PRACTICE EXAM 51: ALGEBRA II REGENTS SIMULATION

PART I — Multiple Choice (24 questions × 2 credits = 48 credits)

1. Express the product $(4 + i)(2 - 3i)$ in $a + bi$ form.

A. $11 + 10i$

B. $5 - 10i$

C. $8 - 3i$

D. $11 - 10i$

2. What is the value of $8^{(4/3)}$?

A. 16

B. 32

C. 10.67

D. 64

3. What are the solutions to the equation $x^2 + 6x + 13 = 0$?

A. $x = 3 \pm 2i$

B. $x = -3 \pm 2i$

C. $x = -3 \pm 4i$

D. $x = -6 \pm 2i$

4. Factored completely, the expression $125x^3 - 8$ is equivalent to

A. $(5x - 2)(25x^2 - 10x + 4)$

B. $(5x + 2)(25x^2 - 10x + 4)$

C. $(5x - 2)(25x^2 + 10x + 4)$

D. $(5x - 2)(5x^2 + 4)$

5. What is the value of $\log_2(64)$?

A. 32

B. 8

C. 5

D. 6

6. What is the sum of the finite geometric series 3 + 6 + 12 + 24 + 48?

A. 48

B. 93

C. 96

D. 81

7. When $p(x) = x^3 - x^2 + 2x - 1$ is divided by $(x - 1)$, the remainder is

A. 1

B. 3

C. −1

D. 0

8. If $f(x) = 2x - 5$ and $g(x) = x + 3$, what is the value of $f(g(1))$?

A. 1

B. 0

C. 3

D. 9

9. What is the solution to the equation $\sqrt{(4x + 1)} = 5$?

A. $x = 5$

B. $x = 6$

C. $x = 24$

D. $x = 25$

10. What is the average rate of change of $f(x) = x^2 + 1$ over the interval [2, 5]?

A. 21

B. 9

C. 5

D. 7

11. A quantity is modeled by $f(t) = 800(1.12)^t$, where t is the time in years. What is the annual percent rate of growth?

A. 12%

B. 112%

C. 1.12%

D. 88%

12. What is the solution to the equation $\log_2(x) + \log_2(5) = 4$?

A. x = 8

B. x = 5

C. x = 3.2

D. x = 11/5

13. Which statement best describes the nature of the solutions of $x^2 - 10x + 25 = 0$?

A. Two distinct rational solutions

B. Two distinct irrational solutions

C. Two complex (non-real) solutions

D. One repeated rational solution

14. What is the exact value of $\cos(\pi/4)$?

A. 1/2

B. $\sqrt{2}/2$

C. $\sqrt{3}/2$

D. 1

15. For the function $y = 6 \sin(3x) - 2$, what is the amplitude?

A. 6

B. 3

C. 2

D. −2

16. What is the 15th term of an arithmetic sequence whose first term is 7 and whose common difference is −2?

A. −28

B. −23

C. -21

D. -14

17. If $f(x) = x^3 + 1$, what is $f^{-1}(x)$?

A. $\sqrt[3]{x} - 1$

B. $\sqrt[3]{(x + 1)}$

C. $(x - 1)^3$

D. $\sqrt[3]{(x - 1)}$

18. A fair coin is flipped 3 times. What is the probability of getting heads on all three flips?

A. 1/2

B. 1/8

C. 1/6

D. 3/8

19. A data set is normally distributed with a mean of 150 and a standard deviation of 20. Approximately what percent of the data is greater than 170?

A. 16%

B. 34%

C. 68%

D. 84%

20. Given that $\cos \theta = -15/17$ and θ is in Quadrant II, what is the value of $\sin \theta$?

A. $-8/17$

B. $15/8$

C. $8/17$

D. $-15/17$

21. Written in vertex form, the expression $x^2 - 12x + 40$ is equivalent to

A. $(x - 6)^2 - 4$

B. $(x + 6)^2 + 4$

C. $(x - 12)^2 - 104$

D. $(x - 6)^2 + 4$

22. What is the sum of the infinite geometric series $64 + 16 + 4 + \ldots$?

A. 85.33

B. 80

C. 96

D. 128

23. What is the solution to the equation $5^x = 200$, rounded to the nearest hundredth?

A. $x = 40.00$

B. $x = 3.50$

C. $x = 3.29$

D. $x = 2.30$

24. In a clinical trial, why are participants randomly assigned to the treatment and placebo groups?

A. To make the experiment faster to conduct

B. To make the groups comparable so outcome differences can be attributed to the treatment

C. To guarantee that the treatment will appear effective

D. To allow participants to choose their preferred group

PART II — Short Constructed Response (8 questions × 2 credits = 16 credits)

Show all work. A correct answer with no supporting work will receive only 1 credit.

25. Solve the equation $x^2 + 2x + 17 = 0$ algebraically. Express your solutions in $a + bi$ form.

26. Solve algebraically for x: $4 / (x - 6) = 1 / x$. State any restrictions on the variable and identify any extraneous solutions that must be rejected.

27. Given $f(x) = x^3 + 2x^2 - x - 2$, determine whether $(x + 1)$ is a factor of $f(x)$. Justify your answer using the Remainder Theorem or the Factor Theorem.

28. Write an equation, in the form $g(x) = a \cdot f(x - h) + k$, for the function obtained by translating $f(x) = \sqrt{x}$ to the left 2 units and down 5 units.

29. Solve algebraically for x: $3^{(2x)} = 81$.

30. Express $3 / (x + 5) + 4 / (x - 5)$ as a single rational expression in simplest form. State any restrictions on the variable.

31. A sequence is defined recursively by $a_1 = 2$ and $a_n = 3a_{n-1} + 1$ for $n \geq 2$. Find the value of a_4.

32. Given that $\tan \theta = 15/8$ and θ terminates in Quadrant III, find the exact value of $\cos \theta$. Show the algebraic work that justifies your answer.

PART III — Extended Constructed Response (3 questions × 4 credits = 12 credits)

Show all work. Partial credit is awarded according to the scoring rubric.

33. The balance of an account is modeled by the function $A(t) = 4000(1.025)^t$, where $A(t)$ is the balance in dollars and t is the time in years.

(a) State the initial balance of the account and the annual percent rate of growth.

(b) Algebraically determine the number of years it will take for the balance to grow to \$5000. Round your answer to the nearest tenth of a year.

34. The table below shows the height h(x), in meters, of a soccer ball at horizontal distance x meters from where it was kicked.

x (meters)	0	1	2	3	4
h(x) (meters)	2	32	50	56	50

(a) Using regression, write a quadratic function $h(x) = ax^2 + bx + c$ that best models the data. Round each coefficient to the nearest tenth.

(b) Use the regression model from part (a) to determine the maximum height of the ball and the horizontal distance at which the maximum height occurs. Round each answer to the nearest tenth.

35. A circle is defined by the equation $x^2 + y^2 = 40$, and a line is defined by the equation $y = x + 2$.

(a) Algebraically determine all points of intersection between the circle and the line. Express coordinates in exact form (radicals permitted).

(b) Verify your answer by substituting one of the intersection points back into both original equations. Show the substitution clearly.

PART IV — Long Constructed Response (1 question × 6 credits = 6 credits)

Show all work. This problem requires multiple steps and integrates concepts from several chapters.

36. A carousel horse moves up and down as the carousel turns. Its height above the platform ranges from a maximum of 1.2 meters to a minimum of 0.4 meters, completing one full up-and-down cycle every 3 seconds. At time $t = 0$, the horse is at its lowest height.

(a) Write a function $h(t)$ that models the height of the carousel horse in meters above the platform as a function of time t in seconds.

(b) State the amplitude, period, and midline of the function $h(t)$, and explain what each represents in the context of the carousel horse.

(c) Algebraically determine all times during the first 3 seconds at which the horse is at a height of 1 meter. Round each answer to the nearest hundredth of a second. Show all algebraic work.

ANSWER KEY WITH EXPLANATIONS – PRACTICE EXAM 51

1. D — Distribute and apply $i^2 = -1$: $(4 + i)(2 - 3i) = 8 - 12i + 2i - 3i^2 = 8 - 10i + 3 = 11 - 10i$. The $-3i^2$ term becomes $+3$, raising the real part to 11. The middle terms combine to $-10i$.

2. A — Apply the cube root, then raise to the fourth: $8^{(4/3)} = (8^{(1/3)})^4 = 2^4 = 16$. The denominator 3 takes the cube root and the numerator 4 raises it to the fourth power. The value is 16.

3. B — Quadratic formula: $x = [-6 \pm \sqrt{(36 - 52)}]/2 = [-6 \pm \sqrt{(-16)}]/2 = [-6 \pm 4i]/2 = -3 \pm 2i$. The negative discriminant produces the imaginary part. Dividing both terms by 2 gives the simplified form.

4. C — This is a difference of cubes, $(5x)^3 - 2^3 = (5x - 2)(25x^2 + 10x + 4)$. The middle term of the quadratic factor is $+ab = +10x$, making it positive. The sign pattern distinguishes it from a sum of cubes.

5. D — A logarithm asks for the exponent on the base: $2^6 = 64$, so $\log_2(64) = 6$. Rewriting in exponential form makes the value clear. The base 2 raised to 6 equals 64.

6. B — Using $S = a_1(r^n - 1)/(r - 1) = 3(2^5 - 1)/(2 - 1) = 3(31) = 93$. The common ratio is 2 and there are five terms. Direct addition of the terms also gives 93.

7. A — By the Remainder Theorem, the remainder is $p(1) = 1 - 1 + 2 - 1 = 1$. Substituting the zero of the divisor avoids long division. The remainder is 1.

8. C — Evaluate the inner function first: $g(1) = 1 + 3 = 4$, then $f(4) = 2(4) - 5 = 3$. Composition works from the inside out. Multiplying 4 by 2 and subtracting 5 gives 3.

9. B — Square both sides: $4x + 1 = 25$, so $4x = 24$ and $x = 6$. Squaring removes the radical. Checking, $\sqrt{4 \cdot 6 + 1} = \sqrt{25} = 5$ confirms the solution.

10. D — Average rate of change is $[f(5) - f(2)]/(5 - 2) = (26 - 5)/3 = 21/3 = 7$. This is the slope of the secant line over the interval. With $f(5) = 26$ and $f(2) = 5$, the result is 7.

11. A — In $a(1 + r)^t$, the base 1.12 equals $1 + 0.12$, so the growth rate is 12%. A base above 1 signals growth, and the base minus 1 gives the rate. The decimal 0.12 converts to 12%.

12. C — Combine the logs: $\log_2(5x) = 4$, so $5x = 2^4 = 16$ and $x = 16/5 = 3.2$. The product rule for logarithms merges the terms. Solving the exponential form gives 3.2.

13. D — The discriminant is $b^2 - 4ac = 100 - 100 = 0$, indicating one repeated real solution. Since $x^2 - 10x + 25 = (x - 5)^2$, the repeated root 5 is rational. A zero discriminant always yields a single repeated solution.

14. B — The cosine of $\pi/4$ (45°) is a standard unit-circle value equal to $\sqrt{2}/2$. In a 45-45-90 triangle the legs are equal, giving this ratio. The value is $\sqrt{2}/2$.

15. A — In $y = a \sin(bx) + d$, the amplitude is $|a|$, which is 6. The amplitude is the coefficient of the sine term. The vertical shift of -2 does not affect it.

16. C — The nth term is $a_n = a_1 + (n - 1)d$, so $a_{15} = 7 + 14(-2) = 7 - 28 = -21$. The common difference applied fourteen times lowers the term. The fifteenth term is -21.

17. D — To invert, solve $y = x^3 + 1$ for x: $x^3 = y - 1$, so $x = \sqrt[3]{y - 1}$ and $f^{-1}(x) = \sqrt[3]{x - 1}$. The inverse undoes cubing and adding 1 in reverse order. The result is $\sqrt[3]{x - 1}$.

18. B — The flips are independent, so multiply the probabilities: $(1/2)(1/2)(1/2) = 1/8$. Each flip has a 1/2 chance of heads. The probability of three heads is 1/8.

19. A — A value of 170 is one standard deviation above the mean of 150. Since 50% lies above the mean and 34% lies between the mean and +1 SD, about 16% lies above +1 SD. The empirical rule gives the tail percentage.

20. C — In Quadrant II sine is positive while cosine is negative, and the 8-15-17 triangle gives the sine ratio. Therefore $\sin \theta = 8/17$. The quadrant makes the sine positive.

21. D — Complete the square: $x^2 - 12x + 40 = (x^2 - 12x + 36) - 36 + 40 = (x - 6)^2 + 4$. Half of -12 squared is 36, added and subtracted to preserve value. The vertex form reveals the vertex at (6, 4).

22. A — The common ratio is 1/4, and an infinite geometric series with $|r| < 1$ sums to $a_1/(1 - r) = 64/(1 - 1/4) = 64/(3/4) \approx 85.33$. The ratio ensures convergence. The series sums to about 85.33.

23. C — Take the log of both sides: $x = \log_5(200) = \ln(200)/\ln(5) \approx 3.29$. The change-of-base formula converts to natural logs. The result rounds to 3.29.

24. B — Random assignment makes the treatment and placebo groups comparable on both known and unknown variables, so any difference in outcome can be attributed to the treatment. This is what gives a clinical trial its causal power. Without randomization, group differences could explain the results.

Part II (Short Constructed Response)

25. $x = -1 \pm 4i$ — Quadratic formula: $x = [-2 \pm \sqrt{(4 - 68)}]/2 = [-2 \pm \sqrt{(-64)}]/2 = [-2 \pm 8i]/2 = -1 \pm 4i$. The negative discriminant produces complex conjugates. Dividing each term by 2 gives the simplified a + bi form.

26. $x = -2$; no extraneous solutions — Cross-multiplying gives $4x = x - 6$, which simplifies to $3x = -6$, so $x = -2$. The restrictions are $x \neq 6$ and $x \neq 0$, and $x = -2$ violates neither, so it is valid.

27. Yes, $(x + 1)$ is a factor — By the Factor Theorem, $f(-1) = -1 + 2 + 1 - 2 = 0$. A remainder of zero confirms that $(x + 1)$ divides $f(x)$ evenly. A nonzero value would have meant it is not a factor.

28. $g(x) = \sqrt{(x + 2)} - 5$ — Translating left 2 units replaces x with $(x + 2)$; translating down 5 units subtracts 5. Combining these gives $\sqrt{(x + 2)} - 5$. The horizontal shift moves opposite the sign inside the radical.

29. $x = 2$ — Write the right side as a power of 3: $81 = 3^4$, so $3^{(2x)} = 3^4$ requires $2x = 4$. Solving gives $x = 2$. Equal bases allow the exponents to be set equal.

30. $(7x + 5) / [(x + 5)(x - 5)]$, $x \neq -5$, $x \neq 5$ — Over the common denominator $(x + 5)(x - 5)$: $3(x - 5) + 4(x + 5) = 3x - 15 + 4x + 20 = 7x + 5$. The numerator combines to $7x + 5$, which does not factor further. Restrictions come from the original denominators.

31. $a_4 = 67$ — Apply the recursion: $a_2 = 3(2) + 1 = 7$, $a_3 = 3(7) + 1 = 22$, $a_4 = 3(22) + 1 = 67$. Each term triples the previous term and adds 1. Building up term by term reaches 67.

32. $\cos\theta = -8/17$ — With $\tan\theta = 15/8$, the 8-15-17 Pythagorean triple gives the hypotenuse 17 and adjacent leg 8. In Quadrant III cosine is negative, so $\cos\theta = -8/17$. The quadrant fixes the negative sign.

Part III (Extended Constructed Response)

33. (a) \$4,000, 2.5% growth; (b) ≈ 9.0 years — The coefficient 4000 is the initial balance, and the base $1.025 = 1 + 0.025$ indicates 2.5% annual growth. For part (b), $5000 = 4000(1.025)^t$ gives $1.025^t = 1.25$, so $t = \ln(1.25)/\ln(1.025) \approx 9.0$ years. Logarithms isolate the exponent.

34. (a) $h(x) = -6.0x^2 + 36.0x + 2.0$; (b) 56.0 m at $x = 3.0$ m — Quadratic regression on the data returns $a = -6.0$, $b = 36.0$, $c = 2.0$. The vertex occurs at $x = -b/(2a) = -36/(-12) = 3.0$ meters, and $h(3) = -54 + 108 + 2 = 56.0$ meters. The negative leading coefficient confirms a maximum.

35. (a) $(-1 + \sqrt{19}, 1 + \sqrt{19})$ and $(-1 - \sqrt{19}, 1 - \sqrt{19})$ — Substituting $y = x + 2$ into the circle gives $x^2 + (x + 2)^2 = 40$, which simplifies to $2x^2 + 4x - 36 = 0$, then $x^2 + 2x - 18 = 0$, so $x = -1 \pm \sqrt{19}$. The y-values follow from $y = x + 2$. **(b)** Checking $(-1 + \sqrt{19}, 1 + \sqrt{19})$: $x^2 + y^2 = (20 - 2\sqrt{19}) + (20 + 2\sqrt{19}) = 40$, and $y = x + 2 = (-1 + \sqrt{19}) + 2 = 1 + \sqrt{19}$, so both equations hold.

Part IV (Long Constructed Response)

36. (a) $h(t) = -0.4 \cos(2\pi t/3) + 0.8$ — The amplitude is $(1.2 - 0.4)/2 = 0.4$ and the midline is $(1.2 + 0.4)/2 = 0.8$. The period of 3 seconds gives $b = 2\pi/3$, and starting at the lowest height at $t = 0$ calls for a negative cosine. This yields $h(t) = -0.4 \cos(2\pi t/3) + 0.8$.

(b) Amplitude 0.4, period 3 s, midline $h = 0.8$ — The amplitude of 0.4 meters is the distance from the midline to the highest or lowest point of the motion. The period of 3 seconds is the time for one complete up-and-down cycle. The midline $h = 0.8$ meters is the average height about which the horse oscillates.

(c) $t \approx 1.15$ and 1.85 seconds — Setting $1 = -0.4 \cos(2\pi t/3) + 0.8$ gives $\cos(2\pi t/3) = -0.75$. Over the first 3 seconds, $2\pi t/3 = 2.4189$ and 3.8643 radians, so $t = 1.15$ and 1.85 seconds. These are the two times the rising and falling horse passes a height of 1 meter.