PRACTICE EXAM 45: ALGEBRA II REGENTS SIMULATION

PART I — Multiple Choice (24 questions × 2 credits = 48 credits)

1. In the expansion of $(a + b)^4$, the coefficient of the a^2b^2 term is

A. 4

B. 6

C. 8

D. 12

2. What is the solution to the inequality $\log_2(x) < 3$?

A. $x < 8$

B. $x > 8$

C. $0 < x < 8$

D. $x \leq 8$

3. Using the quadratic formula, the solutions to $2x^2 + 4x + 5 = 0$, expressed in a + bi form, are

A. $x = -1 \pm (i\sqrt{6})/2$

B. $x = 1 \pm (i\sqrt{6})/2$

C. $x = -1 \pm i\sqrt{6}$

D. $x = -2 \pm i\sqrt{6}$

4. What is the solution set of the equation $(x + 4)/(x + 2) = (x + 6)/(x + 4)$?

A. {2}

B. {−2, 4}

C. {0}

D. { }

5. What is the period of the function $y = 4\cos(\pi x/3)$?

A. 3

B. 6

C. $\pi/3$

D. 4

6. What is the exact solution to the equation $\log_3(x) = 2.5$?

A. $9\sqrt{3}$

B. 7.5

C. 9

D. √7.5

7. If sec θ = 13/5 and θ terminates in Quadrant IV, then sin θ is

A. 12/13

B. 5/12

C. −5/12

D. −12/13

8. $2,000 is invested at an annual interest rate of 5% compounded annually. To the nearest cent, the balance in the account after 10 years is

A. $2,500.00

B. $3,000.00

C. $3,257.79

D. $4,000.00

9. If f(x) = $\log_2(x)$, what is the value of f(32)?

A. 5

B. 4

C. 6

D. 16

10. As $x \to -\infty$, the function $f(x) = 3x^4 - 2x^2 + 5$ behaves as

A. $f(x) \to -\infty$

B. $f(x) \to +\infty$

C. $f(x) \to 0$

D. $f(x) \to 5$

11. Which expression represents the explicit formula for the arithmetic sequence 7, 11, 15, 19, … ?

A. $a_n = 7n + 4$

B. $a_n = 4n + 7$

C. $a_n = 4n + 3$

D. $a_n = 7(4)$^$(n-1)$

12. The exact value of $\sin(5\pi/4)$ is

A. $\sqrt{2}/2$

B. $-\sqrt{3}/2$

C. 1/2

D. $-\sqrt{2}/2$

13. For all values of θ where the expression is defined, $\sin\theta \cdot \sec\theta$ is equivalent to

A. tan θ

B. cot θ

C. cos θ

D. csc θ

14. What is the complete solution set of the equation log(x) + log(x + 21) = 2?

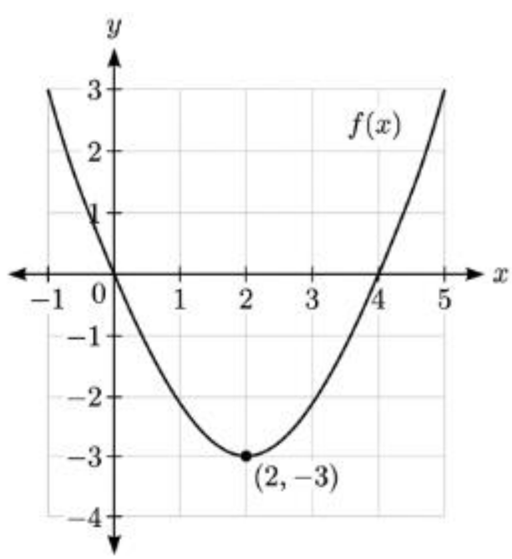
A. {4, −25}

B. {−25}

C. {25}

D. {4}

15. The graph of the function f(x) is shown below. Over which interval is f(x) increasing?



A. (−∞, 2)

B. $(2, \infty)$

C. $(0, 4)$

D. $(-3, \infty)$

16. Which of the following data sets has the largest standard deviation?

A. 10, 10, 10, 10, 10

B. 8, 9, 10, 11, 12

C. 1, 5, 10, 15, 19

D. 9, 10, 10, 10, 11

17. Solve algebraically: 9^(x − 1) = 27^x. The value of x is

A. $x = -2$

B. $x = 2$

C. $x = -1$

D. $x = 1$

18. In how many different ways can 5 different books be arranged on a shelf?

A. 25

B. 120

C. 60

D. 720

19. A 95% confidence interval for the mean of a population is (24.5, 28.7). Which statement best interprets this interval?

A. 95% of the population values lie between 24.5 and 28.7

B. The sample mean is exactly halfway between 24.5 and 28.7

C. 95% of all possible samples have means between 24.5 and 28.7

D. We are 95% confident that the true population mean lies between 24.5 and 28.7

20. For all values of θ where the expression is defined, $(1 + \tan^2\theta) \cdot \cos^2\theta$ is equivalent to

A. $\tan^2\theta$

B. $\cos^2\theta$

C. 1

D. $\sin^2\theta$

21. The exact value of $\arcsin(-1/2)$, expressed in radians, is

A. $-\pi/6$

B. $\pi/6$

C. $7\pi/6$

D. $-\pi/3$

22. When the polynomial $x^4 - 3x^2 + 5x - 2$ is divided by $(x + 2)$, the remainder is

A. 6

B. 0

C. 8

D. −8

23. A researcher wants to estimate the average GPA of all students at a large university and selects 50 students at random to compute their mean GPA. The mean GPA of these 50 students is best described as

A. the population mean of GPAs at the university

B. a sample mean used to estimate the population mean

C. the mode of the GPAs at the university

D. the standard deviation of the sample

24. A polynomial of degree 4 has zeros at $x = -2$ (multiplicity 1), $x = 1$ (multiplicity 2), and $x = 3$ (multiplicity 1), with leading coefficient 2. Which expression represents the polynomial?

A. $(x + 2)(x - 1)^2(x - 3)$

B. $2(x - 2)(x + 1)^2(x + 3)$

C. $2(x + 2)(x - 1)^2(x - 3)$

D. $2(x + 2)^2(x - 1)(x - 3)^2$

PART II — Short Constructed Response (8 questions × 2 credits = 16 credits)

Show all work. A correct answer with no supporting work will receive only 1 credit.

25. Expand and simplify the expression $(2 - 3i)^3$ in a + bi form. Show all algebraic work, including the intermediate squared expression.

26. Solve algebraically for x: $3/(x + 1) = 2/(x - 4)$. State any extraneous solutions and indicate which (if any) are rejected.

27. Use the Factor Theorem to determine whether $(x + 3)$ is a factor of $p(x) = x^3 + 2x^2 - 5x - 6$. Justify your answer with the appropriate evaluation.

28. The function $f(x) = 2^x$ is transformed to produce $g(x) = 3 \cdot 2^{(x - 1)} - 4$. Describe in order each transformation applied to f(x) to obtain g(x), and state the horizontal asymptote of g(x).

29. Solve algebraically for x: $2 \log(x) - \log(3) = \log(12)$. Show all algebraic work, including the use of logarithm properties.

30. Express $x/(x + 1) - 2/(x^2 - 1)$ as a single rational expression in simplest form. State any restrictions on the variable.

31. Find the sum of the first 30 terms of the arithmetic sequence 5, 8, 11, 14, …. Show all algebraic work, including the formula used.

32. Given that $\cos \theta = -3/5$ and $\sin \theta > 0$, find the exact value of $\tan \theta$. Show all algebraic work used to obtain your answer.

PART III — Extended Constructed Response (3 questions × 4 credits = 12 credits)

Show all work. Partial credit is awarded according to the scoring rubric.

33. The population of a town, in residents, is modeled by the function $P(t) = 5{,}000 \cdot e^{(0.03t)}$, where t is the number of years since the year 2020.

(a) State the population of the town in 2020 and the continuous annual growth rate of the population. Justify each value using the structure of the function.

(b) Algebraically determine, to the nearest tenth of a year, the number of years after 2020 at which the population will reach 12,000 residents. Show all algebraic work, including the use of logarithms.

34. The data table below shows the price, in dollars, of a certain commodity over the years 2018 through 2024.

Year (t = years since 2018)	0	1	2	3	4	5	6
Price P(t) ($)	25	32	41	53	68	88	114

(a) Using regression, write an exponential function $P(t) = a \cdot b^t$ that best models the data. Round each coefficient to the nearest hundredth.

(b) Use the regression equation from part (a) to predict the price of the commodity in the year 2027 (t = 9). Round your answer to the nearest dollar.

35. Algebraically solve for all values of x in the interval $0 \leq x \leq 2\pi$:

$\tan^2 x - 3 = 0$

Express each solution in exact form (radians). Show all algebraic work used to obtain each solution.

PART IV — Long Constructed Response (1 question × 6 credits = 6 credits)

Show all work. This problem requires multiple steps and integrates concepts from several chapters.

36. A point on the tip of a wind turbine blade rotates in a vertical circle. The center of the turbine is 40 meters above the ground, and the radius of the rotor circle is 30 meters. The turbine blade completes one full rotation every 4 seconds. At time $t = 0$ seconds, the point on the blade tip is at the lowest position (10 meters above the ground) and begins rotating.

(a) Write a cosine function $h(t)$ that models the height of the point on the blade tip, in meters above the ground, as a function of t, the time in seconds since the rotation began.

(b) State the amplitude, period, and midline of $h(t)$, and explain what each represents in the context of the rotating turbine blade.

(c) Algebraically determine all times during the first 8 seconds at which the point on the blade tip is exactly 50 meters above the ground. Round each answer to the nearest hundredth of a second. Show all algebraic work, including the use of inverse trigonometric functions.

ANSWER KEY WITH EXPLANATIONS – EXAM 45

1. B — The coefficient of $a^{n-k}b^k$ in the binomial expansion of $(a + b)^n$ equals C(n, k). For the a^2b^2 term in $(a + b)^4$, k = 2 and C(4, 2) = 4!/(2! · 2!) = 6.

2. C — Converting the logarithm to exponential form gives $x < 2^3 = 8$. The domain of $\log_2(x)$ also requires x > 0, so the complete solution must satisfy both conditions, producing 0 < x < 8.

3. A — Applying the quadratic formula with a = 2, b = 4, c = 5 yields x = (−4 ± √(16 − 40))/4 = (−4 ± √(−24))/4. Simplifying √(−24) = 2i√6 and reducing by 4 gives x = −1 ± (i√6)/2.

4. D — Cross-multiplying produces $(x + 4)^2 = (x + 2)(x + 6)$, which expands to $x^2 + 8x + 16 = x^2 + 8x + 12$. The variable terms cancel, leaving 16 = 12, a contradiction, so the equation has no solution.

5. B — The period of a cosine function in the form cos(bx) equals 2π/b. With b = π/3, the period is 2π ÷ (π/3) = 2π · (3/π) = 6.

6. A — Converting from logarithmic to exponential form gives x = 3^2.5. Using the product rule for exponents, this becomes 3^2 · 3^0.5 = 9√3.

7. D — Since sec θ = 1/cos θ, cos θ = 5/13. Using a 5-12-13 reference triangle, |sin θ| = 12/13, and in Quadrant IV sine is negative, so sin θ = −12/13.

8. C — The compound interest formula $A = P(1 + r)^t$ applies when interest is compounded annually. Substituting P = 2000, r = 0.05, and t = 10 gives $2000(1.05)^{10}$ ≈ 2000(1.62889) ≈ $3,257.79.

9. A — A logarithm returns the exponent that produces the argument from its base. Since $32 = 2^5$, $\log_2(32)$ = 5 directly by the definition of a logarithm.

10. B — A polynomial's end behavior is governed by its leading term, which here is $3x^4$. An even degree combined with a positive leading coefficient causes the function to approach +∞ as x approaches both negative and positive infinity.

11. C — The explicit formula for an arithmetic sequence is $a_n = a_1 + (n - 1)d$. Substituting $a_1 = 7$ and common difference d = 4 gives $a_n = 7 + 4(n - 1) = 4n + 3$.

12. D — The angle 5π/4 lies in Quadrant III, where sine is negative, with a reference angle of π/4. Since sin(π/4) = √2/2, applying the Quadrant III sign convention yields sin(5π/4) = −√2/2.

13. A — Substituting the reciprocal identity sec θ = 1/cos θ transforms the expression into sin θ · (1/cos θ) = sin θ/cos θ. This quotient is the definition of tan θ.

14. D — Combining the logarithms using the product rule gives log[x(x + 21)] = 2, so $x^2 + 21x = 100$. Factoring (x + 25)(x − 4) = 0 yields x = 4 or x = −25, and x = −25 is rejected because log of a negative number is undefined.

15. B — An upward-opening parabola is decreasing to the left of its vertex and increasing to the right of it. Reading the vertex from the graph at $(2, -3)$, the function is increasing for all $x > 2$, which corresponds to the interval $(2, \infty)$.

16. C — Standard deviation measures how spread out values are from the mean. The set 1, 5, 10, 15, 19 has the widest spread around its mean of 10, with deviations of ±9 and ±5, producing the largest standard deviation among the four sets.

17. A — Rewrite both sides as powers of base 3: $(3^2)^{(x-1)} = (3^3)^x$, giving $3^{(2x-2)} = 3^{(3x)}$. Equating exponents produces $2x - 2 = 3x$, which solves to $x = -2$.

18. B — Arranging n distinct objects in a row is a permutation counted by n!. For 5 different books, the number of arrangements is $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$.

19. D — A confidence interval gives a range of plausible values for the true population parameter, with the confidence level describing the long-run reliability of the procedure. The standard interpretation states that we are 95% confident the true population mean lies within the calculated bounds.

20. C — Using the Pythagorean identity $1 + \tan^2\theta = \sec^2\theta$ converts the expression into $\sec^2\theta \cdot \cos^2\theta$. Since $\sec^2\theta = 1/\cos^2\theta$, the product reduces to 1 by cancellation.

21. A — The principal range of arcsin is $[-\pi/2, \pi/2]$. Since $\sin(-\pi/6) = -1/2$ and $-\pi/6$ falls within that range, $\arcsin(-1/2) = -\pi/6$.

22. D — By the Remainder Theorem, the remainder equals p(c) when dividing by $(x - c)$, where here $c = -2$. Evaluating $p(-2) = 16 - 12 - 10 - 2 = -8$.

23. B — A statistic computed from a sample, such as the mean of 50 randomly selected students, is called a sample mean. Its purpose is to serve as a point estimate of the corresponding population parameter, the true mean GPA of all students.

24. C — Each zero c with multiplicity m contributes a factor of $(x - c)^m$ to the polynomial. Combining the zeros $x = -2$, $x = 1$ (multiplicity 2), and $x = 3$ with leading coefficient 2 produces $2(x + 2)(x - 1)^2(x - 3)$.