PRACTICE EXAM 48: ALGEBRA II REGENTS SIMULATION

PART I — Multiple Choice (24 questions × 2 credits = 48 credits)

1. Express the product $(3 - 2i)(1 + 4i)$ in $a + bi$ form.

A. $11 - 10i$

B. $11 + 10i$

C. $-5 + 10i$

D. $3 - 8i$

2. What is the value of $81^{(3/4)}$?

A. 27

B. 60.75

C. 9

D. 243

3. What are the solutions to the equation $x^2 - 8x + 25 = 0$?

A. $x = -4 \pm 3i$

B. $x = 4 \pm 6i$

C. $x = 4 \pm 3i$

D. $x = 8 \pm 3i$

4. Factored completely, the expression $64x^3 - 1$ is equivalent to

A. $(4x - 1)(16x^2 - 4x + 1)$

B. $(4x + 1)(16x^2 - 4x + 1)$

C. $(4x - 1)(4x^2 + 1)$

D. $(4x - 1)(16x^2 + 4x + 1)$

5. What is the value of $\log_6(36)$?

A. 6

B. 2

C. 1/2

D. 30

6. What is the sum of the finite geometric series 2 + 6 + 18 + 54 + 162?

A. 162

B. 486

C. 242

D. 324

7. When $p(x) = x^3 + 2x^2 - 5$ is divided by $(x - 2)$, the remainder is

A. 11

B. 3

C. −5

D. 16

8. If $f(x) = 4x - 1$ and $g(x) = x + 2$, what is the value of $f(g(0))$?

A. 1

B. −1

C. 3

D. 7

9. What is the solution to the equation $\sqrt{(3x - 2)} = 4$?

A. $x = 4$

B. $x = 18$

C. $x = 6$

D. $x = 16$

10. What is the average rate of change of $f(x) = x^2 - 2x$ over the interval [1, 5]?

A. 16

B. 4

C. 8

D. 3

11. A quantity is modeled by $f(t) = 1200(1.06)^t$, where t is the time in years. What is the annual percent rate of growth?

A. 60%

B. 1.06%

C. 106%

D. 6%

12. What is the solution to the equation $\log_2(x) + \log_2(8) = 5$?

A. x = 4

B. x = 8

C. x = 32

D. x = 2

13. Which statement best describes the nature of the solutions of $x^2 - 6x + 9 = 0$?

A. Two distinct rational solutions

B. Two distinct irrational solutions

C. One repeated rational solution

D. Two complex (non-real) solutions

14. What is the exact value of $\cos(\pi/6)$?

A. 1/2

B. $\sqrt{3}/2$

C. $\sqrt{2}/2$

D. $-\sqrt{3}/2$

15. For the function $y = -4 \sin(2x) + 1$, what is the amplitude?

A. 4

B. 2

C. 1

D. −4

16. What is the 12th term of an arithmetic sequence whose first term is −5 and whose common difference is 3?

A. 31

B. 36

C. 25

D. 28

17. If $f(x) = 2x^3$, what is $f^{-1}(x)$?

A. $\sqrt[3]{(2x)}$

B. $\sqrt[3]{(x/2)}$

C. $(x/2)^3$

D. $x^3/2$

18. In how many ways can a committee of 3 people be chosen from a group of 7 people?

A. 21

B. 210

C. 35

D. 343

19. A data set is normally distributed with a mean of 60 and a standard deviation of 8. Approximately what percent of the data is less than 44?

A. 2.5%

B. 5%

C. 16%

D. 34%

20. Given that $\cos \theta = -12/13$ and θ is in Quadrant III, what is the value of $\sin \theta$?

A. 5/13

B. 12/5

C. −12/13

D. −5/13

21. Written in vertex form, the expression $x^2 - 4x + 1$ is equivalent to

A. $(x - 2)^2 + 1$

B. $(x + 2)^2 - 3$

C. $(x - 2)^2 - 3$

D. $(x - 4)^2 - 15$

22. What is the sum of the infinite geometric series 24 + 12 + 6 + 3 + …?

A. 48

B. 36

C. 45

D. 96

23. What is the solution to the equation $7^x = 300$, rounded to the nearest hundredth?

A. $x = 42.86$

B. $x = 3.50$

C. $x = 2.48$

D. $x = 2.93$

24. A study finds that cities with more firefighters tend to have more fire damage. Which is the best explanation for this association?

A. Hiring more firefighters causes more fire damage

B. A third variable, such as the number or size of fires, influences both

C. Fire damage causes cities to hire fewer firefighters

D. There is no possible relationship between the two variables

PART II — Short Constructed Response (8 questions × 2 credits = 16 credits)

Show all work. A correct answer with no supporting work will receive only 1 credit.

25. Solve the equation $x^2 + 6x + 34 = 0$ algebraically. Express your solutions in $a + bi$ form.

26. Solve algebraically for x: $4 / (x + 5) = 3 / (x + 2)$. State any restrictions on the variable and identify any extraneous solutions that must be rejected.

27. Given $f(x) = x^3 + x^2 - 8x - 6$, determine whether $(x + 3)$ is a factor of $f(x)$. Justify your answer using the Remainder Theorem or the Factor Theorem.

28. Write an equation, in the form $g(x) = a \cdot f(x) + k$, for the function obtained by vertically stretching $f(x) = \sqrt{x}$ by a factor of 3 and then translating the result 1 unit up.

29. Solve algebraically for x: $2^{(x - 3)} = 32$.

30. Express $5 / (x + 4) + 2 / (x - 3)$ as a single rational expression in simplest form. State any restrictions on the variable.

31. A sequence is defined recursively by $a_1 = 2$ and $a_n = 3a_{n-1} - 1$ for $n \geq 2$. Find the value of a_4.

32. Given that $\sin \theta = -8/17$ and θ terminates in Quadrant IV, find the exact value of $\cos \theta$. Show the algebraic work that justifies your answer.

PART III — Extended Constructed Response (3 questions × 4 credits = 12 credits)

Show all work. Partial credit is awarded according to the scoring rubric.

33. The value of an investment is modeled by the function $A(t) = 5000(1.06)^t$, where $A(t)$ is the value in dollars and t is the time in years.

(a) State the initial value of the investment and the annual percent rate of growth.

(b) Algebraically determine the number of years it will take for the investment to grow to \$8000. Round your answer to the nearest tenth of a year.

34. The table below shows the height h(x), in meters, of an arched bridge cable at horizontal distance x meters from the left support.

x (meters)	0	1	2	3	4
h(x) (meters)	4	14	20	22	20

(a) Using regression, write a quadratic function $h(x) = ax^2 + bx + c$ that best models the data. Round each coefficient to the nearest tenth.

(b) Use the regression model from part (a) to determine the maximum height of the cable and the horizontal distance at which the maximum height occurs. Round each answer to the nearest tenth.

35. A circle is defined by the equation $x^2 + y^2 = 46$, and a line is defined by the equation $y = x + 4$.

(a) Algebraically determine all points of intersection between the circle and the line. Express coordinates in exact form (radicals permitted).

(b) Verify your answer by substituting one of the intersection points back into both original equations. Show the substitution clearly.

PART IV — Long Constructed Response (1 question × 6 credits = 6 credits)

Show all work. This problem requires multiple steps and integrates concepts from several chapters.

36. The temperature inside a greenhouse over a 24-hour period varies sinusoidally. The maximum temperature is 30°C and the minimum temperature is 18°C. One complete cycle takes 24 hours. At time t = 0 (midnight), the temperature is at its minimum.

(a) Write a function T(t) that models the temperature in degrees Celsius as a function of time t in hours.

(b) State the amplitude, period, and midline of the function T(t), and explain what each represents in the context of the greenhouse temperature.

(c) Algebraically determine all times during the first 24 hours at which the temperature is 28°C. Round each answer to the nearest hundredth of an hour. Show all algebraic work.

ANSWER KEY WITH EXPLANATIONS – PRACTICE EXAM 48

1. B — Distribute and apply $i^2 = -1$: $(3 - 2i)(1 + 4i) = 3 + 12i - 2i - 8i^2 = 3 + 10i + 8 = 11 + 10i$. The $-8i^2$ term becomes +8, raising the real part to 11. The middle terms combine to +10i.

2. A — Apply the fourth root, then cube: $81^{(3/4)} = (81^{(1/4)})^3 = 3^3 = 27$. The denominator 4 takes the fourth root and the numerator 3 cubes it. The value is 27.

3. C — Quadratic formula: $x = [8 \pm \sqrt{(64 - 100)}]/2 = [8 \pm \sqrt{(-36)}]/2 = [8 \pm 6i]/2 = 4 \pm 3i$. The negative discriminant produces the imaginary part. Dividing both terms by 2 gives the simplified form.

4. D — This is a difference of cubes, $(4x)^3 - 1^3 = (4x - 1)(16x^2 + 4x + 1)$. The middle term of the quadratic factor is $+ab = +4x$, making it positive. The sign pattern distinguishes it from a sum of cubes.

5. B — A logarithm asks for the exponent on the base: $6^2 = 36$, so $\log_6(36) = 2$. Rewriting in exponential form makes the value clear. The base 6 raised to 2 equals 36.

6. C — Using $S = a_1(r^n - 1)/(r - 1) = 2(3^5 - 1)/(3 - 1) = 2(242)/2 = 242$. The common ratio is 3 and there are five terms. Direct addition of the terms also gives 242.

7. A — By the Remainder Theorem, the remainder is $p(2) = 8 + 8 - 5 = 11$. Substituting the zero of the divisor avoids long division. The remainder is 11.

8. D — Evaluate the inner function first: $g(0) = 0 + 2 = 2$, then $f(2) = 4(2) - 1 = 7$. Composition works from the inside out. Multiplying 2 by 4 and subtracting 1 gives 7.

9. C — Square both sides: $3x - 2 = 16$, so $3x = 18$ and $x = 6$. Squaring removes the radical. Checking, $\sqrt{3\cdot6 - 2} = \sqrt{16} = 4$ confirms the solution.

10. B — Average rate of change is $[f(5) - f(1)]/(5 - 1) = (15 - (-1))/4 = 16/4 = 4$. This is the slope of the secant line over the interval. With $f(5) = 15$ and $f(1) = -1$, the result is 4.

11. D — In $a(1 + r)^t$, the base 1.06 equals $1 + 0.06$, so the growth rate is 6%. A base above 1 signals growth, and the base minus 1 gives the rate. The decimal 0.06 converts to 6%.

12. A — Combine the logs: $\log_2(8x) = 5$, so $8x = 2^5 = 32$ and $x = 4$. The product rule for logarithms merges the terms. Solving the exponential form gives 4.

13. C — The discriminant is $b^2 - 4ac = 36 - 36 = 0$, indicating one repeated real solution. Since $x^2 - 6x + 9 = (x - 3)^2$, the repeated root 3 is rational. A zero discriminant always yields a single repeated solution.

14. B — The cosine of $\pi/6$ (30°) is a standard unit-circle value equal to $\sqrt{3}/2$. The 30-60-90 reference triangle gives this ratio. The value is $\sqrt{3}/2$.

15. A — In $y = a \sin(bx) + d$, the amplitude is $|a|$, which is $|-4| = 4$. The amplitude is the absolute value of the sine coefficient. The negative sign reflects the graph but does not change the amplitude.

16. D — The nth term is $a_n = a_1 + (n - 1)d$, so $a_{12} = -5 + 11(3) = -5 + 33 = 28$. The common difference applied eleven times raises the term. The twelfth term is 28.

17. B — To invert, solve $y = 2x^3$ for x: $x^3 = y/2$, so $x = \sqrt[3]{y/2}$ and $f^{-1}(x) = \sqrt[3]{x/2}$. The inverse undoes multiplication by 2 and cubing in reverse order. The result is $\sqrt[3]{x/2}$.

18. C — Order does not matter, so use combinations: $C(7, 3) = (7 \cdot 6 \cdot 5)/(3 \cdot 2 \cdot 1) = 35$. The combination formula counts unordered selections. There are 35 possible committees.

19. A — A value of 44 is two standard deviations below the mean of 60. By the empirical rule, about 2.5% of data lies below −2 SD. The lower tail captures 2.5%.

20. D — In Quadrant III both sine and cosine are negative, and the 5-12-13 triangle gives the sine ratio. Therefore $\sin \theta = -5/13$. The quadrant makes the sine negative.

21. C — Complete the square: $x^2 - 4x + 1 = (x^2 - 4x + 4) - 4 + 1 = (x - 2)^2 - 3$. Half of −4 squared is 4, added and subtracted to preserve value. The vertex form reveals the vertex at $(2, -3)$.

22. A — The common ratio is 1/2, and an infinite geometric series with $|r| < 1$ sums to $a_1/(1 - r) = 24/(1 - 1/2) = 24/(1/2) = 48$. The ratio ensures convergence. The series sums to 48.

23. D — Take the log of both sides: $x = \log_7(300) = \ln(300)/\ln(7) \approx 2.93$. The change-of-base formula converts to natural logs. The result rounds to 2.93.

24. B — A larger city has both more fires and more firefighters, so the size of the fire problem is a confounding variable driving both quantities. The firefighters do not cause the damage; they respond to it. This is a classic case of correlation without causation.

Part II (Short Constructed Response)

25. $x = -3 \pm 5i$ — Quadratic formula: $x = [-6 \pm \sqrt{(36 - 136)}]/2 = [-6 \pm \sqrt{(-100)}]/2 = [-6 \pm 10i]/2 = -3 \pm 5i$. The negative discriminant produces complex conjugates. Dividing each term by 2 gives the simplified $a + bi$ form.

26. $x = 7$; no extraneous solutions — Cross-multiplying gives $4(x + 2) = 3(x + 5)$, which expands to $4x + 8 = 3x + 15$ and simplifies to $x = 7$. The restrictions are $x \neq -5$ and $x \neq -2$, and $x = 7$ violates neither, so it is valid.

27. Yes, $(x + 3)$ is a factor — By the Factor Theorem, $f(-3) = -27 + 9 + 24 - 6 = 0$. A remainder of zero confirms that $(x + 3)$ divides $f(x)$ evenly. A nonzero value would have meant it is not a factor.

28. $g(x) = 3\sqrt{x} + 1$ — A vertical stretch by 3 multiplies the function by 3, giving $3\sqrt{x}$; translating 1 unit up adds 1. Combining these produces $3\sqrt{x} + 1$. The stretch acts on the leading coefficient.

29. $x = 8$ — Write the right side as a power of 2: $32 = 2^5$, so $2^{(x - 3)} = 2^5$ requires $x - 3 = 5$. Solving gives $x = 8$. Equal bases allow the exponents to be set equal.

30. $(7x - 7) / [(x + 4)(x - 3)]$, $x \neq -4$, $x \neq 3$ — Over the common denominator $(x + 4)(x - 3)$: $5(x - 3) + 2(x + 4) = 5x - 15 + 2x + 8 = 7x - 7$. The numerator combines to $7x - 7$, which factors as $7(x - 1)$. Restrictions come from the original denominators.

31. $a_4 = 41$ — Apply the recursion: $a_2 = 3(2) - 1 = 5$, $a_3 = 3(5) - 1 = 14$, $a_4 = 3(14) - 1 = 41$. Each term triples the previous term and subtracts 1. Building up term by term reaches 41.

32. $\cos\theta = 15/17$ — With $\sin\theta = -8/17$, the 8-15-17 Pythagorean triple gives the adjacent leg of 15. In Quadrant IV cosine is positive, so $\cos\theta = 15/17$. The quadrant fixes the positive sign.

Part III (Extended Constructed Response)

33. (a) \$5,000, 6% growth; (b) ≈ 8.1 years — The coefficient 5000 is the initial value, and the base $1.06 = 1 + 0.06$ indicates 6% annual growth. For part (b), $8000 = 5000(1.06)^t$ gives $1.06^t = 1.6$, so $t = \ln(1.6)/\ln(1.06) \approx 8.1$ years. Logarithms isolate the exponent.

34. (a) $h(x) = -2.0x^2 + 12.0x + 4.0$; (b) 22.0 m at $x = 3.0$ m — Quadratic regression on the data returns $a = -2.0$, $b = 12.0$, $c = 4.0$. The vertex occurs at $x = -b/(2a) = -12/(-4) = 3.0$ meters, and $h(3) = -18 + 36 + 4 = 22.0$ meters. The negative leading coefficient confirms a maximum.

35. (a) $(-2 + \sqrt{19}, 2 + \sqrt{19})$ and $(-2 - \sqrt{19}, 2 - \sqrt{19})$ — Substituting $y = x + 4$ into the circle gives $x^2 + (x + 4)^2 = 46$, which simplifies to $2x^2 + 8x - 30 = 0$, then $x^2 + 4x - 15 = 0$, so $x = -2 \pm \sqrt{19}$. The y-values follow from $y = x + 4$. **(b)** Checking $(-2 + \sqrt{19}, 2 + \sqrt{19})$: $x^2 + y^2 = (23 - 4\sqrt{19}) + (23 + 4\sqrt{19}) = 46$, and $y = x + 4 = (-2 + \sqrt{19}) + 4 = 2 + \sqrt{19}$, so both equations hold.

Part IV (Long Constructed Response)

36. (a) $T(t) = -6 \cos(\pi t/12) + 24$ — The amplitude is $(30 - 18)/2 = 6$ and the midline is $(30 + 18)/2 = 24$. The period of 24 hours gives $b = 2\pi/24 = \pi/12$, and starting at the minimum temperature at $t = 0$ calls for a negative cosine. This yields $T(t) = -6 \cos(\pi t/12) + 24$.

(b) Amplitude 6, period 24 h, midline $T = 24$ — The amplitude of 6°C is the distance from the midline to the maximum or minimum temperature. The period of 24 hours is the time for one complete daily temperature cycle. The midline $T = 24$°C is the average temperature about which the greenhouse oscillates.

(c) $t \approx 8.79$ and 15.21 hours — Setting $28 = -6 \cos(\pi t/12) + 24$ gives $\cos(\pi t/12) = -2/3 \approx -0.6667$. Over the first 24 hours, $\pi t/12 = 2.3005$ and 3.9827 radians, so $t = 8.79$ and 15.21 hours. These are the two times the warming and cooling greenhouse passes 28°C.