

# PRACTICE EXAM 52: ALGEBRA II

## REGENTS SIMULATION

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### PART I — Multiple Choice (24 questions $\times$ 2 credits = 48 credits)

1. What is the value of  $i^{25}$ ?

A. 1

B.  $-1$

C.  $-i$

D.  $i$

2. What is the value of  $36^{(3/2)}$ ?

A. 54

B. 108

C. 216

D. 18

3. What are the solutions to the equation  $x^2 - 6x + 10 = 0$ ?

A.  $x = 3 \pm 2i$

B.  $x = 3 \pm i$

C.  $x = -3 \pm i$

D.  $x = 6 \pm i$

4. Factored completely, the expression  $8x^3 + 27$  is equivalent to

A.  $(2x + 3)(4x^2 - 6x + 9)$

B.  $(2x + 3)(4x^2 + 6x + 9)$

C.  $(2x - 3)(4x^2 + 6x + 9)$

D.  $(2x + 3)(2x^2 + 9)$

5. What is the value of  $\log_5(1/25)$ ?

A. 2

B. 25

C.  $-1/2$

D.  $-2$

6. What is the 5th term of a geometric sequence whose first term is 4 and whose common ratio is  $1/2$ ?

A.  $1/2$

B.  $1/4$

C.  $1/8$

D. 1

7. When  $p(x) = x^3 + 4x - 7$  is divided by  $(x - 2)$ , the remainder is

A. 1

B. 7

C. 9

D. 5

8. If  $f(x) = x^2 - 1$  and  $g(x) = x + 4$ , what is the value of  $f(g(-3))$ ?

A. 0

B. 8

C. 12

D. 24

9. What is the solution to the equation  $\sqrt{(2x - 5) + 1} = 4$ ?

A.  $x = 5$

B.  $x = 9$

C.  $x = 7$

D.  $x = 14$

10. What is the average rate of change of  $f(x) = 2^x$  over the interval  $[0, 3]$ ?

A. 8

B. 7

C. 3

D.  $7/3$

11. A car worth  $\$28000$  loses 15% of its value each year. Which function models the car's value after  $t$  years?

A.  $f(t) = 28000(0.85)^t$

B.  $f(t) = 28000(1.15)^t$

C.  $f(t) = 28000(0.15)^t$

D.  $f(t) = 28000 - 0.15t$

12. What is the solution to the equation  $\log_5(x + 3) = 2$ ?

A.  $x = 7$

B.  $x = 22$

C.  $x = 2$

D.  $x = 28$

13. Which statement best describes the nature of the solutions of  $2x^2 - 3x + 4 = 0$ ?

- A. Two distinct rational solutions
- B. Two distinct irrational solutions
- C. One repeated rational solution
- D. Two complex (non-real) solutions

14. What is the exact value of  $\tan(\pi/3)$ ?

- A.  $1/2$
- B.  $\sqrt{3}/3$
- C.  $\sqrt{3}$
- D. 1

15. For the function  $y = 2 \sin(x) + 6$ , what is the equation of the midline?

- A.  $y = 6$
- B.  $y = 2$
- C.  $y = 8$
- D.  $y = 4$

16. What is the sum of the first 10 terms of an arithmetic sequence whose first term is 3 and whose common difference is 6?

- A. 280
- B. 300

C. 330

D. 600

17. If  $f(x) = (2x - 1)/4$ , what is  $f^{-1}(x)$ ?

A.  $(4x - 1)/2$

B.  $(2x + 1)/4$

C.  $(4x + 1)/2$

D.  $4 / (2x - 1)$

18. A box contains 3 defective parts and 7 good parts. If two parts are drawn at random without replacement, what is the probability that both are good?

A.  $49/100$

B.  $7/10$

C.  $21/50$

D.  $7/15$

19. A data set is normally distributed with a mean of 80 and a standard deviation of 5. Approximately what percent of the data falls between 70 and 90?

A. 95%

B. 68%

C. 99.7%

D. 50%

20. Given that  $\cos \theta = 20/29$  and  $\theta$  is in Quadrant IV, what is the value of  $\sin \theta$ ?

A.  $21/29$

B.  $-21/29$

C.  $-20/29$

D.  $21/20$

21. Written in vertex form, the expression  $x^2 + 6x - 2$  is equivalent to

A.  $(x + 3)^2 - 2$

B.  $(x - 3)^2 - 11$

C.  $(x + 3)^2 - 11$

D.  $(x + 6)^2 - 38$

22. What is the sum of the infinite geometric series  $81 + 27 + 9 + \dots$ ?

A. 121.5

B. 108

C. 162

D. 120

23. What is the solution to the equation  $6^x = 400$ , rounded to the nearest hundredth?

A.  $x = 66.67$

B.  $x = 4.00$

C.  $x = 2.60$

D.  $x = 3.34$

24. Two surveys estimate the same population proportion: one uses 50 people and the other uses 500 people. Which is more likely to give an estimate close to the true value, and why?

A. The survey of 50, because smaller samples are easier to control

B. The survey of 500, because larger samples reduce variability and give more reliable estimates

C. Both are equally reliable regardless of sample size

D. Neither can estimate a proportion accurately

**PART II — Short Constructed Response (8 questions  $\times$  2 credits = 16 credits)**

**Show all work. A correct answer with no supporting work will receive only 1 credit.**

25. Solve the equation  $x^2 - 2x + 26 = 0$  algebraically. Express your solutions in a + bi form.

26. Solve algebraically for x:  $2 / (x + 1) = 3 / (x + 4)$ . State any restrictions on the variable and identify any extraneous solutions that must be rejected.

27. Given  $f(x) = x^3 + 2x^2 + 2x - 5$ , determine whether  $(x - 1)$  is a factor of  $f(x)$ . Justify your answer using the Remainder Theorem or the Factor Theorem.

28. Write an equation, in the form  $g(x) = a \cdot f(x) + k$ , for the function obtained by vertically compressing  $f(x) = x^2$  by a factor of  $1/2$  and then translating the result 3 units up.

29. Solve algebraically for  $x$ :  $7^{(x+2)} = 343$ .

30. Express  $5 / (x - 2) - 3 / (x + 2)$  as a single rational expression in simplest form. State any restrictions on the variable.

31. A sequence is defined recursively by  $a_1 = 10$  and  $a_n = a_{n-1} - 3n$  for  $n \geq 2$ . Find the value of  $a_4$ .

32. Given that  $\sin \theta = -9/41$  and  $\theta$  terminates in Quadrant III, find the exact value of  $\tan \theta$ . Show the algebraic work that justifies your answer.

**PART III — Extended Constructed Response (3 questions  $\times$  4 credits = 12 credits)**

**Show all work. Partial credit is awarded according to the scoring rubric.**

33. The population of a town is modeled by the function  $P(t) = 12000(1.02)^t$ , where  $P(t)$  is the population and  $t$  is the time in years.

(a) State the initial population of the town and the annual percent rate of growth.

(b) Algebraically determine the number of years it will take for the population to grow to 15000. Round your answer to the nearest tenth of a year.

34. The table below shows the height  $h(x)$ , in meters, of a dolphin's jump at horizontal distance  $x$  meters from where it left the water.

x (meters)   0   1   2   3   4
--- --- --- --- ---
h(x) (meters)   3   15   19   15   3

(a) Using regression, write a quadratic function  $h(x) = ax^2 + bx + c$  that best models the data. Round each coefficient to the nearest tenth.

(b) Use the regression model from part (a) to determine the maximum height of the jump and the horizontal distance at which the maximum height occurs. Round each answer to the nearest tenth.

35. A circle is defined by the equation  $x^2 + y^2 = 34$ , and a line is defined by the equation  $y = x + 4$ .

(a) Algebraically determine all points of intersection between the circle and the line. Express coordinates in exact form (radicals permitted).

(b) Verify your answer by substituting one of the intersection points back into both original equations. Show the substitution clearly.

**PART IV — Long Constructed Response (1 question × 6 credits = 6 credits)**

**Show all work. This problem requires multiple steps and integrates concepts from several chapters.**

36. The number of hours of daylight in a city varies sinusoidally over the course of a year. The maximum is 15 hours and the minimum is 9 hours, and one full cycle takes 365 days. Let  $t$  be the number of days after the spring equinox, when daylight equals the average of 12 hours and is increasing.

(a) Write a function  $D(t)$  that models the number of hours of daylight as a function of time  $t$  in days.

(b) State the amplitude, period, and midline of the function  $D(t)$ , and explain what each represents in the context of the daylight hours.

(c) Algebraically determine all days during the first 365 days at which there are 14 hours of daylight. Round each answer to the nearest hundredth of a day. Show all algebraic work.

## ANSWER KEY WITH EXPLANATIONS – PRACTICE EXAM 52

**1. D** — Powers of  $i$  cycle every four: 25 divided by 4 leaves remainder 1, so  $i^{25} = i^1 = i$ . The remainder determines the equivalent power. The value is  $i$ .

**2. C** — Apply the square root, then cube:  $36^{(3/2)} = (36^{(1/2)})^3 = 6^3 = 216$ . The denominator 2 takes the square root and the numerator 3 cubes it. The value is 216.

**3. B** — Quadratic formula:  $x = [6 \pm \sqrt{(36 - 40)}]/2 = [6 \pm \sqrt{(-4)}]/2 = [6 \pm 2i]/2 = 3 \pm i$ . The negative discriminant produces the imaginary part. Dividing both terms by 2 gives the simplified form.

**4. A** — This is a sum of cubes,  $(2x)^3 + 3^3 = (2x + 3)(4x^2 - 6x + 9)$ . The middle term of the quadratic factor is  $-6x$ , making it negative. The sign pattern distinguishes it from a difference of cubes.

**5. D** — Rewrite the argument as a power of 5:  $1/25 = 5^{-2}$ , so  $\log_5(1/25) = -2$ . The negative exponent reflects a value below 1. The result is  $-2$ .

**6. B** — The  $n$ th term is  $a_n = a_1 \cdot r^{(n-1)}$ , so  $a_5 = 4(1/2)^4 = 4(1/16) = 1/4$ . The ratio applied four times shrinks the term. The fifth term is  $1/4$ .

**7. C** — By the Remainder Theorem, the remainder is  $p(2) = 8 + 8 - 7 = 9$ . Substituting the zero of the divisor avoids long division. The remainder is 9.

- 8. A** — Evaluate the inner function first:  $g(-3) = -3 + 4 = 1$ , then  $f(1) = 1^2 - 1 = 0$ . Composition works from the inside out. Squaring 1 and subtracting 1 gives 0.
- 9. C** — Subtract 1, then square:  $\sqrt{2x - 5} = 3$ , so  $2x - 5 = 9$  and  $x = 7$ . Isolating the radical before squaring is essential. Checking,  $\sqrt{(2 \cdot 7 - 5) + 1} = \sqrt{9 + 1} = 4$  confirms the solution.
- 10. D** — Average rate of change is  $[f(3) - f(0)]/(3 - 0) = (8 - 1)/3 = 7/3$ . This is the slope of the secant line over the interval. The exponential values 8 and 1 drive the result.
- 11. A** — A quantity that loses 15% uses base  $1 - 0.15 = 0.85$ , with 28000 as the initial value:  $f(t) = 28000(0.85)^t$ . A base below 1 signals depreciation. This models the declining car value correctly.
- 12. B** — Rewrite in exponential form:  $x + 3 = 5^2 = 25$ , so  $x = 22$ . Converting the logarithmic equation isolates  $x$ . The base 5 raised to 2 equals 25.
- 13. D** — The discriminant is  $b^2 - 4ac = 9 - 32 = -23$ , which is negative. A negative discriminant produces two complex conjugate solutions. No real solutions exist.
- 14. C** — The tangent of  $\pi/3$  ( $60^\circ$ ) is a standard unit-circle value equal to  $\sqrt{3}$ . The 30-60-90 reference triangle gives this ratio. The value is  $\sqrt{3}$ .
- 15. A** — In  $y = a \sin(x) + d$ , the midline is  $y = d$ , which is  $y = 6$ . The vertical shift sets the midline. The amplitude of 2 does not affect it.
- 16. B** — Using  $S = n/2(2a_1 + (n - 1)d) = 10/2(6 + 9 \cdot 6) = 5(6 + 54) = 5(60) = 300$ . The formula sums the arithmetic series directly. The sum of the first 10 terms is 300.
- 17. C** — To invert, solve  $y = (2x - 1)/4$  for  $x$ :  $4y = 2x - 1$ , so  $2x = 4y + 1$  and  $f^{-1}(x) = (4x + 1)/2$ . The inverse undoes the operations in reverse order. The result is  $(4x + 1)/2$ .
- 18. D** — Without replacement, multiply the probabilities:  $(7/10)(6/9) = 42/90 = 7/15$ . The second draw has one fewer good part and one fewer total. The probability both are good is  $7/15$ .
- 19. A** — The values 70 and 90 are two standard deviations below and above the mean of 80. By the empirical rule, about 95% of data lies within  $\pm 2$  SD. The interval captures 95%.
- 20. B** — In Quadrant IV cosine is positive while sine is negative, and the 20-21-29 triangle gives the sine ratio. Therefore  $\sin \theta = -21/29$ . The quadrant makes the sine negative.
- 21. C** — Complete the square:  $x^2 + 6x - 2 = (x^2 + 6x + 9) - 9 - 2 = (x + 3)^2 - 11$ . Half of 6 squared is 9, added and subtracted to preserve value. The vertex form reveals the vertex at  $(-3, -11)$ .
- 22. A** — The common ratio is  $1/3$ , and an infinite geometric series with  $|r| < 1$  sums to  $a_1/(1 - r) = 81/(1 - 1/3) = 81/(2/3) = 121.5$ . The ratio ensures convergence. The series sums to 121.5.

**23. D** — Take the log of both sides:  $x = \log_6(400) = \ln(400)/\ln(6) \approx 3.34$ . The change-of-base formula converts to natural logs. The result rounds to 3.34.

**24. B** — A larger sample reduces sampling variability, so the survey of 500 is more likely to produce an estimate close to the true proportion. Bigger samples shrink the margin of error. This is why pollsters favor larger sample sizes.

## Part II (Short Constructed Response)

**25.  $x = 1 \pm 5i$**  — Quadratic formula:  $x = [2 \pm \sqrt{(4 - 104)}]/2 = [2 \pm \sqrt{(-100)}]/2 = [2 \pm 10i]/2 = 1 \pm 5i$ . The negative discriminant produces complex conjugates. Dividing each term by 2 gives the simplified  $a + bi$  form.

**26.  $x = 5$ ; no extraneous solutions** — Cross-multiplying gives  $2(x + 4) = 3(x + 1)$ , which expands to  $2x + 8 = 3x + 3$  and simplifies to  $-x = -5$ , so  $x = 5$ . The restrictions are  $x \neq -1$  and  $x \neq -4$ , and  $x = 5$  violates neither, so it is valid.

**27. Yes,  $(x - 1)$  is a factor** — By the Factor Theorem,  $f(1) = 1 + 2 + 2 - 5 = 0$ . A remainder of zero confirms that  $(x - 1)$  divides  $f(x)$  evenly. A nonzero value would have meant it is not a factor.

**28.  $g(x) = (1/2)x^2 + 3$**  — A vertical compression by a factor of  $1/2$  multiplies the function by  $1/2$ , giving  $(1/2)x^2$ ; translating 3 units up adds 3. Combining these produces  $(1/2)x^2 + 3$ . The compression acts on the leading coefficient.

**29.  $x = 1$**  — Write the right side as a power of 7:  $343 = 7^3$ , so  $7^{(x + 2)} = 7^3$  requires  $x + 2 = 3$ . Solving gives  $x = 1$ . Equal bases allow the exponents to be set equal.

**30.  $(2x + 16) / [(x - 2)(x + 2)]$ ,  $x \neq 2$ ,  $x \neq -2$**  — Over the common denominator  $(x - 2)(x + 2)$ :  $5(x + 2) - 3(x - 2) = 5x + 10 - 3x + 6 = 2x + 16$ . The numerator combines to  $2x + 16$ , which factors as  $2(x + 8)$ . Restrictions come from the original denominators.

**31.  $a_4 = -17$**  — Apply the recursion:  $a_2 = 10 - 3(2) = 4$ ,  $a_3 = 4 - 3(3) = -5$ ,  $a_4 = -5 - 3(4) = -17$ . Each term subtracts three times the index from the previous term. Building up term by term reaches  $-17$ .

**32.  $\tan \theta = 9/40$**  — With  $\sin \theta = -9/41$ , the 9-40-41 Pythagorean triple gives the adjacent leg of 40, and in Quadrant III cosine is negative, so  $\cos \theta = -40/41$ . Then  $\tan \theta = \sin/\cos = (-9/41)/(-40/41) = 9/40$ . The negative signs cancel in the ratio.

## Part III (Extended Constructed Response)

**33. (a) 12,000 people, 2% growth; (b)  $\approx 11.3$  years** — The coefficient 12000 is the initial population, and the base  $1.02 = 1 + 0.02$  indicates 2% annual growth. For part (b),  $15000 = 12000(1.02)^t$  gives  $1.02^t = 1.25$ , so  $t = \ln(1.25)/\ln(1.02) \approx 11.3$  years. Logarithms isolate the exponent.

**34. (a)  $h(x) = -4.0x^2 + 16.0x + 3.0$ ; (b) 19.0 m at  $x = 2.0$  m** — Quadratic regression on the data returns  $a = -4.0$ ,  $b = 16.0$ ,  $c = 3.0$ . The vertex occurs at  $x = -b/(2a) = -16/(-8) = 2.0$  meters, and  $h(2) = -16 + 32 + 3 = 19.0$  meters. The negative leading coefficient confirms a maximum.

**35. (a)  $(-2 + \sqrt{13}, 2 + \sqrt{13})$  and  $(-2 - \sqrt{13}, 2 - \sqrt{13})$**  — Substituting  $y = x + 4$  into the circle gives  $x^2 + (x + 4)^2 = 34$ , which simplifies to  $2x^2 + 8x - 18 = 0$ , then  $x^2 + 4x - 9 = 0$ , so  $x = -2 \pm \sqrt{13}$ . The  $y$ -values follow from  $y = x + 4$ . **(b)** Checking  $(-2 + \sqrt{13}, 2 + \sqrt{13})$ :  $x^2 + y^2 = (17 - 4\sqrt{13}) + (17 + 4\sqrt{13}) = 34$ , and  $y = x + 4 = (-2 + \sqrt{13}) + 4 = 2 + \sqrt{13}$ , so both equations hold.

## Part IV (Long Constructed Response)

**36. (a)  $D(t) = 3 \sin(2\pi t/365) + 12$**  — The amplitude is  $(15 - 9)/2 = 3$  and the midline is  $(15 + 9)/2 = 12$ . The period of 365 days gives  $b = 2\pi/365$ , and starting at the average value increasing at  $t = 0$  calls for a positive sine. This yields  $D(t) = 3 \sin(2\pi t/365) + 12$ .

**(b) Amplitude 3, period 365 days, midline  $D = 12$**  — The amplitude of 3 hours is the maximum departure from the average daylight length. The period of 365 days is the time for one complete yearly cycle. The midline  $D = 12$  hours is the average daylight length about which the value oscillates.

**(c)  $t \approx 42.39$  and  $140.11$  days** — Setting  $14 = 3 \sin(2\pi t/365) + 12$  gives  $\sin(2\pi t/365) = 2/3 \approx 0.6667$ . Over the first 365 days,  $2\pi t/365 = 0.7297$  and  $2.4119$  radians, so  $t = 42.39$  and  $140.11$  days. These are the two days, on the rising and falling sides of the summer peak, when daylight reaches 14 hours.