PRACTICE EXAM 50: ALGEBRA II REGENTS SIMULATION

PART I — Multiple Choice (24 questions × 2 credits = 48 credits)

1. Express the difference (5 + 2i) − (3 − 7i) in a + bi form.

A. 2 − 5i

B. 2 + 9i

C. 8 − 5i

D. 2 − 9i

2. What is the value of 16^(5/4)?

A. 20

B. 64

C. 80

D. 32

3. What are the solutions to the equation $x^2 - 2x + 5 = 0$?

A. x = 1 ± 2i

B. $x = -1 \pm 2i$

C. $x = 1 \pm 4i$

D. $x = 2 \pm 2i$

4. Factored completely, the expression $2x^3 + x^2 - 8x - 4$ is equivalent to

A. $(2x + 1)(x^2 - 4)$

B. $(2x - 1)(x - 2)(x + 2)$

C. $(2x + 1)(x - 2)(x + 2)$

D. $(2x + 1)(x - 4)(x + 1)$

5. What is the value of $\log_9(3)$?

A. 3

B. 1/2

C. 2

D. 1/3

6. What is the sum of the finite geometric series 1 + 4 + 16 + 64 + 256?

A. 341

B. 256

C. 1024

D. 320

7. When $p(x) = 2x^3 + 3x^2 - x + 4$ is divided by $(x + 1)$, the remainder is

A. 8

B. 4

C. 2

D. 6

8. If $f(x) = x + 5$ and $g(x) = 3x$, what is the value of $g(f(-2))$?

A. 1

B. −1

C. 9

D. 13

9. What is the solution to the equation $2\sqrt{(x + 3)} = 10$?

A. $x = 22$

B. $x = 25$

C. $x = 5$

D. $x = 2$

10. What is the average rate of change of $f(x) = x^3 - x$ over the interval [0, 2]?

A. 6

B. 3

C. 2

D. 4

11. \$3000 is invested at an annual interest rate of 3% compounded annually. What is the value of the investment after 4 years?

A. \$3360.00

B. \$3090.00

C. \$3376.53

D. \$3450.00

12. What is the solution to the equation $\log_6(2x) = 2$?

A. $x = 6$

B. $x = 12$

C. $x = 9$

D. $x = 18$

13. Which statement best describes the nature of the solutions of $x^2 + x + 1 = 0$?

A. Two distinct rational solutions

B. Two complex (non-real) solutions

C. Two distinct irrational solutions

D. One repeated rational solution

14. What is the exact value of $\sin(\pi/3)$?

A. $\sqrt{3}/2$

B. 1/2

C. $\sqrt{2}/2$

D. $-\sqrt{3}/2$

15. For the function $y = 4 \sin(\pi x)$, what is the period?

A. π

B. 4

C. 2

D. 2π

16. In an arithmetic sequence, the fourth term is 14 and the ninth term is 34. What is the first term?

A. 6

B. 4

C. 10

D. 2

17. If $f(x) = \log_5(x)$, what is $f^{-1}(x)$?

A. 5^x

B. x^5

C. log_x(5)

D. $(1/5)^x$

18. In how many ways can a starting team of 5 players be chosen from a roster of 10 players?

A. 30240

B. 252

C. 50

D. 100000

19. A data set is normally distributed with a mean of 200 and a standard deviation of 25. Approximately what percent of the data falls between 175 and 225?

A. 95%

B. 34%

C. 50%

D. 68%

20. Given that $\sin \theta = 9/41$ and θ is in Quadrant II, what is the value of $\tan \theta$?

A. 9/40

B. 40/9

C. −9/40

D. −40/9

21. Written in vertex form, the expression $3x^2 + 12x + 5$ is equivalent to

A. $3(x + 2)^2 - 7$

B. $3(x + 2)^2 + 5$

C. $(x + 6)^2 - 31$

D. $3(x + 4)^2 - 43$

22. What is the sum of the infinite geometric series 100 + 20 + 4 + …?

A. 120

B. 150

C. 100

D. 125

23. What is the solution to the equation $2^x = 45$, rounded to the nearest hundredth?

A. $x = 22.50$

B. $x = 6.00$

C. $x = 5.49$

D. $x = 4.50$

24. A company surveys customers by asking, "Don't you agree that our excellent product is worth the price?" What is the problem with this survey question?

A. The sample is too small

B. It is a leading question that biases respondents toward agreement

C. The question is too short to be valid

D. There is no problem with this question

PART II — Short Constructed Response (8 questions × 2 credits = 16 credits)

Show all work. A correct answer with no supporting work will receive only 1 credit.

25. Solve the equation $x^2 - 8x + 20 = 0$ algebraically. Express your solutions in $a + bi$ form.

26. Solve algebraically for x: $3 / (x + 2) = 6 / (x + 8)$. State any restrictions on the variable and identify any extraneous solutions that must be rejected.

27. Given $f(x) = x^3 - 4x^2 + 5x - 2$, determine whether $(x - 2)$ is a factor of $f(x)$. Justify your answer using the Remainder Theorem or the Factor Theorem.

28. Write an equation, in the form $g(x) = a \cdot f(x) + k$, for the function obtained by reflecting $f(x) = |x|$ across the x-axis and then translating the result 4 units down.

29. Solve algebraically for x: $8^x = 4$.

30. Express $2x / (x - 5) + 3 / (x + 1)$ as a single rational expression in simplest form. State any restrictions on the variable.

31. A sequence is defined recursively by $a_1 = 3$ and $a_n = 2a_{n-1} - 5$ for $n \geq 2$. Find the value of a_4.

32. Given that $\sin \theta = 12/13$ and θ terminates in Quadrant II, find the exact value of $\cos \theta$. Show the algebraic work that justifies your answer.

PART III — Extended Constructed Response (3 questions × 4 credits = 12 credits)

Show all work. Partial credit is awarded according to the scoring rubric.

33. The value of a piece of equipment is modeled by the function $V(t) = 25000(0.88)^t$, where $V(t)$ is the value in dollars and t is the time in years.

(a) State the initial value of the equipment and the annual percent rate of depreciation.

(b) Algebraically determine the number of years it will take for the value to fall to \$10000. Round your answer to the nearest tenth of a year.

34. The table below shows the height $h(x)$, in meters, of a thrown javelin at horizontal distance x meters from the thrower.

x (meters)	0	1	2	3	4
h(x) (meters)	4	9	12	13	12

(a) Using regression, write a quadratic function $h(x) = ax^2 + bx + c$ that best models the data. Round each coefficient to the nearest tenth.

(b) Use the regression model from part (a) to determine the maximum height of the javelin and the horizontal distance at which the maximum height occurs. Round each answer to the nearest tenth.

35. A circle is defined by the equation $x^2 + y^2 = 68$, and a line is defined by the equation $y = x + 2$.

(a) Algebraically determine all points of intersection between the circle and the line. Express coordinates in exact form (radicals permitted).

(b) Verify your answer by substituting one of the intersection points back into both original equations. Show the substitution clearly.

PART IV — Long Constructed Response (1 question × 6 credits = 6 credits)

Show all work. This problem requires multiple steps and integrates concepts from several chapters.

36. A wind turbine blade tip rotates in a circle. The tip's height above the ground ranges from a minimum of 15 meters to a maximum of 65 meters, completing one full rotation every 8 seconds. At time $t = 0$, the blade tip is at its maximum height.

(a) Write a function $h(t)$ that models the height of the blade tip in meters above the ground as a function of time t in seconds.

(b) State the amplitude, period, and midline of the function $h(t)$, and explain what each represents in the context of the rotating blade tip.

(c) Algebraically determine all times during the first 8 seconds at which the blade tip is at a height of 50 meters. Round each answer to the nearest hundredth of a second. Show all algebraic work.

ANSWER KEY WITH EXPLANATIONS – PRACTICE EXAM 50

1. B — Combine real and imaginary parts, distributing the subtraction: $(5 - 3) + (2 - (-7))i = 2 + 9i$. Subtracting $-7i$ adds $7i$, raising the imaginary part to $9i$. The result is $2 + 9i$.

2. D — Apply the fourth root, then raise to the fifth: $16^{(5/4)} = (16^{(1/4)})^5 = 2^5 = 32$. The denominator 4 takes the fourth root and the numerator 5 raises it to the fifth power. The value is 32.

3. A — Quadratic formula: $x = [2 \pm \sqrt{(4 - 20)}]/2 = [2 \pm \sqrt{(-16)}]/2 = [2 \pm 4i]/2 = 1 \pm 2i$. The negative discriminant produces the imaginary part. Dividing both terms by 2 gives the simplified form.

4. C — Group and factor: $x^2(2x + 1) - 4(2x + 1) = (2x + 1)(x^2 - 4)$, then the difference of squares gives $(2x + 1)(x - 2)(x + 2)$. Complete factoring requires breaking down $x^2 - 4$. The fully factored form has three factors.

5. B — A logarithm asks for the exponent on the base: $9^{(1/2)} = 3$, so $\log_9(3) = 1/2$. The square root of 9 is 3. The result is 1/2.

6. A — Using $S = a_1(r^n - 1)/(r - 1) = (4^5 - 1)/(4 - 1) = 1023/3 = 341$. The common ratio is 4 and there are five terms. Direct addition of the terms also gives 341.

7. D — By the Remainder Theorem, the remainder is $p(-1) = -2 + 3 + 1 + 4 = 6$. Substituting the zero of the divisor avoids long division. The remainder is 6.

8. C — Evaluate the inner function first: $f(-2) = -2 + 5 = 3$, then $g(3) = 3(3) = 9$. Composition works from the inside out. Multiplying 3 by 3 gives 9.

9. A — Divide by 2, then square: $\sqrt{x + 3} = 5$, so $x + 3 = 25$ and $x = 22$. Isolating the radical before squaring is essential. Checking, $2\sqrt{25} = 2(5) = 10$ confirms the solution.

10. B — Average rate of change is $[f(2) - f(0)]/(2 - 0) = (6 - 0)/2 = 3$. This is the slope of the secant line over the interval. With $f(2) = 6$ and $f(0) = 0$, the result is 3.

11. C — Apply $A = P(1 + r)^t = 3000(1.03)^4 = 3000(1.1255) \approx 3376.53$. Compounding multiplies by 1.03 each year for four years. The value grows to about \$3376.53.

12. D — Rewrite in exponential form: $2x = 6^2 = 36$, so $x = 18$. Converting the logarithmic equation isolates x. The base 6 raised to 2 equals 36.

13. B — The discriminant is $b^2 - 4ac = 1 - 4 = -3$, which is negative. A negative discriminant produces two complex conjugate solutions. No real solutions exist.

14. A — The sine of $\pi/3$ (60°) is a standard unit-circle value equal to $\sqrt{3}/2$. The 30-60-90 reference triangle gives this ratio. The value is $\sqrt{3}/2$.

15. C — The period of $\sin(bx)$ is $2\pi/b$; with $b = \pi$, the period is $2\pi/\pi = 2$. The coefficient of x sets the period. The period is 2.

16. D — The common difference is $d = (34 - 14)/(9 - 4) = 20/5 = 4$. Working back, $a_1 = a_4 - 3d = 14 - 12 = 2$. Subtracting three common differences recovers the first term.

17. A — The inverse of a logarithm with base 5 is the exponential function with base 5: $f^{-1}(x) = 5^x$. Exponentials and logarithms are inverse operations. The inverse is 5^x.

18. B — Order does not matter for a team, so use combinations: $C(10, 5) = 252$. The combination formula counts unordered selections. There are 252 possible teams.

19. D — The values 175 and 225 are one standard deviation below and above the mean of 200. By the empirical rule, about 68% of data lies within ±1 SD. The interval captures 68%.

20. C — In Quadrant II sine is positive and cosine is negative, and the 9-40-41 triangle gives $\cos\theta = -40/41$. Then $\tan\theta = \sin/\cos = (9/41)/(-40/41) = -9/40$. The quadrant makes the tangent negative.

21. A — Factor 3 from the variable terms, then complete the square: $3(x^2 + 4x) + 5 = 3(x^2 + 4x + 4) - 12 + 5 = 3(x + 2)^2 - 7$. The subtracted 12 accounts for the factor of 3 times the added 4. The vertex form reveals the vertex at $(-2, -7)$.

22. D — The common ratio is 1/5, and an infinite geometric series with $|r| < 1$ sums to $a_1/(1 - r) = 100/(1 - 1/5) = 100/(4/5) = 125$. The ratio ensures convergence. The series sums to 125.

23. C — Take the log of both sides: $x = \log_2(45) = \ln(45)/\ln(2) \approx 5.49$. The change-of-base formula converts to natural logs. The result rounds to 5.49.

24. B — The wording "our excellent product is worth the price" presupposes the conclusion and pressures respondents to agree. A leading question biases the responses toward the desired answer. Neutral phrasing is required for valid survey data.

Part II (Short Constructed Response)

25. $x = 4 \pm 2i$ — Quadratic formula: $x = [8 \pm \sqrt{(64 - 80)}]/2 = [8 \pm \sqrt{(-16)}]/2 = [8 \pm 4i]/2 = 4 \pm 2i$. The negative discriminant produces complex conjugates. Dividing each term by 2 gives the simplified $a + bi$ form.

26. $x = 4$; no extraneous solutions — Cross-multiplying gives $3(x + 8) = 6(x + 2)$, which expands to $3x + 24 = 6x + 12$ and simplifies to $-3x = -12$, so $x = 4$. The restrictions are $x \neq -2$ and $x \neq -8$, and $x = 4$ violates neither, so it is valid.

27. Yes, $(x - 2)$ is a factor — By the Factor Theorem, $f(2) = 8 - 16 + 10 - 2 = 0$. A remainder of zero confirms that $(x - 2)$ divides $f(x)$ evenly. A nonzero value would have meant it is not a factor.

28. $g(x) = -|x| - 4$ — Reflecting across the x-axis negates the function to $-|x|$; translating 4 units down subtracts 4. Combining these produces $-|x| - 4$. The reflection flips the V-shape to open downward.

29. $x = 2/3$ — Write both sides as powers of 2: $8^x = 2^{(3x)}$ and $4 = 2^2$, so $2^{(3x)} = 2^2$ requires $3x = 2$. Solving gives $x = 2/3$. Equal bases allow the exponents to be set equal.

30. $(2x^2 + 5x - 15) / [(x - 5)(x + 1)]$, $x \neq 5$, $x \neq -1$ — Over the common denominator $(x - 5)(x + 1)$: $2x(x + 1) + 3(x - 5) = 2x^2 + 2x + 3x - 15 = 2x^2 + 5x - 15$. The numerator combines to $2x^2 + 5x - 15$, which does not factor further. Restrictions come from the original denominators.

31. $a_4 = -11$ — Apply the recursion: $a_2 = 2(3) - 5 = 1$, $a_3 = 2(1) - 5 = -3$, $a_4 = 2(-3) - 5 = -11$. Each term doubles the previous term and subtracts 5. Building up term by term reaches -11.

32. $\cos \theta = -5/13$ — With $\sin \theta = 12/13$, the 5-12-13 Pythagorean triple gives the adjacent leg of 5. In Quadrant II cosine is negative, so $\cos \theta = -5/13$. The quadrant fixes the negative sign.

Part III (Extended Constructed Response)

33. (a) \$25,000, 12% depreciation; (b) ≈ 7.2 years — The coefficient 25000 is the initial value, and the base $0.88 = 1 - 0.12$ indicates 12% annual depreciation. For part (b), $10000 = 25000(0.88)^t$ gives $0.88^t = 0.4$, so $t = \ln(0.4)/\ln(0.88) \approx 7.2$ years. Logarithms isolate the exponent.

34. (a) $h(x) = -1.0x^2 + 6.0x + 4.0$; (b) 13.0 m at $x = 3.0$ m — Quadratic regression on the data returns $a = -1.0$, $b = 6.0$, $c = 4.0$. The vertex occurs at $x = -b/(2a) = -6/(-2) = 3.0$ meters, and $h(3) = -9 + 18 + 4 = 13.0$ meters. The negative leading coefficient confirms a maximum.

35. (a) $(-1 + \sqrt{33}, 1 + \sqrt{33})$ and $(-1 - \sqrt{33}, 1 - \sqrt{33})$ — Substituting $y = x + 2$ into the circle gives $x^2 + (x + 2)^2 = 68$, which simplifies to $2x^2 + 4x - 64 = 0$, then $x^2 + 2x - 32 = 0$, so $x = -1 \pm \sqrt{33}$. The y-values follow from $y = x + 2$. **(b)** Checking $(-1 + \sqrt{33}, 1 + \sqrt{33})$: $x^2 + y^2 = (34 - 2\sqrt{33}) + (34 + 2\sqrt{33}) = 68$, and $y = x + 2 = (-1 + \sqrt{33}) + 2 = 1 + \sqrt{33}$, so both equations hold.

Part IV (Long Constructed Response)

36. (a) $h(t) = 25 \cos(\pi t/4) + 40$ — The amplitude is $(65 - 15)/2 = 25$ and the midline is $(65 + 15)/2 = 40$. The period of 8 seconds gives $b = 2\pi/8 = \pi/4$, and starting at the maximum height at $t = 0$ calls for a positive cosine. This yields $h(t) = 25 \cos(\pi t/4) + 40$.

(b) Amplitude 25, period 8 s, midline $h = 40$ — The amplitude of 25 meters equals the blade's rotational radius, the distance from the hub to the tip. The period of 8 seconds is the time for one full rotation. The midline $h = 40$ meters is the hub height above the ground, about which the tip's height oscillates.

(c) $t \approx 1.48$ and 6.52 seconds — Setting $50 = 25 \cos(\pi t/4) + 40$ gives $\cos(\pi t/4) = 0.4$. Over the first 8 seconds, $\pi t/4 = 1.1593$ and 5.1239 radians, so $t = 1.48$ and 6.52 seconds. These are the two times the descending and ascending blade tip passes a height of 50 meters.