

# PRACTICE EXAM 44: ALGEBRA II

## REGENTS SIMULATION

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### PART I — Multiple Choice (24 questions $\times$ 2 credits = 48 credits)

1. For which value of  $k$  does the equation  $x^2 + 6x + k = 0$  have two distinct complex (non-real) solutions?

- A.  $k = 9$
- B.  $k = 0$
- C.  $k = 5$
- D.  $k = 12$

2. What is the solution set of the equation  $\sqrt{2x + 1} = 5$ ?

- A.  $\{12.5\}$
- B.  $\{6\}$
- C.  $\{12\}$
- D.  $\{2\}$

3. The product  $(2i)(-3i)(4i)$  is equivalent to

- A. 24

B.  $24i$

C.  $-24i$

D.  $-24$

4. For the function  $g(x) = 2x^3 - x$ , the average rate of change on the interval from  $x = 1$  to  $x = 3$  is

A. 25

B. 26

C. 24

D. 50

5. The graph of the function  $y = a \sin(bx)$  is shown below. What are the values of  $a$  and  $b$ ?

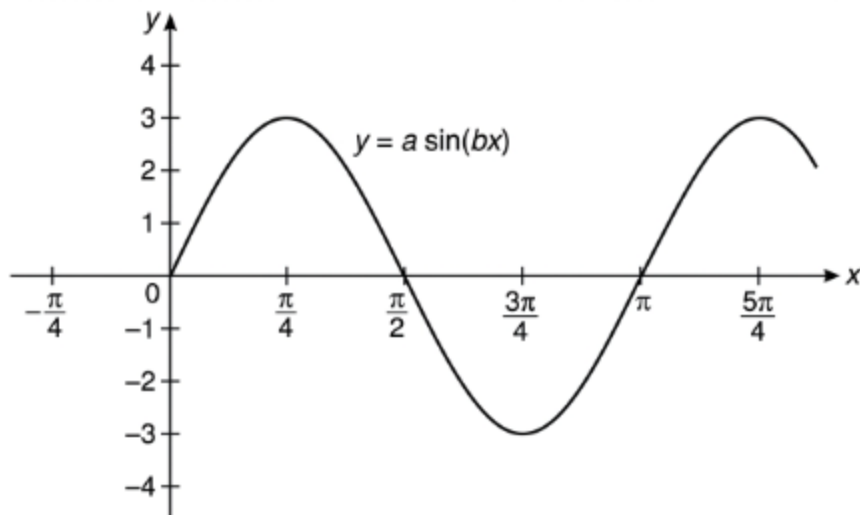


Figure PQ-1

A.  $a = 2, b = 3$

B.  $a = 3, b = \pi$

C.  $a = 3, b = 2$

D.  $a = 2, b = \pi$

6. If  $z = 4 - 7i$ , then  $z \cdot \bar{z}$  (the product of  $z$  and its complex conjugate) is equal to

A.  $16 - 49i$

B.  $49$

C.  $16 + 49i$

D.  $65$

7. If  $\log(2) = a$  and  $\log(3) = b$ , then  $\log(72)$  is equal to

A.  $3a + 2b$

B.  $2a + 3b$

C.  $6ab$

D.  $a + b$

8. If  $f(x) = x^2$  with the restricted domain  $x \geq 0$ , then  $f^{-1}(x)$  is

A.  $\pm\sqrt{x}$

B.  $\sqrt{x}$

C.  $-\sqrt{x}$

D.  $1/x^2$

9. Solve algebraically:  $x^3 + 2x^2 - 9x - 18 = 0$ . The complete solution set is

- A.  $x = -2, 9$
- B.  $x = -2$  only
- C.  $x = 0, 3, -3$
- D.  $x = -3, -2, 3$

10. \$1,000 is invested at a 6% annual interest rate compounded continuously. To the nearest cent, the balance in the account after 5 years is

- A. \$1,200.00
- B. \$1,318.08
- C. \$1,349.86
- D. \$1,500.00

11. The exact value of  $\cos(11\pi/6)$  is

- A.  $\sqrt{3}/2$
- B.  $-\sqrt{3}/2$
- C.  $1/2$
- D.  $-1/2$

12. The sum of the first 6 terms of the geometric sequence 3, 6, 12, 24, ... is

- A. 96
- B. 189
- C. 192
- D. 378

13. The function  $f(x) = 200 \cdot (1.07)^x$  represents a quantity that

- A. decreases by 7% each year
- B. increases by 1.07% each year
- C. decreases by 1.07% each year
- D. increases by 7% each year

14. What is the complete solution to the equation  $\log_3(x^2) = 4$ ?

- A.  $x = \pm 9$
- B.  $x = 9$  only
- C.  $x = \pm 12$
- D.  $x = \pm 16$

15. A box contains 6 red balls and 4 blue balls. Two balls are drawn from the box without replacement. What is the probability that both balls drawn are red?

- A.  $3/5$
- B.  $9/25$

C.  $1/3$

D.  $2/5$

16. The value of the summation  $\Sigma$  (from  $k = 1$  to  $5$ ) of  $2^k$  is

A. 32

B. 62

C. 60

D. 50

17. The domain of the function  $f(x) = \log(x - 5)$  is

A.  $x > 5$

B.  $x \geq 5$

C.  $x < 5$

D. all real numbers except 5

18. A sequence is defined by  $a_1 = 2$  and  $a_n = a_{n-1} + 5$  for  $n \geq 2$ . The sequence is best described as

A. geometric with common ratio 5

B. exponential with base 5

C. quadratic with leading coefficient 5

D. arithmetic with common difference 5

19. A polynomial  $p(x)$  has a double root at  $x = 2$  and simple roots at  $x = -1$  and  $x = 4$ . At the point  $x = 2$ , the graph of  $p(x)$

- A. crosses the  $x$ -axis from above to below
- B. crosses the  $x$ -axis from below to above
- C. touches the  $x$ -axis but does not cross it
- D. has a vertical asymptote at  $x = 2$

20. A poll of 1,000 randomly selected voters found that 58% favor a certain candidate, with a margin of error of 3% at the 95% confidence level. The 95% confidence interval for the true proportion of voters who favor the candidate is

- A. (50%, 60%)
- B. (55%, 61%)
- C. (57%, 59%)
- D. (52%, 64%)

21. For all values of  $\theta$  where the expression is defined,  $\sin^2\theta + \cos^2\theta + \tan^2\theta$  is equivalent to

- A.  $\sec^2\theta$
- B.  $\csc^2\theta$
- C. 1
- D. 2

22. What is the solution to the equation  $4^{(x+1)} = 8^{(2x-1)}$ ?

A.  $x = 1$

B.  $x = 3$

C.  $x = 1/2$

D.  $x = 5/4$

23. If  $h(x) = (x + 5)^2$ , which pair of functions  $f$  and  $g$  satisfies the composition  $f(g(x)) = h(x)$ ?

A.  $f(x) = x + 5$  and  $g(x) = x^2$

B.  $f(x) = x^2$  and  $g(x) = x + 5$

C.  $f(x) = \sqrt{x}$  and  $g(x) = (x + 5)^2$

D.  $f(x) = x + 5$  and  $g(x) = \sqrt{x}$

24. An observational study finds that students who eat breakfast score higher on standardized tests. To establish that eating breakfast causes higher test scores, researchers should

A. survey a larger random sample of students

B. analyze additional observational data carefully

C. conduct a randomized controlled experiment

D. conclude causation directly from the observed correlation

**PART II — Short Constructed Response (8 questions  $\times$  2 credits = 16 credits)**

**Show all work. A correct answer with no supporting work will receive only 1 credit.**

25. Express the expression  $(3 + 2i)^2 - (1 - i)(1 + i)$  in  $a + bi$  form. Show all algebraic work used to obtain the result.

26. Solve algebraically for  $x$ :  $(x + 3)/(x - 2) + 1 = 5/(x - 2)$ . State any extraneous solutions and indicate which (if any) are rejected.

27. Using the Rational Root Theorem, determine all rational roots of  $f(x) = x^3 - 4x^2 + x + 6$ . Show each candidate value you test and the corresponding evaluation.

28. Write the equation of the function  $g(x)$  obtained by reflecting  $f(x) = 2^x$  across the  $y$ -axis and then translating the resulting graph 4 units up.

29. Solve algebraically for  $x$ :  $\log_2(x + 3) + \log_2(x - 1) = 5$ . State any extraneous solutions and indicate which (if any) are rejected.

30. Express  $1/(x^2 - 9) + 2/(x - 3)$  as a single rational expression in simplest form. State any restrictions on the variable.

31. An arithmetic sequence has first term  $a_1 = -3$  and common difference  $d = 4$ . Write the explicit formula for the  $n$ th term  $a_n$ , and use it to find the value of  $a_{12}$ .

32. Given that  $\sin \theta = 3/5$  and  $\pi/2 < \theta < \pi$ , find the exact value of  $\tan \theta$ . Show all algebraic work used to obtain your answer.

**PART III — Extended Constructed Response (3 questions  $\times$  4 credits = 12 credits)**

**Show all work. Partial credit is awarded according to the scoring rubric.**

33. The temperature of a cup of coffee, in degrees Fahrenheit, after  $t$  minutes is modeled by the function  $T(t) = 70 + 90(0.85)^t$ .

(a) State the room temperature surrounding the coffee and the initial temperature of the coffee at  $t = 0$ . Justify each value using the structure of the function.

(b) Algebraically determine, to the nearest tenth of a minute, the amount of time required for the coffee to cool to  $100^\circ\text{F}$ . Show all algebraic work, including the use of logarithms.

34. A linear regression analysis is performed on data collected from 50 students. The variable  $x$  represents the number of hours each student studied for an exam, and the variable  $y$  represents the student's score on the exam. The regression equation is  $\hat{y} = 60 + 3.2x$ , with a correlation coefficient of  $r = 0.78$ .

(a) Interpret the slope of the regression equation in the context of this study.

(b) Use the regression equation to predict the exam score for a student who studied 8 hours. Round your answer to the nearest whole point.

35. Algebraically solve for all values of  $x$  in the interval  $0 \leq x \leq 2\pi$ :

$$2 \cos^2 x - \cos x - 1 = 0$$

Express each solution in exact form (radians). Show all algebraic work used to obtain each solution, including any substitution.

**PART IV — Long Constructed Response (1 question  $\times$  6 credits = 6 credits)**

**Show all work. This problem requires multiple steps and integrates concepts from several chapters.**

36. The number of hours of daylight in a certain town varies sinusoidally throughout the year. The maximum number of daylight hours is 15.5, which occurs on the summer solstice (day 172 of the year). The minimum number of daylight hours is 8.5, which occurs on the winter solstice (day 355 of the year). Assume the year has 365 days, with day  $d = 1$  corresponding to January 1.

(a) Write a cosine function  $H(d)$  that models the number of hours of daylight in the town on day  $d$  of the year.

(b) State the amplitude, period, and midline of  $H(d)$ , and explain what each represents in the context of the town's annual pattern of daylight.

(c) Algebraically determine all days of the year on which the town has exactly 12 hours of daylight. Round each answer to the nearest whole day. Show all algebraic work, including the use of inverse trigonometric functions.

## ANSWER KEY WITH EXPLANATIONS – EXAM 44

**1. D** — Complex solutions occur when the discriminant  $b^2 - 4ac$  is negative. Computing  $6^2 - 4(1)(k) = 36 - 4k < 0$  requires  $k > 9$ , so among the choices only  $k = 12$  produces two distinct complex solutions. The other values yield either real or repeated roots.

**2. C** — Squaring both sides eliminates the radical, giving  $2x + 1 = 25$ . Solving yields  $2x = 24$ , so  $x = 12$ . Substituting back confirms  $\sqrt{25} = 5$ , validating the solution.

**3. B** — Group the coefficients and powers of  $i$  separately:  $(2)(-3)(4) = -24$  and  $i \cdot i \cdot i = i^3 = -i$ . Multiplying the two results gives  $(-24)(-i) = 24i$ .

**4. A** — The average rate of change equals  $[g(b) - g(a)] / (b - a)$ . Computing  $g(1) = 2 - 1 = 1$  and  $g(3) = 54 - 3 = 51$  yields  $(51 - 1)/(3 - 1) = 50/2 = 25$ .

**5. C** — The amplitude equals the maximum displacement from the midline, which the graph shows is 3, so  $|a| = 3$ . The period from peak to peak is  $\pi$ , and using period =  $2\pi/b$  gives  $b = 2$ .

- 6. D** — The product of a complex number and its conjugate equals  $a^2 + b^2$ , a purely real number. Substituting  $a = 4$  and  $b = -7$  gives  $16 + 49 = 65$ , with the imaginary cross-terms cancelling exactly.
- 7. A** — Express 72 as a product of powers of 2 and 3:  $72 = 8 \cdot 9 = 2^3 \cdot 3^2$ . Applying the product and power rules of logarithms gives  $\log(72) = 3 \cdot \log(2) + 2 \cdot \log(3) = 3a + 2b$ .
- 8. B** — Restricting the domain to  $x \geq 0$  makes  $f(x) = x^2$  one-to-one, so an inverse exists. Swapping variables and solving  $y = x^2$  for the non-negative root yields  $f^{-1}(x) = \sqrt{x}$ .
- 9. D** — Group the polynomial in pairs:  $x^2(x + 2) - 9(x + 2) = (x + 2)(x^2 - 9)$ . Factoring the difference of squares completes the factorization as  $(x + 2)(x - 3)(x + 3)$ , giving roots  $x = -2, 3$ , and  $-3$ .
- 10. C** — Continuous compounding uses the formula  $A = P \cdot e^{rt}$ . Substituting  $P = 1000$ ,  $r = 0.06$ , and  $t = 5$  gives  $1000 \cdot e^{0.30} \approx 1000(1.34986) \approx \$1,349.86$ .
- 11. A** — The angle  $11\pi/6$  lies in Quadrant IV, where cosine is positive, with a reference angle of  $\pi/6$ . Since  $\cos(\pi/6) = \sqrt{3}/2$ , applying the Quadrant IV sign convention gives  $\cos(11\pi/6) = \sqrt{3}/2$ .
- 12. B** — Apply the finite geometric series formula  $S_n = a_1(1 - r^n)/(1 - r)$  with  $a_1 = 3$ ,  $r = 2$ , and  $n = 6$ . Computing  $3(1 - 64)/(1 - 2) = 3(-63)/(-1) = 189$ .
- 13. D** — In an exponential function  $f(x) = a \cdot b^x$ , a base  $b > 1$  indicates growth, and the percent rate equals  $(b - 1) \cdot 100\%$ . With  $b = 1.07$ , the function grows by  $(1.07 - 1) \cdot 100\% = 7\%$  per year.
- 14. A** — Converting the logarithmic equation to exponential form yields  $x^2 = 3^4 = 81$ , so  $x = \pm 9$ . Both values are valid because  $x^2$  is positive regardless of the sign of  $x$ , keeping the logarithm's argument inside its domain.
- 15. C** — Without replacement, the two draws are dependent events. Computing  $P(\text{red first}) = 6/10$  and  $P(\text{red second given red first}) = 5/9$ , then multiplying, gives  $(6/10)(5/9) = 30/90 = 1/3$ .
- 16. B** — Expand the summation by substituting  $k = 1$  through 5:  $2^1 + 2^2 + 2^3 + 2^4 + 2^5 = 2 + 4 + 8 + 16 + 32$ . Adding these values produces a sum of 62.
- 17. A** — A logarithm's argument must be strictly positive for the function to be defined. Setting  $x - 5 > 0$  gives  $x > 5$ , so the domain consists of all real numbers greater than 5.
- 18. D** — Adding the same constant to obtain each successive term defines an arithmetic sequence, and that constant is the common difference. The recursion adds 5 at every step, making the sequence arithmetic with common difference 5.
- 19. C** — Even multiplicity at a root produces a graph that touches the x-axis and turns around rather than crossing it. The double root at  $x = 2$  has multiplicity 2, which is even, so the graph touches but does not cross at that point.

**20. B** — A confidence interval is constructed by adding and subtracting the margin of error from the sample proportion. Computing  $58\% - 3\% = 55\%$  and  $58\% + 3\% = 61\%$  produces the interval (55%, 61%).

**21. A** — The Pythagorean identity  $\sin^2\theta + \cos^2\theta = 1$  reduces the expression to  $1 + \tan^2\theta$ . The companion identity  $1 + \tan^2\theta = \sec^2\theta$  then gives the simplified equivalent form.

**22. D** — Rewrite both sides as powers of 2:  $(2^2)^{(x+1)} = (2^3)^{(2x-1)}$ , which gives  $2^{2(x+1)} = 2^{3(2x-1)}$ . Equating exponents produces  $2x + 2 = 6x - 3$ , which solves to  $x = 5/4$ .

**23. B** — In the composition  $f(g(x))$ , the inner function  $g$  acts first. Setting  $g(x) = x + 5$  and then applying  $f(x) = x^2$  produces  $f(x + 5) = (x + 5)^2$ , matching  $h(x)$  exactly.

**24. C** — A randomized controlled experiment is the only study design that can establish a cause-and-effect relationship by isolating the variable of interest through random treatment assignment. Observational studies and larger samples can demonstrate association but cannot rule out confounding variables that could explain the relationship.