PRACTICE EXAM 43: ALGEBRA II REGENTS SIMULATION

PART I — Multiple Choice (24 questions × 2 credits = 48 credits)

1. The range of the function $f(x) = \sqrt[3]{(x + 1)} - 3$ is

A. $y \geq -3$

B. $y \leq -3$

C. all real numbers

D. $y \neq -3$

2. What is the solution to the rational equation $2x/(x + 1) = 3$?

A. $x = 1$

B. $x = 3/5$

C. $x = 3$

D. $x = -3$

3. The value of i^{102} is

A. -1

B. 1

C. i

D. −i

4. What is the vertex of the parabola defined by $y = 2(x - 3)^2 + 5$?

A. $(-3, 5)$

B. $(3, 5)$

C. $(3, -5)$

D. $(-3, -5)$

5. The roots of the equation $3x^2 + bx + 12 = 0$ have a sum of -5. The value of b is

A. -15

B. 5

C. -5

D. 15

6. The expression $\log_8(64)$ is equivalent to

A. 2

B. 3

C. 8

D. 1/2

7. For all values of θ where $\sin\theta \neq 0$, the expression $\sin\theta \cdot \cot\theta$ is equivalent to

A. $\sin\theta$

B. $\tan\theta$

C. $\cos\theta$

D. 1

8. In a class of 30 students, 18 are taking math and 12 are taking science. If 8 students are taking both subjects, what is the probability that a randomly selected student takes science, given that the student takes math?

A. 8/30

B. 4/9

C. 12/30

D. 2/3

9. Factored completely, the expression $8x^3 + 27$ is equivalent to

A. $(2x + 3)(4x^2 - 6x + 9)$

B. $(2x - 3)(4x^2 + 6x + 9)$

C. $(2x + 3)(4x^2 + 6x + 9)$

D. $(2x + 3)^3$

10. The mean of five numbers is 12. Four of the numbers are 8, 11, 14, and 16. The fifth number is

A. 8

B. 12

C. 13

D. 11

11. The polynomial $p(x) = (x + 2)(x - 1)^2(x - 4)$ has how many distinct real zeros?

A. 2

B. 3

C. 4

D. 5

12. For the system of equations $y = 3^x$ and $y = 9$, the value of x is

A. 0

B. 3

C. 2

D. 6

13. If $\tan \theta = 3/4$ and θ terminates in Quadrant III, then $\sin \theta$ is

A. 4/5

B. 3/5

C. −4/5

D. −3/5

14. What is the solution to the equation $\log(x) + \log(x - 3) = 1$?

A. $x = 5$

B. $x = -2$ or $x = 5$

C. $x = -2$

D. $x = 10$

15. If $f(x) = 2 \cdot 3^x$, then $f^{-1}(x)$ is

A. $\log_3(x - 2)$

B. $\log_3(x/2)$

C. $\log_2(x/3)$

D. $\log_2(x - 3)$

16. What is the population variance of the data set {3, 5, 7, 5, 5}, which has a mean of 5?

A. 0

B. 1.0

C. 1.6

D. 2.0

17. The 20th term of the arithmetic sequence 4, 10, 16, 22, … is

A. 118

B. 124

C. 100

D. 130

18. What is the solution to the equation $\sqrt[3]{(x - 2)} = 3$?

A. x = 5

B. x = 25

C. x = 11

D. x = 29

19. In how many ways can a committee of 3 students be chosen from a group of 8 students?

A. 24

B. 56

C. 336

D. 512

20. How many solutions does the equation $\sin x = 1/2$ have in the interval $0 \leq x \leq 2\pi$?

A. 1

B. 0

C. 2

D. 4

21. The equation $x^2 - 6x + 11 = 0$, when rewritten by completing the square, becomes

A. $(x + 3)^2 = 2$

B. $(x - 3)^2 = 11$

C. $(x - 3)^2 = 2$

D. $(x - 3)^2 = -2$

22. The solution to the equation $2\log(x) = \log(36)$ is

A. $x = 18$

B. $x = 9$

C. $x = 6$

D. $x = 36$

23. If $f(x) = x^2 - 1$ and $g(x) = \sqrt{x}$, what is the value of $(f \circ g)(9)$?

A. 8

B. 80

C. 3

D. $\sqrt{8}$

24. A scatter plot of monthly sales (y) versus advertising spending (x) has a strong positive linear correlation coefficient of r = 0.92. Which statement best describes the relationship between the two variables?

A. Increasing advertising spending causes monthly sales to increase

B. As advertising spending increases, monthly sales tend to increase

C. The two variables are completely unrelated to each other

D. The amount of monthly sales determines the advertising spending

PART II — Short Constructed Response (8 questions × 2 credits = 16 credits)

Show all work. A correct answer with no supporting work will receive only 1 credit.

25. Solve the equation $3x^2 + 27 = 0$ algebraically. Express both solutions in a + bi form.

26. Solve algebraically for x: $1 + 2/x = 15/x^2$. State any extraneous solutions and indicate which (if any) are rejected.

27. Use the Factor Theorem to determine whether $(x - 4)$ is a factor of $f(x) = 2x^3 - 9x^2 + 5x - 4$. Justify your answer with the appropriate evaluation.

28. The function f(x) is transformed to produce g(x) = f(x + 3) − 2. Describe in order each transformation applied to the graph of f(x) to obtain the graph of g(x).

29. Solve algebraically for x: 3^(2x − 1) = 27^(x − 2). Show all algebraic work.

30. Express the result of (5 + 3i)(2 − 4i) − (7 + i) in a + bi form. Show all algebraic work used to obtain the result.

31. A geometric sequence has a first term $a_1 = 6$ and a common ratio r = 1/2. Write the explicit formula for the nth term a_n, and use it to find the value of a_7.

32. Given that cos θ = 5/13 and sin θ < 0, find the exact values of sin θ and tan θ. Show the algebraic work used to obtain each value.

PART III — Extended Constructed Response (3 questions × 4 credits = 12 credits)

Show all work. Partial credit is awarded according to the scoring rubric.

33. The amount, A, of a medication remaining in a patient's bloodstream is modeled by the function $A(t) = 50(0.75)^t$, where A is measured in milligrams and t is the number of hours since the medication was administered.

(a) State the initial amount of medication, in milligrams, and the percent rate of decrease per hour.

(b) Algebraically determine, to the nearest tenth of an hour, the time at which only 10 milligrams of the medication remains in the bloodstream. Show all algebraic work, including the use of logarithms.

34. A school administrator wants to determine whether students prefer Option A or Option B for a new school policy. A simulation based on the responses of 250 randomly selected students produced a sample proportion of 0.46 in favor of Option A, with a margin of error of 0.06 at the 95% confidence level.

(a) Construct the 95% confidence interval for the true proportion of all students at the school who prefer Option A.

(b) Based on the confidence interval, can the administrator conclude with 95% confidence that the majority of all students prefer Option A? Justify your answer using the values in the interval.

35. Algebraically solve the equation $x^4 - 5x^2 + 4 = 0$ over the real numbers. Show all algebraic work, including any substitution used, and list every real solution.

PART IV — Long Constructed Response (1 question × 6 credits = 6 credits)

Show all work. This problem requires multiple steps and integrates concepts from several chapters.

36. A spotlight at a concert oscillates horizontally to track a performer who paces back and forth across the stage. The horizontal position of the spotlight beam relative to the center of the stage is modeled by a sinusoidal function. The beam reaches a maximum displacement of 12 meters to the right of center and a minimum displacement of 12 meters to the left of center. One complete back-and-forth cycle takes 20 seconds. At time $t = 0$ seconds, the spotlight beam is centered on the stage and moving toward the right.

(a) Write a sine function $p(t)$ that models the horizontal position of the spotlight beam, in meters from the center of the stage, as a function of t, the time in seconds since the show began. Use the convention that positions to the right of center are positive.

(b) State the amplitude, period, and midline of $p(t)$, and explain what each represents in the context of the spotlight's motion across the stage.

(c) Algebraically determine all times during the first 40 seconds at which the spotlight beam is exactly 8 meters to the right of center. Round each answer to the nearest hundredth of a second. Show all algebraic work, including the use of inverse trigonometric functions.

ANSWER KEY WITH EXPLANATIONS — EXAM 43

1. C — The cube root function $\sqrt[3]{x}$ has a range of all real numbers because cube roots are defined for every real input and produce every real output. Vertical translations like subtracting 3 shift the graph downward but never restrict the set of output values.

2. D — Cross-multiplying gives $2x = 3(x + 1)$, which simplifies to $2x = 3x + 3$. Solving for x produces $-x = 3$, so $x = -3$, which does not make the denominator zero and is therefore a valid solution.

3. A — Powers of i cycle through the values {i, −1, −i, 1} with period 4. Dividing 102 by 4 leaves a remainder of 2, so $i^{102} = i^2 = -1$.

4. B — A parabola written in vertex form $y = a(x - h)^2 + k$ has its vertex located at (h, k). Reading the equation directly with $h = 3$ and $k = 5$ identifies the vertex as (3, 5).

5. D — By Vieta's formulas, the sum of the roots of $ax^2 + bx + c = 0$ equals $-b/a$. Setting $-b/3 = -5$ gives $-b = -15$, so $b = 15$.

6. A — Rewriting the argument as a power of the base gives $64 = 8^2$. Therefore $\log_8(64) = \log_8(8^2) = 2$, since the logarithm returns the exponent required to produce the argument.

7. C — Substituting the quotient identity $\cot\theta = \cos\theta/\sin\theta$ transforms the expression into $\sin\theta \cdot (\cos\theta/\sin\theta)$. The $\sin\theta$ factors cancel, leaving the simplified result $\cos\theta$.

8. B — Conditional probability is defined as $P(S \mid M) = P(S \cap M) / P(M)$. Substituting the 8 students taking both subjects and the 18 students taking math gives $8/18 = 4/9$.

9. A — The sum-of-cubes formula $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$ applies with $a = 2x$ and $b = 3$. Substituting gives $(2x + 3)((2x)^2 - (2x)(3) + 3^2) = (2x + 3)(4x^2 - 6x + 9)$.

10. D — If the mean of five numbers is 12, their total must equal $12 \cdot 5 = 60$. Subtracting the four known values gives $60 - (8 + 11 + 14 + 16) = 60 - 49 = 11$.

11. B — A distinct zero is counted once regardless of its multiplicity in the factorization. Reading the factors gives zeros at $x = -2$, $x = 1$, and $x = 4$, producing 3 distinct real zeros.

12. C — Setting the two expressions for y equal gives $3^x = 9$, and rewriting 9 as 3^2 yields $3^x = 3^2$. Since the exponential function with base 3 is one-to-one, equating exponents gives $x = 2$.

13. D — In Quadrant III, both sine and cosine are negative. A 3-4-5 reference triangle gives $|\sin\theta| = 3/5$, and applying the Quadrant III sign convention yields $\sin\theta = -3/5$.

14. A — Combining the logarithms using the product rule gives $\log[x(x-3)] = 1$, so $x^2 - 3x = 10$. Factoring $(x-5)(x+2) = 0$ gives $x = 5$ or $x = -2$, and $x = -2$ is rejected because log of a negative number is undefined.

15. B — Swap x and y in $y = 2 \cdot 3^x$ to obtain $x = 2 \cdot 3^y$. Dividing both sides by 2 gives $x/2 = 3^y$, and converting to logarithmic form yields $y = \log_3(x/2)$.

16. C — Population variance is the mean of the squared deviations from the mean. The squared deviations are 4, 0, 4, 0, 0, summing to 8, and dividing by $n = 5$ gives a variance of 1.6.

17. A — The explicit formula for an arithmetic sequence is $a_n = a_1 + (n-1)d$, with $a_1 = 4$ and common difference $d = 6$. Substituting $n = 20$ gives $a_{20} = 4 + 19(6) = 4 + 114 = 118$.

18. D — Cubing both sides eliminates the cube root, producing $x - 2 = 27$. Adding 2 to both sides yields $x = 29$, and substituting back verifies the solution.

19. B — A committee selection where order does not matter is a combination, computed as $C(n, k) = n!/[k!(n-k)!]$. Computing $C(8, 3) = 8!/(3! \cdot 5!) = 56$ gives the number of possible committees.

20. C — Sine equals 1/2 in Quadrant I at $x = \pi/6$ and in Quadrant II at $x = 5\pi/6$ within one full period. Both values fall within the interval $[0, 2\pi]$, producing exactly 2 solutions.

21. D — Completing the square requires adding $(b/2)^2 = 9$ to both sides of $x^2 - 6x = -11$. This produces $x^2 - 6x + 9 = -2$, which factors as the perfect square $(x-3)^2 = -2$.

22. C — Applying the power rule converts the equation to $\log(x^2) = \log(36)$, so $x^2 = 36$ and $x = \pm 6$. The solution $x = -6$ is rejected because the original logarithm $\log(x)$ requires $x > 0$, leaving $x = 6$ as the only valid solution.

23. A — The composition $(f \circ g)(x)$ means evaluate g first and then apply f. Computing $g(9) = \sqrt{9} = 3$ and then $f(3) = 3^2 - 1 = 8$ yields the final value.

24. B — A correlation coefficient measures the strength and direction of a linear association but does not establish causation. A strong positive r near 0.92 indicates that as advertising spending increases, monthly sales tend to increase together.