PRACTICE EXAM 46: ALGEBRA II REGENTS SIMULATION

PART I — Multiple Choice (24 questions × 2 credits = 48 credits)

1. Which value of the discriminant of a quadratic equation indicates that the equation has two distinct irrational solutions?

A. 12

B. 25

C. 0

D. −8

2. What is the solution to the equation $\sqrt{(x^2 + 9)} = x + 3$?

A. $x = -3$

B. $x = 3$

C. $x = 6$

D. $x = 0$

3. The product $(1 + 4i)(1 - 4i)$ is equal to

A. 1

B. 17

C. -15

D. $17i$

4. One root of the polynomial $f(x) = x^3 + 2x^2 - 5x - 6$ is $x = 2$. The other two roots are

A. $x = 1$ and $x = 3$

B. $x = -1$ and $x = 3$

C. $x = -1$ and $x = -3$

D. $x = -2$ and $x = -3$

5. The vertical asymptote of the function $f(x) = \log_3(x - 5)$ is the line

A. $x = -5$

B. $x = 5$

C. $y = 5$

D. $y = 0$

6. Which transformation describes the graph of $y = \sin(x - \pi/2)$ compared to the graph of $y = \sin(x)$?

A. Shift right $\pi/2$ units

B. Shift left $\pi/2$ units

C. Shift up $\pi/2$ units

D. Reflect across the x-axis

7. What is the solution to the equation 5^(x + 2) = 125?

A. x = 3

B. x = −3

C. x = 25

D. x = 1

8. The sum of the infinite geometric series 8 + 4 + 2 + 1 + … is

A. unbounded (no finite sum)

B. 8

C. 16

D. 12

9. If f(x) = e^(2x), then $f^{-1}(x)$ is

A. ln(2x)

B. (1/2) ln(x)

C. e^(x/2)

D. ln(x) − 2

10. A survey of 100 students found that 60 students like math, 40 students like science, and 20 students like both subjects. What is the probability that a randomly selected student likes science, given that the student likes math?

A. 1/5

B. 2/5

C. 1/2

D. 1/3

11. Over the set of real numbers, the expression $x^2 + 9$ is

A. not factorable over the real numbers

B. equal to $(x + 3)^2$

C. equal to $(x - 3)(x + 3)$

D. equal to $(x - 3)^2$

12. What is the complete solution to the equation $\log(x - 2) + \log(x - 5) = 1$?

A. $x = 0$

B. $x = 7$ or $x = 0$

C. $x = 7$

D. $x = 2$ or $x = 5$

13. The exact value of $\cos(7\pi/4)$ is

A. $-\sqrt{2}/2$

B. $1/2$

C. $-1/2$

D. $\sqrt{2}/2$

14. The average rate of change of the function $f(x) = 2^x$ on the interval $[0, 3]$ is

A. $7/3$

B. 7

C. 3

D. $1/3$

15. Which sampling method is most likely to produce biased results?

A. Simple random sampling

B. Voluntary response sampling

C. Stratified random sampling

D. Systematic sampling with random start

16. What is the maximum number of x-intercepts that the graph of a fifth-degree polynomial function can have?

A. 1

B. 4

C. 5

D. 6

17. For all values of θ where the expression is defined, $\csc^2\theta - \cot^2\theta$ is equivalent to

A. 1

B. $\sin^2\theta$

C. $\cos^2\theta$

D. $\tan^2\theta$

18. Solve algebraically for x: e^(x + 1) = 8. Rounded to the nearest hundredth, x is approximately

A. 7.00

B. 2.08

C. 0.90

D. 1.08

19. If $f(x) = (x + 1)/(x - 2)$, then $f^{-1}(x)$ is

A. $(2x - 1)/(x + 1)$

B. $(2x + 1)/(x - 1)$

C. $(x - 2)/(x + 1)$

D. $(x + 1)/(2x - 1)$

20. How many different 3-letter codes can be made from the letters A, B, C, D, E if no letter can be repeated and the order of letters matters?

A. 10

B. 20

C. 60

D. 125

21. In a standard normal distribution, approximately what percent of values fall between $z = -1$ and $z = 1$?

A. 68%

B. 95%

C. 50%

D. 34%

22. When the polynomial $x^3 + 2x^2 - 3x + 1$ is divided by $(x^2 + 1)$, the remainder is

A. -1

B. 1

C. $4x$

D. $-4x - 1$

23. Two events A and B are independent. If $P(A) = 0.4$ and $P(B) = 0.3$, then $P(A \text{ and } B)$ is

A. 0.7

B. 0.5

C. 0.12

D. 0.1

24. In the geometric sequence 12, −6, 3, −3/2, …, the common ratio is

A. −2

B. −1/2

C. 1/2

D. −6

PART II — Short Constructed Response (8 questions × 2 credits = 16 credits)

Show all work. A correct answer with no supporting work will receive only 1 credit.

25. Express $(1 + i)^4$ in a + bi form. Show all algebraic work, including any intermediate squared expression used to obtain the result.

26. Solve algebraically for x: $4/(x - 1) - 2/(x + 1) = 0$. State any extraneous solutions and indicate which (if any) are rejected.

27. Use the Factor Theorem to determine whether $(x - 1)$ is a factor of $p(x) = 3x^3 - 4x^2 - 5x + 6$. Show all work and state your conclusion.

28. The function g(x) is obtained from f(x) = √x by a horizontal stretch by a factor of 4 followed by a reflection across the x-axis. Write the equation that defines g(x).

29. Solve algebraically for x: $\log_2(x + 4) = 5$. Show all work used to arrive at the solution.

30. Express $3/(x - 2) + x/(x + 1)$ as a single rational expression in simplest form. State any restrictions on the variable.

31. A geometric sequence has first term $a_1 = 1024$ and common ratio $r = 1/2$. Write the explicit formula for the nth term a_n, and use it to find the value of a_8.

32. Given that $\tan \theta = -8/15$ and θ terminates in Quadrant II, find the exact values of $\sin \theta$ and $\cos \theta$. Show all algebraic work used to obtain each value.

PART III — Extended Constructed Response (3 questions × 4 credits = 12 credits)

Show all work. Partial credit is awarded according to the scoring rubric.

33. A medication is administered to a patient with an initial dose of 200 milligrams. The amount remaining in the patient's bloodstream is modeled by the function $A(t) = 200(0.85)^t$, where A is measured in milligrams and t is the number of hours since the medication was administered.

(a) State the percent rate of decrease in the amount of medication per hour, and explain how this rate can be determined from the structure of the equation.

(b) Algebraically determine, to the nearest tenth of an hour, the half-life of the medication — the time required for the amount remaining to reach half of the original dose. Show all algebraic work, including the use of logarithms.

34. A simple random sample of 200 voters in a certain city is taken, and 88 of these voters indicate that they support a proposed ballot measure.

(a) Calculate the sample proportion of voters who support the proposed ballot measure.

(b) Using a margin of error of 0.07 at the 95% confidence level, construct the 95% confidence interval for the true proportion of all voters in the city who support the measure. State the interval and explain what it indicates about voter support for the measure.

35. Algebraically solve for all values of x in the interval $0 \leq x \leq 2\pi$:

$\sqrt{3} \sin x + \cos x = 0$

Express each solution in exact form (radians). Show all algebraic work used to obtain each solution.

PART IV — Long Constructed Response (1 question × 6 credits = 6 credits)

Show all work. This problem requires multiple steps and integrates concepts from several chapters.

36. A weighted pendulum swings back and forth with sinusoidal motion. The horizontal displacement of the weight from its rest (equilibrium) position is modeled by a sine function. The maximum displacement of the weight is 25 centimeters to the right of equilibrium, and the maximum displacement is 25 centimeters to the left of equilibrium. One complete swing (back and forth) takes 1.5 seconds to complete. At time t = 0 seconds, the weight is at its equilibrium position (0 centimeters) and is moving to the right.

(a) Write a sine function d(t) that models the horizontal displacement of the weight, in centimeters from equilibrium, as a function of t, the time in seconds since the pendulum was released. Use the convention that displacements to the right of equilibrium are positive.

(b) State the amplitude, period, and midline of d(t), and explain what each represents in the context of the pendulum's motion.

(c) Algebraically determine all times during the first 3 seconds at which the pendulum weight is exactly 15 centimeters to the right of equilibrium. Round each answer to the nearest hundredth of a second. Show all algebraic work, including the use of inverse trigonometric functions.

ANSWER KEY WITH EXPLANATIONS — EXAM 46

1. A — A positive discriminant indicates two distinct real solutions, and when that discriminant is not a perfect square, the solutions are irrational. Of the choices, 12 is positive but not a perfect square, so it produces two distinct irrational solutions.

2. D — Squaring both sides eliminates the radical: $x^2 + 9 = (x + 3)^2 = x^2 + 6x + 9$. The x^2 and constant terms cancel, leaving $6x = 0$, so $x = 0$, which satisfies the original equation since $\sqrt{9} = 0 + 3$.

3. B — The product of a complex number and its conjugate equals $a^2 + b^2$, a purely real number. Computing $(1)^2 + (4)^2 = 1 + 16 = 17$.

4. C — Since $x = 2$ is a root, $(x - 2)$ is a factor; dividing the polynomial by $(x - 2)$ produces the quotient $x^2 + 4x + 3$. Factoring this further as $(x + 1)(x + 3)$ yields the remaining roots $x = -1$ and $x = -3$.

5. B — A logarithmic function has a vertical asymptote where its argument equals zero. Setting $x - 5 = 0$ gives $x = 5$, the location of the vertical asymptote of f(x).

6. A — Replacing x with $(x - h)$ in a function shifts its graph horizontally by h units to the right. Since $y = \sin(x - \pi/2)$ has $h = \pi/2 > 0$, the graph shifts right by $\pi/2$ units.

7. D — Rewriting 125 as 5^3 gives $5^{(x + 2)} = 5^3$. Since the exponential function with base 5 is one-to-one, equating exponents yields $x + 2 = 3$, so $x = 1$.

8. C — An infinite geometric series converges when $|r| < 1$, with sum $S = a/(1 - r)$. Substituting $a = 8$ and $r = 1/2$ gives $8/(1 - 1/2) = 8/(1/2) = 16$.

9. B — Swap x and y in $y = e^{(2x)}$ to obtain $x = e^{(2y)}$. Taking the natural logarithm of both sides gives $\ln(x) = 2y$, so $y = (1/2) \ln(x)$.

10. D — Conditional probability is defined as $P(S \mid M) = P(S \cap M) / P(M)$. Substituting the 20 students who like both subjects and the 60 students who like math gives $20/60 = 1/3$.

11. A — The expression $x^2 + 9$ is a sum of squares, which has no factorization over the real numbers because no real value squared equals -9. It factors only over the complex numbers as $(x + 3i)(x - 3i)$.

12. C — Combining the logarithms using the product rule gives $\log[(x - 2)(x - 5)] = 1$, leading to $x^2 - 7x = 0$ with roots $x = 0$ or $x = 7$. The value $x = 0$ is rejected because it makes the original logarithm arguments negative, leaving $x = 7$ as the valid solution.

13. D — The angle $7\pi/4$ lies in Quadrant IV, where cosine is positive, with a reference angle of $\pi/4$. Since $\cos(\pi/4) = \sqrt{2}/2$, applying the Quadrant IV sign convention gives $\cos(7\pi/4) = \sqrt{2}/2$.

14. A — The average rate of change equals $[f(b) - f(a)] / (b - a)$. Computing $f(0) = 1$ and $f(3) = 8$ gives $(8 - 1)/(3 - 0) = 7/3$.

15. B — Voluntary response sampling allows individuals to self-select into the sample, which systematically favors those with strong opinions on the topic. This selection mechanism over-represents certain viewpoints and produces results that do not reliably reflect the broader population.

16. C — The Fundamental Theorem of Algebra guarantees that a polynomial of degree n has exactly n zeros in the complex number system, counted with multiplicity. Among real zeros that produce x-intercepts on the graph, a fifth-degree polynomial can have at most 5.

17. A — Dividing the Pythagorean identity $\sin^2\theta + \cos^2\theta = 1$ by $\sin^2\theta$ produces $1 + \cot^2\theta = \csc^2\theta$. Rearranging this gives $\csc^2\theta - \cot^2\theta = 1$, a Pythagorean identity that holds for all θ where the functions are defined.

18. D — Taking the natural logarithm of both sides gives $x + 1 = \ln(8)$. Subtracting 1 yields $x = \ln(8) - 1 \approx 2.0794 - 1 \approx 1.08$.

19. B — Swap x and y in $y = (x + 1)/(x - 2)$, giving $x = (y + 1)/(y - 2)$. Multiplying through and grouping y-terms produces $y(x - 1) = 2x + 1$, so $y = (2x + 1)/(x - 1)$.

20. C — Order matters and no repetition is allowed, so the count is a permutation: $P(n, r) = n!/(n - r)!$. Computing $P(5, 3) = 5 \cdot 4 \cdot 3 = 60$ gives the number of possible 3-letter codes.

21. A — The empirical rule states that approximately 68% of values in a normal distribution lie within one standard deviation of the mean. For the standard normal distribution, this interval corresponds exactly to $z = -1$ to $z = 1$.

22. D — Performing polynomial long division, $x^3/x^2 = x$, and $x(x^2 + 1) = x^3 + x$. Subtracting leaves $2x^2 - 4x + 1$; continuing with $2x^2/x^2 = 2$ and subtracting $2x^2 + 2$ yields the final remainder $-4x - 1$.

23. C — For independent events, the probability of both occurring equals the product of their individual probabilities. Multiplying $P(A) \cdot P(B) = 0.4 \cdot 0.3 = 0.12$ gives the joint probability.

24. B — The common ratio of a geometric sequence equals any term divided by the preceding term. Computing $-6/12 = -1/2$ confirms that each subsequent term is obtained by multiplying the previous one by $-1/2$.