PRACTICE EXAM 41: ALGEBRA II REGENTS SIMULATION

PART I — Multiple Choice (24 questions × 2 credits = 48 credits)

1. As $x \to \infty$, which best describes the end behavior of the polynomial function $f(x) = -2x^5 + 3x^3 - x + 7$?

A. $f(x) \to \infty$

B. $f(x) \to -\infty$

C. $f(x) \to 7$

D. $f(x) \to 0$

2. The modulus (absolute value) of the complex number $5 - 12i$ is

A. 7

B. 17

C. −7

D. 13

3. When the polynomial $3x^3 - 7x^2 + 4x - 9$ is divided by $(x - 2)$, the remainder is

A. −5

B. 5

C. −9

D. 13

4. What is the solution set of the equation $x/(x - 4) = 12/(x - 4) + 3$?

A. {4}

B. {12}

C. {0}

D. { }

5. Solve the radical equation $\sqrt{2x + 7} = x - 4$. The solution set is

A. {1, 9}

B. {1}

C. { }

D. {9}

6. Which expression is equivalent to $\log(x^2y / z^3)$, where x, y, and z are positive real numbers?

A. $2 \log x \cdot \log y / 3 \log z$

B. $2 \log x + \log y - 3 \log z$

C. $\log(2x) + \log y - \log(3z)$

D. $2 \log x - \log y + 3 \log z$

7. Which equation results from completing the square on $x^2 + 8x - 5 = 0$?

A. $(x + 4)^2 = 21$

B. $(x - 4)^2 = 21$

C. $(x + 4)^2 = 11$

D. $(x - 8)^2 = 21$

8. A sequence is defined recursively by $a_1 = 5$ and $a_n = 3a_{n-1} - 4$ for $n \geq 2$. What is the value of a_3?

A. 11

B. 23

C. 29

D. 41

9. The domain of the function $f(x) = \sqrt{(x - 3)} / (x - 7)$ is

A. $(-\infty, 3] \cup (7, \infty)$

B. $(3, 7) \cup (7, \infty)$

C. all real numbers except 7

D. $[3, 7) \cup (7, \infty)$

10. In the binomial expansion of $(x + 2)^5$, what is the coefficient of the x^3 term?

A. 40

B. 20

C. 80

D. 10

11. If $2000 is invested at an annual interest rate of 5% compounded quarterly, how much will be in the account after 8 years?

A. $2,800.00

B. $2,976.26

C. $2,953.71

D. $3,011.04

12. On a normally distributed exam, the mean score is 76 and the standard deviation is 6. The interval that captures approximately 95% of scores is

A. 70 to 82

B. 76 to 88

C. 64 to 88

D. 58 to 94

13. A value of 88 occurs in a normal distribution with a mean of 75 and a standard deviation of 6.5. The z-score for this value is approximately

A. 2.00

B. 1.50

C. −2.00

D. 0.50

14. The graph of the function f(x) is shown below. For what value(s) of x does f(x) = 0?

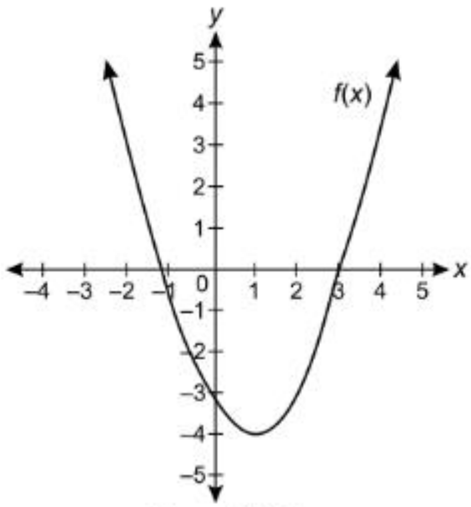


Figure PQ-1

A. x = −4 only

B. x = 1 only

C. x = −1 only

D. x = −1 and x = 3

15. What is the smallest positive solution to the equation 2 cos x − 1 = 0?

A. π/6

B. $\pi/3$

C. $\pi/2$

D. $2\pi/3$

16. A polynomial function p(x) has degree 5. According to the Fundamental Theorem of Algebra, p(x) must have exactly

A. 5 distinct real zeros

B. 5 rational zeros

C. 5 zeros in the complex number system, counting multiplicity

D. at most 5 zeros, possibly fewer

17. What is the exact value of arctan(−1), expressed in radians?

A. $-\pi/4$

B. $\pi/4$

C. $3\pi/4$

D. $-3\pi/4$

18. Solve the equation 2^x = 50. Rounded to the nearest hundredth, x is approximately

A. 25.00

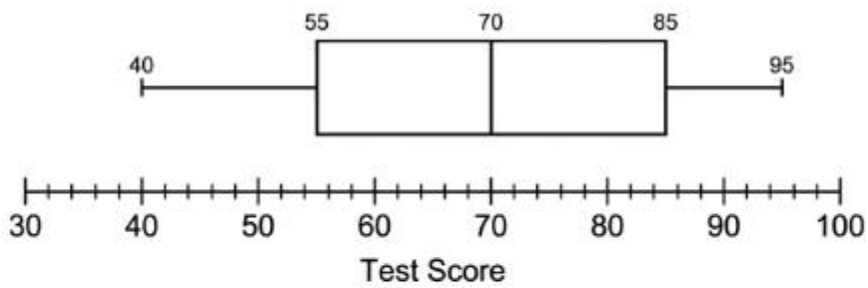
B. 1.70

C. 4.64

D. 5.64

19. The box-and-whisker plot below summarizes the test scores of 50 students. What is the interquartile range (IQR) of the data?

Figure PQ-2: Box Plot Analysis



A. 25

B. 40

C. 30

D. 55

20. What is the sum of the first 20 terms of the arithmetic sequence 7, 11, 15, 19, … ?

A. 540

B. 900

C. 1900

D. 1140

21. If $\log_5(x) = 3$, then the value of x is

A. 125

B. 15

C. 243

D. 8

22. A fair coin is flipped 4 times. What is the probability of obtaining exactly 3 heads?

A. 1/2

B. 3/4

C. 1/8

D. 1/4

23. A simulation of 500 trials produces a sample proportion of 0.6 with a margin of error of 0.04 at the 95% confidence level. The 95% confidence interval for the true population proportion is

A. (0.50, 0.70)

B. (0.56, 0.64)

C. (0.58, 0.62)

D. (0.52, 0.68)

24. Solve the equation e^(2x) = 7 algebraically for x. Rounded to the nearest hundredth, x is approximately

A. 1.95

B. 3.50

C. 0.97

D. 1.25

PART II — Short Constructed Response (8 questions × 2 credits = 16 credits)

Show all work. A correct answer with no supporting work will receive only 1 credit.

25. Express the quotient $(3 + i) / (2 - i)$ in $a + bi$ form. Show all algebraic work, including multiplication by the conjugate of the denominator.

26. Solve the equation $x^3 - 27 = 0$ algebraically over the complex numbers. Express any non-real solutions in $a + bi$ form.

27. Verify algebraically that the Pythagorean identity $\sin^2\theta + \cos^2\theta = 1$ holds for the angle $\theta = 5\pi/4$. Substitute exact trigonometric values and simplify to justify your answer.

28. Solve the equation $2x^2 + 5x - 12 = 0$ algebraically. Show all algebraic work used to obtain both solutions.

29. An exponential function of the form $f(x) = a \cdot 2^x$ passes through the point (3, 40). Determine the value of a, and then write the explicit equation for f(x).

30. Express $(3x)/(x - 1) - 2/(x + 2)$ as a single rational expression in simplest form. State any restrictions on the variable.

31. Given the polynomial $p(x) = 2x^3 - 7x^2 + 2x + 3$, list all possible rational roots according to the Rational Root Theorem. Then test the candidates and identify one actual rational root of $p(x)$. Show all work.

32. Given that $\cos \theta = 7/25$ and θ terminates in Quadrant IV, find the exact value of $\tan \theta$. Show the algebraic work that justifies your answer.

PART III — Extended Constructed Response (3 questions × 4 credits = 12 credits)

Show all work. Partial credit is awarded according to the scoring rubric.

33. A new automobile is purchased for $28,000. The value of the automobile depreciates by 12% each year.

(a) Write an exponential function $V(t)$ that models the value of the automobile, in dollars, after t years of ownership.

(b) Algebraically determine the first whole year in which the value of the automobile drops below $10,000. Show all algebraic work, including the use of logarithms.

34. The data set below records the heights, in centimeters, of 10 randomly selected high school students:

158, 162, 165, 170, 171, 174, 175, 178, 182, 185

(a) Calculate the mean and the sample standard deviation of the data. Round each value to the nearest hundredth.

(b) Using the values from part (a), determine the number of students whose heights fall within one sample standard deviation of the mean. Show the interval used in your reasoning.

35. Algebraically solve for all values of x in the interval $0 \leq x \leq 2\pi$:

$$4\cos^2 x - 3 = 0$$

Express each solution in exact form (radians). Show all algebraic work used to obtain each solution.

PART IV — Long Constructed Response (1 question × 6 credits = 6 credits)

Show all work. This problem requires multiple steps and integrates concepts from several chapters.

36. The average daily temperature, in degrees Fahrenheit, in a certain northern city varies sinusoidally throughout the year. The maximum average daily temperature of 82°F occurs on July 21 (day 202 of the year), and the minimum average daily temperature of 28°F occurs on January 21 (day 21 of the year). Assume the year has 365 days, with day 1 corresponding to January 1.

(a) Write a cosine function T(d) that models the average daily temperature, in degrees Fahrenheit, as a function of d, the day of the year.

(b) State the amplitude, period, and midline of the function T(d), and explain what each represents in the context of the city's seasonal temperature pattern.

(c) Algebraically determine all days of the year on which the average daily temperature is exactly 60°F. Round each answer to the nearest whole day. Show all algebraic work, including the use of inverse trigonometric functions.

ANSWER KEY WITH EXPLANATIONS – EXAM 41

1. B — A polynomial's end behavior is determined entirely by its leading term, which here is $-2x^5$. With an odd degree and a negative leading coefficient, the function approaches $-\infty$ as x grows to positive infinity. Lower-degree terms become negligible as $|x|$ becomes large.

2. D — The modulus of a complex number a + bi equals $\sqrt{(a^2 + b^2)}$, representing the distance from the origin in the complex plane. Substituting a = 5 and b = −12 gives $\sqrt{(25 + 144)} = \sqrt{169} = 13$. The sign of b does not affect the modulus because it is squared.

3. A — By the Remainder Theorem, the remainder when p(x) is divided by (x − c) equals p(c). Evaluating $p(2) = 3(8) - 7(4) + 4(2) - 9 = 24 - 28 + 8 - 9 = -5$. This shortcut avoids long division entirely.

4. C — Multiplying both sides by (x − 4) yields $x = 12 + 3(x - 4)$, which simplifies to $x = 3x$, so $x = 0$. Substituting back gives $0/(-4) = 0$ and $12/(-4) + 3 = 0$, confirming the solution without creating extraneous values.

5. D — Squaring both sides produces $2x + 7 = x^2 - 8x + 16$, which rearranges to $x^2 - 10x + 9 = 0$ with factors $(x - 1)(x - 9)$. Testing x = 1 gives $\sqrt{9} = 3 \neq -3$, so it is extraneous; only x = 9 satisfies the original radical equation.

6. B — Logarithm rules convert products to sums, quotients to differences, and exponents to multipliers. Applying all three to $\log(x^2y/z^3)$ gives $\log(x^2) + \log(y) - \log(z^3) = 2 \log x + \log y - 3 \log z$.

7. A — Completing the square requires isolating the constant and adding $(b/2)^2$ to both sides. Starting from $x^2 + 8x = 5$, adding 16 gives $x^2 + 8x + 16 = 21$, which factors into the perfect square $(x + 4)^2 = 21$.

8. C — Apply the recursion in sequence: $a_2 = 3(5) - 4 = 11$, then $a_3 = 3(11) - 4 = 29$. Each term depends only on the previous term, so the sequence must be evaluated in order.

9. D — The radical $\sqrt{(x - 3)}$ requires $x - 3 \geq 0$, so $x \geq 3$, and the denominator (x − 7) must not equal zero, so $x \neq 7$. Combining both restrictions produces the domain $[3, 7) \cup (7, \infty)$.

10. A — In the binomial expansion of $(x + 2)^5$, the general term is $C(5, k) \cdot x^{(5 - k)} \cdot 2^k$. The x^3 term corresponds to k = 2, giving the coefficient $C(5, 2) \cdot 2^2 = 10 \cdot 4 = 40$.

11. B — Apply the compound interest formula $A = P(1 + r/n)^{(nt)}$ with P = 2000, r = 0.05, n = 4 quarterly periods, and t = 8 years. Computing $2000(1.0125)^{32} \approx 2000(1.48813) \approx \$2{,}976.26$.

12. C — The empirical rule states that approximately 95% of data in a normal distribution lies within two standard deviations of the mean. With mean 76 and standard deviation 6, the interval is 76 ± 12, producing 64 to 88.

13. A — The z-score formula $z = (x - \mu)/\sigma$ converts a raw value into the number of standard deviations it lies from the mean. Substituting yields $(88 - 75)/6.5 = 13/6.5 = 2.00$, indicating the value is two standard deviations above the mean.

14. D — The values at which $f(x) = 0$ correspond exactly to the x-intercepts of the graph. The parabola shown crosses the x-axis at $x = -1$ and $x = 3$, so both values are roots of the function.

15. B — Solving $2 \cos x - 1 = 0$ isolates $\cos x = 1/2$. The smallest positive angle whose cosine equals 1/2 is $\pi/3$, since $\cos(\pi/3) = 1/2$ in the first quadrant.

16. C — The Fundamental Theorem of Algebra guarantees that every polynomial of degree n has exactly n zeros in the complex number system when each zero is counted with its multiplicity. A polynomial of degree 5 therefore has exactly 5 such zeros, though some may repeat or be non-real.

17. A — The principal range of arctan is $(-\pi/2, \pi/2)$, and $\tan(-\pi/4) = -1$. Since $-\pi/4$ falls within the principal range, $\arctan(-1) = -\pi/4$. The output is negative because the input is negative.

18. D — Taking the natural logarithm of both sides gives $x \cdot \ln(2) = \ln(50)$, so $x = \ln(50)/\ln(2) \approx 3.912/0.693 \approx 5.64$. The change-of-base relationship is essential when the equation cannot be solved by equating like bases.

19. C — The interquartile range measures the spread of the middle 50% of data and equals Q3 − Q1. Reading the box-and-whisker plot gives Q3 = 85 and Q1 = 55, so IQR = 85 − 55 = 30.

20. B — Apply the arithmetic series formula $S_n = (n/2)[2a_1 + (n - 1)d]$ with $n = 20$, $a_1 = 7$, and common difference $d = 4$. This produces $S_{20} = 10[14 + 76] = 10(90) = 900$.

21. A — Converting $\log_5(x) = 3$ from logarithmic to exponential form gives $x = 5^3 = 125$. The two forms are inverse operations and can always be rewritten using base^exponent = argument.

22. D — The binomial probability formula gives P(k successes) = C(n, k) · p^k · (1 − p)^(n − k). With $n = 4$, $k = 3$, and $p = 1/2$, the result is $C(4, 3) \cdot (1/2)^4 = 4 \cdot 1/16 = 1/4$.

23. B — A confidence interval is constructed by subtracting and adding the margin of error to the sample proportion. Computing 0.6 − 0.04 = 0.56 and 0.6 + 0.04 = 0.64 produces the 95% confidence interval (0.56, 0.64).

24. C — Taking the natural logarithm of both sides yields $2x = \ln(7)$, so $x = \ln(7)/2 \approx 1.9459/2 \approx 0.97$. Natural log is the standard tool for solving exponential equations whose base is e.