It measures how many standard deviations the value lies from the mean. A value of 57 is 2 standard deviations below the mean.

20. B — In Quadrant III both sine and cosine are negative, and the 8-15-17 triangle gives the cosine ratio. Therefore $\cos\theta = -8/17$. The quadrant makes the cosine negative.

21. A — Complete the square: $x^2 - 10x + 30 = (x^2 - 10x + 25) - 25 + 30 = (x - 5)^2 + 5$. Half of −10 squared is 25, added and subtracted to preserve value. The vertex form reveals the vertex at (5, 5).

22. D — The common ratio is 1/4, and an infinite geometric series with $|r| < 1$ sums to $a_1/(1 - r) = 40/(1 - 1/4) = 40/(3/4) \approx 53.33$. The ratio ensures convergence. The series sums to about 53.33.

23. B — Take the log of both sides: $x = \log_4(250) = \ln(250)/\ln(4) \approx 3.98$. The change-of-base formula converts to natural logs. The result rounds to 3.98.

24. C — Because the researcher only recorded behavior without assigning who ate breakfast, the study is observational and can establish association but not causation. Confounding variables such as home environment could explain the link. Only a randomized experiment can support a causal claim.

Part II (Short Constructed Response)

25. $x = 3 \pm 3i$ — Quadratic formula: $x = [6 \pm \sqrt{(36 - 72)}]/2 = [6 \pm \sqrt{(-36)}]/2 = [6 \pm 6i]/2 = 3 \pm 3i$. The negative discriminant produces complex conjugates. Dividing each term by 2 gives the simplified $a + bi$ form.

26. $x = 7$; no extraneous solutions — Cross-multiplying gives $5(x - 3) = 2(x + 3)$, which expands to $5x - 15 = 2x + 6$ and simplifies to $3x = 21$, so $x = 7$. The restrictions are $x \neq -3$ and $x \neq 3$, and $x = 7$ violates neither, so it is valid.

27. Yes, $(x - 2)$ is a factor — By the Factor Theorem, $f(2) = 8 - 4 - 8 + 4 = 0$. A remainder of zero confirms that $(x - 2)$ divides $f(x)$ evenly. A nonzero value would have meant it is not a factor.

28. $g(x) = (x - 2)^3 + 5$ — Translating right 2 units replaces x with $(x - 2)$; translating up 5 units adds 5. Combining these gives $(x - 2)^3 + 5$. The horizontal shift moves opposite the sign inside.

29. $x = 3/2$ — Write the right side as a power of 2: $64 = 2^6$, so $2^{(4x)} = 2^6$ requires $4x = 6$. Solving gives $x = 3/2$. Equal bases allow the exponents to be set equal.

30. $(x + 11) / [(x - 1)(x + 2)]$, $x \neq 1$, $x \neq -2$ — Over the common denominator $(x - 1)(x + 2)$: $4(x + 2) - 3(x - 1) = 4x + 8 - 3x + 3 = x + 11$. The numerator combines to $x + 11$, which does not factor further. Restrictions come from the original denominators.

31. $a_4 = 58$ — Apply the recursion: $a_2 = 2(1) + 2^2 = 6$, $a_3 = 2(6) + 3^2 = 21$, $a_4 = 2(21) + 4^2 = 58$. Each term doubles the previous term and adds the square of the index. Building up term by term reaches 58.

32. $\sin \theta = 24/25$ — With $\cos \theta = -7/25$, the 7-24-25 Pythagorean triple gives the opposite leg of 24. In Quadrant II sine is positive, so $\sin \theta = 24/25$. The quadrant fixes the positive sign.

Part III (Extended Constructed Response)

33. (a) 240 mg, half-life 6 hours; (b) ≈ 13.6 hours — The coefficient 240 is the initial amount, and the exponent t/6 with base 1/2 shows the substance halves every 6 hours. For part (b), $50 = 240(1/2)^{(t/6)}$ gives $(1/2)^{(t/6)} = 0.2083$, so $t/6 = \log_{0.5}(0.2083) \approx 2.263$ and $t \approx 13.6$ hours. Logarithms isolate the exponent.

34. (a) $h(x) = -7.0x^2 + 42.0x + 1.0$; (b) 64.0 m at $x = 3.0$ m — Quadratic regression on the data returns $a = -7.0$, $b = 42.0$, $c = 1.0$. The vertex occurs at $x = -b/(2a) = -42/(-14) = 3.0$ meters, and $h(3) = -63 + 126 + 1 = 64.0$ meters. The negative leading coefficient confirms a maximum.

35. (a) $(-1 + 2\sqrt{5}, 1 + 2\sqrt{5})$ and $(-1 - 2\sqrt{5}, 1 - 2\sqrt{5})$ — Substituting $y = x + 2$ into the circle gives $x^2 + (x + 2)^2 = 42$, which simplifies to $2x^2 + 4x - 38 = 0$, then $x^2 + 2x - 19 = 0$, so $x = -1 \pm 2\sqrt{5}$. The y-values follow from $y = x + 2$. **(b)** Checking $(-1 + 2\sqrt{5}, 1 + 2\sqrt{5})$: $x^2 + y^2 = (21 - 4\sqrt{5}) + (21 + 4\sqrt{5}) = 42$, and $y = x + 2 = (-1 + 2\sqrt{5}) + 2 = 1 + 2\sqrt{5}$, so both equations hold.

Part IV (Long Constructed Response)

36. (a) $h(t) = 5\cos(4\pi t) + 7$ — The amplitude is $(12 - 2)/2 = 5$ and the midline is $(12 + 2)/2 = 7$. The period of 0.5 seconds gives $b = 2\pi/0.5 = 4\pi$, and starting at the highest point at $t = 0$ calls for a positive cosine. This yields $h(t) = 5\cos(4\pi t) + 7$.

(b) Amplitude 5, period 0.5 s, midline $h = 7$ — The amplitude of 5 cm is the distance from the midline to the highest or lowest point of the piston's travel. The period of 0.5 seconds is the time for one complete up-and-down cycle. The midline $h = 7$ cm is the average height about which the piston oscillates.

(c) $t \approx 0.09$ and 0.41 seconds — Setting $9 = 5\cos(4\pi t) + 7$ gives $\cos(4\pi t) = 0.4$. Over the first 0.5 seconds, $4\pi t = 1.1593$ and 5.1239 radians, so $t = 0.09$ and 0.41 seconds. These are the two times the descending and ascending piston passes a height of 9 cm.