PRACTICE EXAM 42: ALGEBRA II REGENTS SIMULATION

PART I — Multiple Choice (24 questions × 2 credits = 48 credits)

1. The polynomial $p(x) = x^2 + bx + c$ has real coefficients. If $3 + 2i$ is one root of $p(x)$, the other root must be

A. $3 - 2i$

B. $-3 + 2i$

C. $-3 - 2i$

D. $2 + 3i$

2. The expression $(16x^8)^{(3/4)}$, where $x > 0$, is equivalent to

A. $12x^6$

B. $8x^{11}$

C. $8x^6$

D. $12x^{11}$

3. When the polynomial $x^3 - 4x^2 + x + 6$ is divided by $x^2 - x - 2$, the quotient is

A. $x + 2$

B. $x + 3$

C. $x - 2$

D. $x - 3$

4. What is the solution to the equation $\log_2(x + 3) - \log_2(x - 1) = 2$?

A. $x = 2$

B. $x = 7/3$

C. $x = 1$

D. $x = 5/3$

5. If $f(x) = \sqrt[3]{(x - 4)} + 2$, then $f^{-1}(x)$ is equal to

A. $(x - 2)^3 + 4$

B. $(x + 2)^3 - 4$

C. $(x - 4)^3 + 2$

D. $(x - 2)^3 - 4$

6. The value of the summation Σ (from $k = 1$ to 10) of $(3k - 2)$ is

A. 130

B. 155

C. 140

D. 145

7. A radioactive isotope has a half-life of 12 years. If a sample initially weighs 80 grams, approximately how much remains after 30 years?

A. 11.3 g

B. 14.1 g

C. 16.0 g

D. 20.0 g

8. If the equation $x^2 - 7x + k = 0$ has two roots whose product is -10, what is the value of k?

A. 7

B. 10

C. -10

D. -7

9. Expressed in a + bi form, the quotient $(4 + 2i)/(1 - i)$ is equivalent to

A. $4 + 2i$

B. $2 + 6i$

C. $3 + i$

D. $1 + 3i$

10. The exact value of $\tan(2\pi/3)$ is

A. $-\sqrt{3}$

B. $\sqrt{3}$

C. $-1/\sqrt{3}$

D. $1/\sqrt{3}$

11. If a data set has a standard deviation of 4.5, the variance of the data set is

A. 4.5

B. 9.0

C. 20.25

D. 2.12

12. The sum of the infinite geometric series $1 + 1/3 + 1/9 + 1/27 + \ldots$ is

A. 4/3

B. 3/2

C. 2

D. unbounded (no finite sum)

13. Solve the equation $5(2)^x = 60$ algebraically for x. Rounded to the nearest hundredth, x is approximately

A. 6.00

B. 12.00

C. 2.49

D. 3.58

14. The function g(x) is obtained from $f(x) = x^2$ by a vertical stretch by a factor of 2 followed by a translation 5 units to the left. Which equation defines g(x)?

A. $g(x) = 2(x + 5)^2$

B. $g(x) = 2(x - 5)^2$

C. $g(x) = (2x + 5)^2$

D. $g(x) = (1/2)(x + 5)^2$

15. A survey of 200 randomly selected high school students found that 124 of the students had a part-time job. The sample proportion of students with a part-time job is

A. 0.38

B. 0.62

C. 0.31

D. 0.50

16. For all values of θ where $\sin \theta \neq 0$, the expression $(1 - \cos^2\theta)/\sin \theta$ is equivalent to

A. $\cos \theta$

B. 1

C. sin θ

D. tan θ

17. A sinusoidal function f(x) is shown in the graph below. What is the period of f(x)?

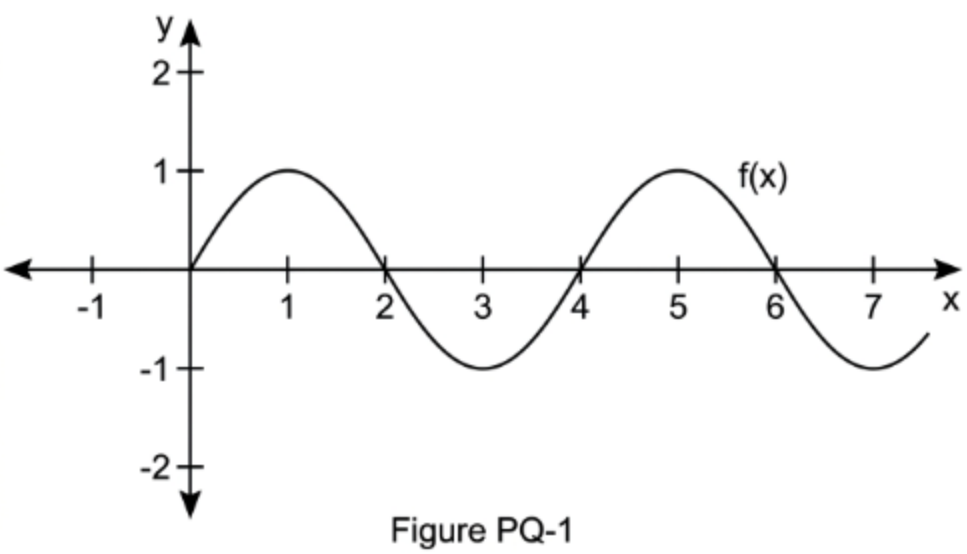

Figure PQ-1

A. 4

B. 2

C. 8

D. 1

18. The vertical asymptote(s) of the rational function $f(x) = (x + 3)/(x^2 - 4)$ are located at

A. $x = -3$ only

B. $x = 2$ only

C. $y = 0$

D. $x = 2$ and $x = -2$

19. How many real solutions does the system $y = x^2 + 1$ and $y = 2x$ have?

A. 0

B. 1

C. 2

D. infinitely many

20. Which sequence has the explicit formula $a_n = 2 \cdot 5^{(n-1)}$?

A. 2, 7, 12, 17, …

B. 5, 10, 15, 20, …

C. 2, 10, 50, 250, …

D. 5, 25, 125, 625, …

21. The probability that a certain event occurs is 3/7. The probability that the event does not occur is

A. 3/7

B. 7/3

C. 1/7

D. 4/7

22. For the function $f(x) = 5\sin(x) + 3$, the maximum value of $f(x)$ is

A. 8

B. 5

C. 3

D. −2

23. A normal distribution has a mean of 50 and a standard deviation of 10. Approximately what percent of data values fall below 40?

A. 32%

B. 34%

C. 16%

D. 84%

24. The expression i^{47} is equivalent to

A. 1

B. −i

C. i

D. −1

PART II — Short Constructed Response (8 questions × 2 credits = 16 credits)

Show all work. A correct answer with no supporting work will receive only 1 credit.

25. Solve the equation $x^2 + 8x + 25 = 0$ algebraically. Express both solutions in a + bi form.

26. Solve algebraically for x: $2x/(x - 3) - 4 = 6/(x - 3)$. State any extraneous solutions and indicate which (if any) are rejected.

27. Given the polynomial $p(x) = (x - 2)^2(x + 1)(x - 5)^3$, state each zero of p(x) and its multiplicity. Then state the total number of zeros of p(x), counting multiplicity.

28. Describe in order the sequence of transformations applied to $f(x) = \sqrt{x}$ to obtain $g(x) = -2\sqrt{(x - 3)} + 1$. Then state the domain and range of g(x).

29. Solve algebraically for x: $4 \log_2(x) = 12$. Show all work.

30. Express $x/(x + 2) + 3/(x - 4)$ as a single rational expression in simplest form. State any restrictions on the variable.

31. Calculate the average rate of change of the function $f(x) = 3x^2 - 2x + 1$ over the interval $[-1, 4]$. Show all algebraic work used to obtain your answer.

32. Given that $\sin \theta = \sqrt{3}/2$ and $\pi/2 < \theta < \pi$, find the exact value of $\cos(2\theta)$. Show all algebraic work, including any identity used.

PART III — Extended Constructed Response (3 questions × 4 credits = 12 credits)

Show all work. Partial credit is awarded according to the scoring rubric.

33. A bank account earns an annual interest rate of 4% compounded monthly. An initial deposit of $5,000 is made, and no further deposits or withdrawals are made.

(a) Write a function A(t) that models the account balance, in dollars, after t years.

(b) Algebraically determine, to the nearest tenth of a year, the amount of time required for the balance in the account to double. Show all algebraic work, including the use of logarithms.

34. Scores on a standardized college admissions test are normally distributed with a mean of 500 and a standard deviation of 100.

(a) Determine the z-score for a student who scored 650 on the test. Round your answer to the nearest hundredth.

(b) Using the empirical (68–95–99.7) rule, estimate the percentage of test-takers who scored between 400 and 700. Show the reasoning that justifies your estimate.

35. Algebraically solve for all values of x in the interval $0 \leq x \leq 2\pi$:

$2 \sin x \cos x = \sin x$

Express each solution in exact form (radians). Show all algebraic work used to obtain each solution.

PART IV — Long Constructed Response (1 question × 6 credits = 6 credits)

Show all work. This problem requires multiple steps and integrates concepts from several chapters.

36. A buoy floats in the open ocean, and its vertical position above and below the average sea level varies sinusoidally as ocean waves pass beneath it. The buoy reaches a maximum height of 1.5 meters above average sea level and a minimum height of 1.5 meters below average sea level. One complete wave cycle takes 8 seconds. At time t = 0 seconds, the buoy is at the average sea level position and rising.

(a) Write a sine function h(t) that models the height of the buoy, in meters, above average sea level as a function of t, the time in seconds since the buoy was first observed.

(b) State the amplitude, period, and midline of h(t), and explain what each represents in the context of the buoy's motion through the passing waves.

(c) Algebraically determine all times during the first 16 seconds at which the buoy is exactly 1 meter above the average sea level. Round each answer to the nearest hundredth of a second. Show all algebraic work, including the use of inverse trigonometric functions.

ANSWER KEY WITH EXPLANATIONS — EXAM 42

1. A — The Complex Conjugate Root Theorem states that if a polynomial with real coefficients has a complex root a + bi, then its conjugate a − bi must also be a root. Since p(x) has real coefficients and 3 + 2i is one root, its conjugate 3 − 2i must also be a root.

2. C — Apply the power rule for exponents by raising each factor inside the parentheses to the 3/4 power. Computing 16^(3/4) = (2⁴)^(3/4) = 2³ = 8, and (x⁸)^(3/4) = x^(8 · 3/4) = x⁶. Combining these factors yields 8x⁶.

3. D — Performing polynomial long division, the first term of the quotient is $x^3/x^2 = x$; multiplying back and subtracting leaves $-3x^2 + 3x + 6$. The next term $-3x^2/x^2 = -3$ multiplied back gives a remainder of 0, so the full quotient is x − 3.

4. B — Combining the logarithms using the quotient rule gives $\log_2[(x + 3)/(x - 1)] = 2$. Converting to exponential form yields $(x + 3)/(x - 1) = 4$, which simplifies to $x + 3 = 4x - 4$ and solves to $x = 7/3$.

5. A — Finding the inverse requires swapping x and y in $y = \sqrt[3]{(x - 4)} + 2$, producing $x = \sqrt[3]{(y - 4)} + 2$. Isolating the cube root, cubing both sides, and solving for y yields $f^{-1}(x) = (x - 2)^3 + 4$.

6. D — Apply linearity of summation: $\Sigma(3k - 2) = 3 \cdot \Sigma k - \Sigma 2$. Substituting Σk from 1 to 10 = 55 and the constant sum $2 \cdot 10 = 20$ gives $3(55) - 20 = 165 - 20 = 145$.

7. B — The half-life formula $A(t) = A_0 \cdot (1/2)^{(t/h)}$ models exponential decay where h is the half-life. Substituting $A_0 = 80$, $h = 12$, and $t = 30$ gives $80(0.5)^{2.5} \approx 80(0.1768) \approx 14.1$ grams.

8. C — By Vieta's formulas, the product of the roots of $ax^2 + bx + c = 0$ equals c/a. Here a = 1 and c = k, so the product of the roots equals k itself, making $k = -10$.

9. D — Multiply the numerator and denominator by the conjugate of the denominator, $1 + i$: $[(4 + 2i)(1 + i)] / [(1 - i)(1 + i)] = (2 + 6i)/2$. Dividing each term by 2 yields $1 + 3i$ with the imaginary part eliminated from the denominator.

10. A — The angle $2\pi/3$ lies in Quadrant II, where tangent is negative, with a reference angle of $\pi/3$. Since $\tan(\pi/3) = \sqrt{3}$, applying the Quadrant II sign convention gives $\tan(2\pi/3) = -\sqrt{3}$.

11. C — Standard deviation is defined as the positive square root of variance, so variance equals the square of standard deviation. Squaring 4.5 yields 20.25, the variance of the data set.

12. B — An infinite geometric series converges when $|r| < 1$, with sum $S = a/(1 - r)$. Substituting $a = 1$ and $r = 1/3$ gives $S = 1/(1 - 1/3) = 1/(2/3) = 3/2$.

13. D — Divide both sides by 5 to isolate the exponential: $2^x = 12$. Applying the change-of-base formula gives $x = \ln(12)/\ln(2) \approx 2.4849/0.6931 \approx 3.58$.

14. A — A vertical stretch by factor a multiplies the output by a, transforming x^2 into $2x^2$. A horizontal translation 5 units to the left replaces x with $(x + 5)$, producing $g(x) = 2(x + 5)^2$.

15. B — A sample proportion equals the number of successes divided by the sample size. Computing 124/200 yields 0.62, representing 62% of the surveyed students.

16. C — The Pythagorean identity $\sin^2\theta + \cos^2\theta = 1$ rearranges to $1 - \cos^2\theta = \sin^2\theta$. Substituting into the expression gives $\sin^2\theta/\sin\theta$, which simplifies to $\sin\theta$ when divided by the common factor.

17. A — The period of a sinusoidal function is the horizontal distance between two consecutive maxima. The graph shows maxima at $x = 1$ and $x = 5$, so the period equals $5 - 1 = 4$.

18. D — Vertical asymptotes occur where the denominator equals zero and the numerator does not. Solving $x^2 - 4 = 0$ gives $x = \pm 2$, and neither value makes the numerator $(x + 3)$ zero, so both $x = 2$ and $x = -2$ are vertical asymptotes.

19. B — Substituting $y = 2x$ into $y = x^2 + 1$ yields $x^2 - 2x + 1 = 0$, which factors as $(x - 1)^2$. The repeated root indicates the line is tangent to the parabola, producing exactly one point of intersection.

20. C — Substituting n = 1 into $a_n = 2 \cdot 5^{(n-1)}$ gives $a_1 = 2 \cdot 5^0 = 2$, and each subsequent term multiplies the previous one by the common ratio 5. The resulting sequence 2, 10, 50, 250, … matches the explicit formula exactly.

21. D — The Complement Rule states that the probability an event does not occur equals 1 minus the probability that it does occur. Computing $1 - 3/7 = 4/7$ yields the complementary probability.

22. A — For a sinusoidal function of the form $a\sin(x) + d$, the maximum value occurs when $\sin(x) = 1$ and equals $a + d$. Substituting $a = 5$ and $d = 3$ gives a maximum value of 8.

23. C — Using the empirical rule, about 68% of values lie within one standard deviation of the mean, leaving roughly 32% split evenly between the two tails. The lower tail below $\mu - \sigma = 40$ holds half of that, or approximately 16%.

24. B — Powers of i cycle through the values $\{i, -1, -i, 1\}$ with period 4. Dividing 47 by 4 leaves a remainder of 3, so $i^{47} = i^3 = -i$.