PRACTICE EXAM 47: ALGEBRA II REGENTS SIMULATION

PART I — Multiple Choice (24 questions × 2 credits = 48 credits)

1. What is the complete solution set of the equation $x^3 + 5x^2 + 6x = 0$?

A. {0, 2, 3}

B. {0, −5, −6}

C. {0, −2, −3}

D. {−1, −2, −3}

2. Expressed in a + bi form, the expression $(2 - i)^2$ is equivalent to

A. 3 + 4i

B. 3 − 4i

C. 5 − 4i

D. 4 − 4i

3. As $x \to \infty$, the function $f(x) = -x^3 + 4x^2 - 7$ behaves as

A. $f(x) \to \infty$

B. $f(x) \to 0$

C. $f(x) \to -7$

D. $f(x) \to -\infty$

4. The exact value of $\sec(\pi/3)$ is

A. 2

B. 1/2

C. $\sqrt{3}$

D. $2/\sqrt{3}$

5. The expression $\log(100x)$, where $x > 0$, is equivalent to

A. $100 + \log(x)$

B. $\log(100) \cdot \log(x)$

C. $2 + \log(x)$

D. $100 \cdot \log(x)$

6. What is the value of x in the equation $\log_6(x) = 3$?

A. 18

B. 9

C. 36

D. 216

7. The y-intercept of the function $f(x) = x^3 - 5x^2 + 2x - 8$ is

A. −8

B. −5

C. 8

D. 0

8. If $f(x) = (x - 3)/2$, then $f^{-1}(x)$ is

A. $2x - 3$

B. $2x + 3$

C. $(x + 3)/2$

D. $(x - 2)/3$

9. For all values of x, the expression $\sin(x + \pi)$ is equivalent to

A. $\sin(x)$

B. $\cos(x)$

C. $-\sin(x)$

D. $-\cos(x)$

10. A sequence is defined recursively by $a_1 = 3$ and $a_n = a_{n-1}^2 - 1$ for $n \geq 2$. The value of a_3 is

A. 8

B. 24

C. 47

D. 63

11. A population grows at a rate of 2.5% per year. If the initial population is P_0, which function models the population after t years?

A. $P(t) = P_0(0.025)^t$

B. $P(t) = P_0(1.025)^t$

C. $P(t) = P_0(2.5)^t$

D. $P(t) = P_0(0.975)^t$

12. The domain of the function $f(x) = \sqrt{(x + 2)}$ is

A. $x \geq -2$

B. $x > -2$

C. $x \geq 2$

D. all real numbers except -2

13. The exact value of the population standard deviation of the data set {4, 6, 8, 10} is

A. 2

B. 1.58

C. √5

D. 2.5

14. The exact value of cos(−π/4) is

A. √2/2

B. −√2/2

C. 1/2

D. −1/2

15. Solve the equation $2^x = 7$ algebraically. Rounded to the nearest hundredth, x is approximately

A. 3.50

B. 1.95

C. 2.50

D. 2.81

16. The sum of the first 4 terms of the geometric sequence 1, 3, 9, 27, … is

A. 27

B. 40

C. 81

D. 120

17. A fair six-sided die is rolled twice. What is the probability of rolling a 6 on both rolls?

A. 1/36

B. 1/12

C. 1/3

D. 2/6

18. Compared to the graph of $f(x) = \log(x)$, the graph of $g(x) = \log(x - 2) + 3$ is shifted

A. left 2 units and up 3 units

B. left 2 units and down 3 units

C. right 2 units and up 3 units

D. right 2 units and down 3 units

19. For which values of x is the function $f(x) = (x - 1)(x + 3)$ positive?

A. $-3 < x < 1$

B. $x < -3$ or $x > 1$

C. $x > 1$ only

D. $x < 1$ only

20. How many solutions does the equation $\cos x = 0$ have in the interval $0 \leq x \leq 2\pi$?

A. 0

B. 1

C. 3

D. 2

21. Which of the following is a possible rational root of the polynomial $f(x) = 2x^3 + 5x^2 - x - 4$, according to the Rational Root Theorem?

A. 1/2

B. 1/4

C. 2/5

D. 3

22. To estimate the proportion of voters in favor of a candidate, a researcher polls a random sample of 500 voters and computes the proportion who favor the candidate. This computed proportion is best classified as

A. a population mean

B. a population proportion

C. a parameter of the voting population

D. a sample statistic

23. For all values of θ where the expression is defined, $\sec \theta / \csc \theta$ is equivalent to

A. cot θ

B. tan θ

C. sin θ

D. cos θ

24. The graph of the function f(x) is shown below. What is the value of f(2)?

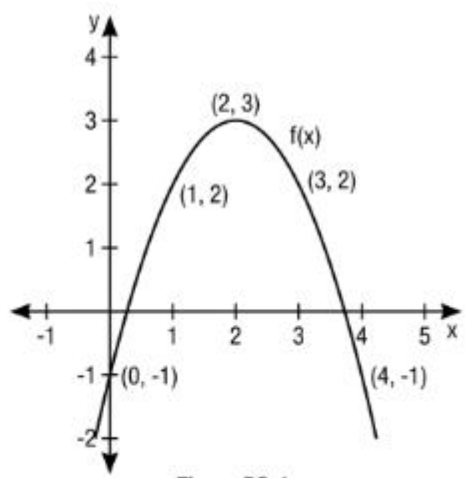

Figure PQ-1:

A. 0

B. 1

C. 3

D. −1

PART II — Short Constructed Response (8 questions × 2 credits = 16 credits)

Show all work. A correct answer with no supporting work will receive only 1 credit.

25. Express the quotient $(5 + 2i)/(3 - i)$ in $a + bi$ form. Show all algebraic work, including the multiplication by the conjugate of the denominator.

26. Solve algebraically for x: $2x/(x - 5) = 10/(x - 5) + 3$. State any extraneous solutions and indicate which (if any) are rejected.

27. Without solving the equation, determine the nature of the solutions of $4x^2 - 12x + 9 = 0$ by using the discriminant. Justify your answer using the value of the discriminant.

28. Describe in order each transformation applied to $f(x) = x^2$ to obtain $g(x) = -(x + 1)^2 + 6$. Then state the vertex of g(x).

29. Solve algebraically for x: $\log(x) + \log(x + 9) = 1$. State any extraneous solutions and indicate which (if any) are rejected.

30. Express $x/(x^2 - 1) + 2/(x + 1)$ as a single rational expression in simplest form. State any restrictions on the variable.

31. An arithmetic sequence has third term $a_3 = 11$ and seventh term $a_7 = 27$. Determine the value of the first term a_1 and the common difference d. Then find the value of a_{15}.

32. Given that $\sin \theta = -5/13$ and $\tan \theta > 0$, find the exact value of $\cos \theta$. Show all algebraic work used to obtain your answer.

PART III — Extended Constructed Response (3 questions × 4 credits = 12 credits)

Show all work. Partial credit is awarded according to the scoring rubric.

33. The number of users of a new mobile application is increasing each month. The number of users, N, after t months can be modeled by the function $N(t) = 5{,}000 \cdot (1.18)^t$.

(a) State the initial number of users at t = 0 and the monthly percent rate of increase in the number of users. Justify each value using the structure of the function.

(b) Algebraically determine, to the nearest tenth of a month, the amount of time required for the number of users to reach 25,000. Show all algebraic work, including the use of logarithms.

34. A simulation is conducted to estimate the probability that a roll of a fair six-sided die results in a multiple of 3 (that is, a 3 or a 6). The simulation is run for 1,000 trials.

(a) Determine the theoretical probability of rolling a multiple of 3 on a single roll of a fair six-sided die.

(b) After the simulation, 320 of the 1,000 trials produce a multiple of 3. State the experimental probability obtained from the simulation, and explain why this experimental value may differ from the theoretical probability calculated in part (a).

35. Algebraically solve for all values of x in the interval $0 \leq x \leq 2\pi$:

$$\sin(2x) = \sin(x)$$

Express each solution in exact form (radians). Show all algebraic work used to obtain each solution, including the use of a double-angle identity.

PART IV — Long Constructed Response (1 question × 6 credits = 6 credits)

Show all work. This problem requires multiple steps and integrates concepts from several chapters.

36. A mass attached to a vertical spring oscillates up and down according to a sinusoidal function. The maximum displacement of the mass is 12 centimeters above its equilibrium position, and the minimum displacement is 12 centimeters below equilibrium. One complete oscillation takes 2 seconds. At time $t = 0$ seconds, the mass is at its maximum height of 12 centimeters above the equilibrium position.

(a) Write a cosine function $h(t)$ that models the height of the mass, in centimeters above its equilibrium position, as a function of t, the time in seconds since the oscillation began.

(b) State the amplitude, period, and midline of $h(t)$, and explain what each represents in the context of the oscillating spring-mass system.

(c) Algebraically determine all times during the first 4 seconds at which the mass is exactly 5 centimeters above the equilibrium position. Round each answer to the nearest hundredth of a second. Show all algebraic work, including the use of inverse trigonometric functions.

ANSWER KEY WITH EXPLANATIONS – EXAM 47

1. C — Factoring out x gives $x(x^2 + 5x + 6) = 0$, and the quadratic factors further as $x(x + 2)(x + 3) = 0$. Setting each factor equal to zero produces the complete solution set $\{0, -2, -3\}$.

2. B — Squaring a binomial uses $(a - b)^2 = a^2 - 2ab + b^2$. Computing $(2)^2 - 2(2)(i) + i^2 = 4 - 4i + (-1) = 3 - 4i$, using the identity $i^2 = -1$ to simplify.

3. D — End behavior is governed by the leading term, which here is $-x^3$. An odd-degree polynomial with a negative leading coefficient approaches $-\infty$ as x grows to positive infinity.

4. A — Secant is the reciprocal of cosine, so $\sec(\pi/3) = 1/\cos(\pi/3)$. Since $\cos(\pi/3) = 1/2$, taking the reciprocal gives 2.

5. C — Applying the product rule of logarithms gives $\log(100x) = \log(100) + \log(x)$. Since $100 = 10^2$, $\log(100) = 2$, producing the equivalent form $2 + \log(x)$.

6. D — Converting from logarithmic form to exponential form gives $x = 6^3$. Computing this power yields $x = 216$.

7. A — The y-intercept of a function occurs where $x = 0$. Substituting gives $f(0) = 0 - 0 + 0 - 8 = -8$.

8. B — Swap x and y in $y = (x - 3)/2$ to obtain $x = (y - 3)/2$. Multiplying both sides by 2 and adding 3 yields $f^{-1}(x) = 2x + 3$.

9. C — Adding π to an angle reflects its terminal point through the origin, reversing the sign of the sine value. This produces the identity $\sin(x + \pi) = -\sin(x)$, which holds for every real x.

10. D — Apply the recursion sequentially: $a_2 = (3)^2 - 1 = 8$, and then $a_3 = (8)^2 - 1 = 63$. Each term depends only on the previous one, so the sequence must be computed in order.

11. B — An exponential growth model has the form $P(t) = P_0(1 + r)^t$. Substituting $r = 0.025$ gives the growth factor $1 + 0.025 = 1.025$, producing $P(t) = P_0(1.025)^t$.

12. A — A square root function is defined only when its radicand is non-negative. Setting $x + 2 \geq 0$ gives $x \geq -2$, the complete domain of the function.

13. C — Computing deviations from the mean of 7 gives −3, −1, 1, 3, with squared values 9, 1, 1, 9 summing to 20. Population variance equals $20/4 = 5$, so the population standard deviation is $\sqrt{5}$.

14. A — Cosine is an even function, so $\cos(-\theta) = \cos(\theta)$. Substituting gives $\cos(-\pi/4) = \cos(\pi/4) = \sqrt{2}/2$.

15. D — Taking the natural logarithm of both sides yields $x \cdot \ln(2) = \ln(7)$. Dividing gives $x = \ln(7)/\ln(2) \approx 1.9459/0.6931 \approx 2.81$.

16. B — Apply the finite geometric series formula $S_n = a_1(1 - r^n)/(1 - r)$ with $a_1 = 1$, $r = 3$, and $n = 4$. Computing $1(1 - 81)/(1 - 3) = (-80)/(-2) = 40$.

17. A — Successive die rolls are independent, so the probabilities of each outcome multiply. Multiplying $P(6) \cdot P(6) = (1/6)(1/6) = 1/36$.

18. C — Replacing x with $(x - h)$ in a function shifts its graph h units to the right when $h > 0$, and adding k shifts it k units up. With $h = 2$ and $k = 3$, the graph shifts right 2 units and up 3 units.

19. B — A product of two factors is positive when both share the same sign. The product $(x - 1)(x + 3)$ is positive either when both factors are positive ($x > 1$) or when both are negative ($x < -3$).

20. D — Cosine equals zero at $x = \pi/2$ and $x = 3\pi/2$ within one period. Both values fall within the interval $[0, 2\pi]$, producing exactly 2 solutions.

21. A — The Rational Root Theorem states that any rational root has the form p/q, where p divides the constant term and q divides the leading coefficient. With p dividing 4 and q dividing 2, the value 1/2 satisfies both conditions and is a valid candidate.

22. D — A numerical summary computed from a sample is called a sample statistic, while a value summarizing an entire population is a parameter. The sample proportion from 500 polled voters is a statistic used to estimate the unknown population parameter.

23. B — Substituting the reciprocal identities gives $\sec\theta / \csc\theta = (1/\cos\theta) / (1/\sin\theta)$. Simplifying the complex fraction by multiplying numerator by the reciprocal of the denominator yields $\sin\theta / \cos\theta = \tan\theta$.

24. C — A function value f(a) is read directly from the graph by finding the y-coordinate of the point at x = a. The graph shows the vertex of the parabola at (2, 3), so f(2) = 3.