PRACTICE EXAM 49: ALGEBRA II REGENTS SIMULATION

PART I — Multiple Choice (24 questions × 2 credits = 48 credits)

1. What is the value of i^{100}?

A. i

B. −1

C. 1

D. −i

2. What is the value of 27^(4/3)?

A. 36

B. 108

C. 12

D. 81

3. What are the solutions to the equation $x^2 + 10x + 29 = 0$?

A. x = 5 ± 2i

B. $x = -5 \pm 2i$

C. $x = -5 \pm 4i$

D. $x = -10 \pm 2i$

4. Factored completely, the expression $x^3 + 125$ is equivalent to

A. $(x + 5)(x^2 - 5x + 25)$

B. $(x + 5)(x^2 + 5x + 25)$

C. $(x - 5)(x^2 + 5x + 25)$

D. $(x + 5)(x^2 - 25)$

5. What is the value of $\log_4(1/16)$?

A. 2

B. −1/2

C. −2

D. 4

6. What is the 4th term of a geometric sequence whose first term is 5 and whose common ratio is 3?

A. 405

B. 135

C. 45

D. 60

7. When $p(x) = x^3 - 5x + 6$ is divided by $(x + 2)$, the remainder is

A. 0

B. 4

C. −4

D. 8

8. If $f(x) = x^2 + 1$ and $g(x) = 2x - 3$, what is the value of $f(g(2))$?

A. 2

B. 5

C. 10

D. 1

9. What is the solution to the equation $\sqrt{(x - 4)} + 3 = 8$?

A. $x = 21$

B. $x = 29$

C. $x = 25$

D. $x = 9$

10. What is the average rate of change of $f(x) = 2x^2$ over the interval $[-1, 3]$?

A. 16

B. 8

C. 4

D. 5

11. A population of 600 deer decreases by 8% each year. Which function models the deer population after t years?

A. $f(t) = 600(0.92)^t$

B. $f(t) = 600(1.08)^t$

C. $f(t) = 600(0.08)^t$

D. $f(t) = 600 - 8t$

12. What is the solution to the equation $\log_3(x) - \log_3(2) = 2$?

A. $x = 6$

B. $x = 11$

C. $x = 9$

D. $x = 18$

13. Which statement best describes the nature of the solutions of $4x^2 + 4x + 1 = 0$?

A. Two distinct rational solutions

B. Two distinct irrational solutions

C. One repeated rational solution

D. Two complex (non-real) solutions

14. What is the exact value of $\tan(\pi/6)$?

A. $\sqrt{3}$

B. $\sqrt{3}/3$

C. 1/2

D. 1

15. For the function $y = 3\cos(x) - 5$, what is the equation of the midline?

A. $y = 3$

B. $y = 5$

C. $y = -2$

D. $y = -5$

16. What is the sum of the first 12 terms of an arithmetic sequence whose first term is 4 and whose common difference is 5?

A. 378

B. 384

C. 372

D. 756

17. If $f(x) = 5x - 2$, what is $f^{-1}(x)$?

A. $5x + 2$

B. $(x - 2)/5$

C. $(x + 2)/5$

D. $1 / (5x - 2)$

18. A jar contains 5 red marbles and 3 green marbles. If two marbles are drawn at random without replacement, what is the probability that both are red?

A. 25/64

B. 5/14

C. 5/8

D. 1/2

19. A data set is normally distributed with a mean of 75 and a standard deviation of 6. What value corresponds to a z-score of -1?

A. 69

B. 81

C. 74

D. 6

20. Given that $\tan \theta = -24/7$ and θ is in Quadrant IV, what is the value of $\cos \theta$?

A. $-7/25$

B. $24/25$

C. $-24/25$

D. $7/25$

21. Written in vertex form, the expression $x^2 + 8x + 10$ is equivalent to

A. $(x + 4)^2 + 10$

B. $(x + 4)^2 - 6$

C. $(x - 4)^2 - 6$

D. $(x + 8)^2 - 54$

22. What is the sum of the infinite geometric series $50 + 10 + 2 + \ldots$?

A. 60

B. 75

C. 62.5

D. 250

23. What is the solution to the equation $3^x = 50$, rounded to the nearest hundredth?

A. $x = 3.56$

B. $x = 16.67$

C. $x = 2.30$

D. $x = 4.00$

24. To test whether a new fertilizer increases crop yield, which study design gives the strongest evidence of a cause-and-effect relationship?

A. Surveying farmers who already use the fertilizer

B. Comparing this year's yield to last year's with no other controls

C. Observing fields that happen to use the fertilizer

D. A randomized experiment assigning plots to fertilizer and control groups

PART II — Short Constructed Response (8 questions × 2 credits = 16 credits)

Show all work. A correct answer with no supporting work will receive only 1 credit.

25. Solve the equation $x^2 - 4x + 53 = 0$ algebraically. Express your solutions in $a + bi$ form.

26. Solve algebraically for x: $5 / (x - 1) = 2 / (x + 5)$. State any restrictions on the variable and identify any extraneous solutions that must be rejected.

27. Given $f(x) = x^3 - 3x^2 - 6x + 8$, determine whether $(x - 4)$ is a factor of $f(x)$. Justify your answer using the Remainder Theorem or the Factor Theorem.

28. Write an equation, in the form $g(x) = a \cdot f(x) + k$, for the function obtained by reflecting $f(x) = x^2$ across the x-axis and then translating the result 6 units up.

29. Solve algebraically for x: 5^(3x) = 125.

30. Express $6 / (x^2 - 1) - 1 / (x - 1)$ as a single rational expression in simplest form. State any restrictions on the variable.

31. A sequence is defined recursively by $a_1 = 1$ and $a_n = 4a_{n-1} + 2$ for $n \geq 2$. Find the value of a_4.

32. Given that $\cos \theta = 40/41$ and θ terminates in Quadrant I, find the exact value of $\tan \theta$. Show the algebraic work that justifies your answer.

PART III — Extended Constructed Response (3 questions × 4 credits = 12 credits)

Show all work. Partial credit is awarded according to the scoring rubric.

33. The balance of a savings account is modeled by the function $A(t) = 1500(1.05)^t$, where $A(t)$ is the balance in dollars and t is the time in years.

(a) State the initial balance of the account and the annual percent rate of growth.

(b) Algebraically determine the number of years it will take for the balance to double to \$3000. Round your answer to the nearest tenth of a year.

34. The table below shows the height h(x), in meters, of a fireworks shell at horizontal distance x meters from the launch point.

x (meters)	0	1	2	3	4
h(x) (meters)	1	26	41	46	41

(a) Using regression, write a quadratic function $h(x) = ax^2 + bx + c$ that best models the data. Round each coefficient to the nearest tenth.

(b) Use the regression model from part (a) to determine the maximum height of the shell and the horizontal distance at which the maximum height occurs. Round each answer to the nearest tenth.

35. A circle is defined by the equation $x^2 + y^2 = 58$, and a line is defined by the equation $y = x + 6$.

(a) Algebraically determine all points of intersection between the circle and the line. Express coordinates in exact form (radicals permitted).

(b) Verify your answer by substituting one of the intersection points back into both original equations. Show the substitution clearly.

PART IV — Long Constructed Response (1 question × 6 credits = 6 credits)

Show all work. This problem requires multiple steps and integrates concepts from several chapters.

36. A buoy bobs up and down on the waves. Its height relative to the average water level ranges from a maximum of 1.5 meters above to a minimum of 1.5 meters below that level, completing one full bob every 5 seconds. At time t = 0, the buoy is at the average water level and moving upward.

(a) Write a function h(t) that models the height of the buoy in meters relative to the average water level as a function of time t in seconds.

(b) State the amplitude, period, and midline of the function h(t), and explain what each represents in the context of the bobbing buoy.

(c) Algebraically determine all times during the first 5 seconds at which the buoy is 1 meter above the average water level. Round each answer to the nearest hundredth of a second. Show all algebraic work.

ANSWER KEY WITH EXPLANATIONS – PRACTICE EXAM 49

1. C — Powers of i cycle every four: 100 divided by 4 leaves remainder 0, so $i^{100} = i^0 = 1$. A remainder of zero corresponds to a full cycle. The value is 1.

2. D — Apply the cube root, then raise to the fourth: $27^{(4/3)} = (27^{(1/3)})^4 = 3^4 = 81$. The denominator 3 takes the cube root and the numerator 4 raises it to the fourth power. The value is 81.

3. B — Quadratic formula: $x = [-10 \pm \sqrt{(100 - 116)}]/2 = [-10 \pm \sqrt{(-16)}]/2 = [-10 \pm 4i]/2 = -5 \pm 2i$. The negative discriminant produces the imaginary part. Dividing both terms by 2 gives the simplified form.

4. A — This is a sum of cubes, $x^3 + 5^3 = (x + 5)(x^2 - 5x + 25)$. The middle term of the quadratic factor is $-5x$, making it negative. The sign pattern distinguishes it from a difference of cubes.

5. C — Rewrite the argument as a power of 4: $1/16 = 4^{-2}$, so $\log_4(1/16) = -2$. The negative exponent reflects a value below 1. The result is −2.

6. B — The nth term is $a_n = a_1 \cdot r^{(n-1)}$, so $a_4 = 5(3^3) = 5(27) = 135$. The ratio applied three times scales the term. The fourth term is 135.

7. D — By the Remainder Theorem, the remainder is $p(-2) = -8 + 10 + 6 = 8$. Substituting the zero of the divisor avoids long division. The remainder is 8.

8. A — Evaluate the inner function first: $g(2) = 2(2) - 3 = 1$, then $f(1) = 1^2 + 1 = 2$. Composition works from the inside out. Squaring 1 and adding 1 gives 2.

9. B — Subtract 3, then square: $\sqrt{(x - 4)} = 5$, so $x - 4 = 25$ and $x = 29$. Isolating the radical before squaring is essential. Checking, $\sqrt{25} + 3 = 5 + 3 = 8$ confirms the solution.

10. C — Average rate of change is $[f(3) - f(-1)]/(3 - (-1)) = (18 - 2)/4 = 16/4 = 4$. This is the slope of the secant line over the interval. With $f(3) = 18$ and $f(-1) = 2$, the result is 4.

11. A — A quantity that decreases by 8% uses base $1 - 0.08 = 0.92$, with 600 as the initial population: $f(t) = 600(0.92)^t$. A base below 1 signals decay. This models the declining deer population correctly.

12. D — Combine using the quotient rule: $\log_3(x/2) = 2$, so $x/2 = 3^2 = 9$ and $x = 18$. The difference of logs becomes a quotient. Solving the exponential form gives 18.

13. C — The discriminant is $b^2 - 4ac = 16 - 16 = 0$, indicating one repeated real solution. Since $4x^2 + 4x + 1 = (2x + 1)^2$, the repeated root $-1/2$ is rational. A zero discriminant always yields a single repeated solution.

14. B — The tangent of $\pi/6$ (30°) is $1/\sqrt{3}$, which rationalizes to $\sqrt{3}/3$. The 30-60-90 reference triangle gives this ratio. The value is $\sqrt{3}/3$.

15. D — In $y = a \cos(x) + d$, the midline is $y = d$, which is $y = -5$. The vertical shift sets the midline. The amplitude of 3 does not affect it.

16. A — Using $S = n/2(2a_1 + (n - 1)d) = 12/2(8 + 11 \cdot 5) = 6(8 + 55) = 6(63) = 378$. The formula sums the arithmetic series directly. The sum of the first 12 terms is 378.

17. C — To invert, solve $y = 5x - 2$ for x: $x = (y + 2)/5$, so $f^{-1}(x) = (x + 2)/5$. The inverse undoes multiplication by 5 and subtraction of 2 in reverse order. The result is $(x + 2)/5$.

18. B — Without replacement, multiply the probabilities: $(5/8)(4/7) = 20/56 = 5/14$. The second draw has one fewer red marble and one fewer total. The probability both are red is 5/14.

19. A — Rearrange the z-score formula: value = mean + z · SD = $75 + (-1)(6) = 69$. A z-score of -1 sits one standard deviation below the mean. The value is 69.

20. D — In Quadrant IV cosine is positive while sine is negative, and the 7-24-25 triangle gives the cosine ratio. Therefore $\cos \theta = 7/25$. The quadrant makes the cosine positive.

21. B — Complete the square: $x^2 + 8x + 10 = (x^2 + 8x + 16) - 16 + 10 = (x + 4)^2 - 6$. Half of 8 squared is 16, added and subtracted to preserve value. The vertex form reveals the vertex at $(-4, -6)$.

22. C — The common ratio is 1/5, and an infinite geometric series with $|r| < 1$ sums to $a_1/(1 - r) = 50/(1 - 1/5) = 50/(4/5) = 62.5$. The ratio ensures convergence. The series sums to 62.5.

23. A — Take the log of both sides: $x = \log_3(50) = \ln(50)/\ln(3) \approx 3.56$. The change-of-base formula converts to natural logs. The result rounds to 3.56.

24. D — A randomized experiment randomly assigns plots to fertilizer and control groups, balancing out confounding variables and isolating the fertilizer's effect. Random assignment is what permits a causal conclusion. The observational options cannot rule out other explanations.

Part II (Short Constructed Response)

25. $x = 2 \pm 7i$ — Quadratic formula: $x = [4 \pm \sqrt{16 - 212}]/2 = [4 \pm \sqrt{-196}]/2 = [4 \pm 14i]/2 = 2 \pm 7i$. The negative discriminant produces complex conjugates. Dividing each term by 2 gives the simplified $a + bi$ form.

26. $x = -9$; no extraneous solutions — Cross-multiplying gives $5(x + 5) = 2(x - 1)$, which expands to $5x + 25 = 2x - 2$ and simplifies to $3x = -27$, so $x = -9$. The restrictions are $x \neq 1$ and $x \neq -5$, and $x = -9$ violates neither, so it is valid.

27. Yes, $(x - 4)$ is a factor — By the Factor Theorem, $f(4) = 64 - 48 - 24 + 8 = 0$. A remainder of zero confirms that $(x - 4)$ divides $f(x)$ evenly. A nonzero value would have meant it is not a factor.

28. $g(x) = -x^2 + 6$ — Reflecting across the x-axis negates the function to $-x^2$; translating 6 units up adds 6. Combining these produces $-x^2 + 6$. The reflection flips the parabola to open downward.

29. $x = 1$ — Write the right side as a power of 5: $125 = 5^3$, so $5^{3x} = 5^3$ requires $3x = 3$. Solving gives $x = 1$. Equal bases allow the exponents to be set equal.

30. $(5 - x) / [(x - 1)(x + 1)]$, $x \neq 1$, $x \neq -1$ — Factor the first denominator as $(x - 1)(x + 1)$, then use it as the common denominator: $6/[(x - 1)(x + 1)] - (x + 1)/[(x - 1)(x + 1)] = (6 - x - 1)/[(x - 1)(x + 1)] = (5 - x)/[(x - 1)(x + 1)]$. The numerator combines to $5 - x$.

31. $a_4 = 106$ — Apply the recursion: $a_2 = 4(1) + 2 = 6$, $a_3 = 4(6) + 2 = 26$, $a_4 = 4(26) + 2 = 106$. Each term quadruples the previous term and adds 2. Building up term by term reaches 106.

32. $\tan\theta = 9/40$ — With $\cos\theta = 40/41$, the 9-40-41 Pythagorean triple gives the opposite leg of 9, and in Quadrant I sine is positive, so $\sin\theta = 9/41$. Then $\tan\theta = \sin/\cos = (9/41)/(40/41) = 9/40$. The 41s cancel in the ratio.

Part III (Extended Constructed Response)

33. (a) \$1,500, 5% growth; (b) ≈ 14.2 years — The coefficient 1500 is the initial balance, and the base $1.05 = 1 + 0.05$ indicates 5% annual growth. For part (b), $3000 = 1500(1.05)^t$ gives $1.05^t = 2$, so $t = \ln(2)/\ln(1.05) \approx 14.2$ years. Logarithms isolate the exponent.

34. (a) $h(x) = -5.0x^2 + 30.0x + 1.0$; (b) 46.0 m at $x = 3.0$ m — Quadratic regression on the data returns $a = -5.0$, $b = 30.0$, $c = 1.0$. The vertex occurs at $x = -b/(2a) = -30/(-10) = 3.0$ meters, and $h(3) = -45 + 90 + 1 = 46.0$ meters. The negative leading coefficient confirms a maximum.

35. (a) $(-3 + 2\sqrt{5}, 3 + 2\sqrt{5})$ and $(-3 - 2\sqrt{5}, 3 - 2\sqrt{5})$ — Substituting $y = x + 6$ into the circle gives $x^2 + (x + 6)^2 = 58$, which simplifies to $2x^2 + 12x - 22 = 0$, then $x^2 + 6x - 11 = 0$, so $x = -3 \pm 2\sqrt{5}$. The y-values follow from $y = x + 6$. **(b)** Checking $(-3 + 2\sqrt{5}, 3 + 2\sqrt{5})$: $x^2 + y^2 = (29 - 12\sqrt{5}) + (29 + 12\sqrt{5}) = 58$, and $y = x + 6 = (-3 + 2\sqrt{5}) + 6 = 3 + 2\sqrt{5}$, so both equations hold.

Part IV (Long Constructed Response)

36. (a) $h(t) = 1.5 \sin(2\pi t/5)$ — The amplitude is 1.5 meters and the midline is 0 since the motion is centered on the average water level. The period of 5 seconds gives $b = 2\pi/5$, and starting at the midline moving upward at $t = 0$ calls for a positive sine. This yields $h(t) = 1.5 \sin(2\pi t/5)$.

(b) Amplitude 1.5, period 5 s, midline $h = 0$ — The amplitude of 1.5 meters is the maximum rise or fall from the average water level. The period of 5 seconds is the time for one complete bob up and down. The midline $h = 0$ is the average water level about which the buoy oscillates.

(c) $t \approx 0.58$ and 1.92 seconds — Setting $1 = 1.5 \sin(2\pi t/5)$ gives $\sin(2\pi t/5) = 2/3 \approx 0.6667$. Over the first 5 seconds, $2\pi t/5 = 0.7297$ and 2.4119 radians, so $t = 0.58$ and 1.92 seconds. These are the two times the rising and falling buoy passes 1 meter above the average level.