PRACTICE EXAM 40: ALGEBRA II REGENTS SIMULATION

PART I — Multiple Choice (24 questions × 2 credits = 48 credits)

1. When the polynomial $p(x) = 2x^3 + 5x^2 - 4x + 3$ is divided by $(x + 2)$, the remainder is

A. −15

B. 7

C. 11

D. 15

2. Expressed in $a + bi$ form, the expression $(3 - 2i)^2$ is equivalent to

A. $5 + 12i$

B. $5 - 12i$

C. $13 - 12i$

D. $9 - 12i$

3. The solution to the rational equation $2/(x + 1) = 5/(x + 10)$ is

A. $x = 5$

B. $x = -5$

C. $x = 15$

D. $x = 3$

4. What is the solution to the equation $\log_3(x) + \log_3(x - 6) = 3$?

A. $x = 3$

B. $x = 6$

C. $x = 9$

D. $x = 27$

5. The exact value of $\sin(7\pi/6)$ is

A. $-1/2$

B. $1/2$

C. $-\sqrt{3}/2$

D. $\sqrt{3}/2$

6. Factored completely, the expression $x^3 - 8$ is equivalent to

A. $(x - 2)^3$

B. $(x - 2)(x^2 - 2x + 4)$

C. $(x + 2)(x^2 - 2x + 4)$

D. $(x - 2)(x^2 + 2x + 4)$

7. What is the sum of the first 10 terms of the arithmetic sequence 3, 7, 11, 15, … ?

A. 39

B. 165

C. 210

D. 240

8. The function $f(t) = 1500(0.92)^t$ models the value, in dollars, of an asset over time in years. The annual depreciation rate is

A. 0.92%

B. 8%

C. 92%

D. 108%

9. Which statement best describes the nature of the solutions of $9x^2 - 12x + 4 = 0$?

A. Two distinct irrational solutions

B. One repeated rational solution

C. Two distinct complex solutions

D. Two distinct rational solutions

10. If $f(x) = x^2 + 1$ and $g(x) = 2x - 3$, what is the value of $f(g(2))$?

A. 2

B. 5

C. 10

D. −2

11. If $f(x) = (x - 4)/2$, then $f^{-1}(x)$ is equal to

A. $(x + 4)/2$

B. $2/(x - 4)$

C. $2x - 4$

D. $2x + 4$

12. What is the solution to the equation $\log_5(2x + 3) = 2$?

A. $x = 7.5$

B. $x = 5$

C. $x = 11$

D. $x = 23$

13. The amplitude of the function $f(x) = -4\cos(3x) + 2$ is

A. 4

B. −4

C. 3

D. 2

14. When $p(x) = x^4 - 2x^3 + x - 5$ is divided by $(x - 1)$, the remainder is

A. 1

B. −3

C. −5

D. 5

15. On a standardized test, scores are normally distributed with a mean of 100 and a standard deviation of 15. Approximately what percent of scores fall above 130?

A. 16%

B. 13.5%

C. 5%

D. 2.5%

16. The first three terms of a geometric sequence are 3, 12, 48. Which expression represents the explicit formula for the nth term?

A. $a_n = 3 + 4(n - 1)$

B. $a_n = 3 \cdot 4^{(n-1)}$

C. $a_n = 4 \cdot 3^{(n-1)}$

D. $a_n = 12 \cdot 4^{(n-1)}$

17. The solutions to the equation $x^2 - 6x + 25 = 0$ are

A. $x = 3 \pm 4i$

B. $x = -3 \pm 4i$

C. $x = 6 \pm 8i$

D. $x = 3 \pm 2i$

18. Which expression is equivalent to $(x^2 + 5x + 6) / (x^2 - 4)$?

A. $(x + 3)/(x + 2)$, $x \neq -2$, $x \neq 2$

B. $(x - 3)/(x - 2)$, $x \neq -2$, $x \neq 2$

C. $(x + 2)/(x - 2)$, $x \neq -2$, $x \neq 2$

D. $(x + 3)/(x - 2)$, $x \neq -2$, $x \neq 2$

19. What is the solution to the equation $8^{(2x)} = 32$?

A. $x = 5/3$

B. $x = 5/6$

C. $x = 3/5$

D. $x = 6/5$

20. Given that $\cos \theta = -3/5$ and θ terminates in Quadrant III, the value of $\sin \theta$ is

A. 4/5

B. 3/4

C. −4/5

D. −3/4

21. A bag contains 4 red, 3 blue, and 5 green marbles. A marble is drawn at random, replaced, and a second marble is drawn. What is the probability that the first is red and the second is green?

A. 5/12

B. 20/132

C. 1/3

D. 5/36

22. Which method best describes a simple random sample of 50 students chosen from a high school of 1,000 students?

A. Survey the first 50 students who enter the cafeteria at lunchtime

B. Assign each student a number and use a random number generator to pick 50

C. Survey 10 students randomly selected from each grade level

D. Survey the 50 students with the highest grade point averages

23. A scatter plot shows that as the value of x increases by 1, the value of y consistently changes by a constant amount. Which type of regression model best fits the data?

A. Linear regression

B. Quadratic regression

C. Exponential regression

D. Logarithmic regression

24. The graph of $f(x) = |x|$ is transformed to produce $g(x) = -|x - 2| + 5$. Which best describes the transformation?

A. Shift left 2, reflect across the x-axis, shift up 5

B. Shift right 2, reflect across the y-axis, shift down 5

C. Shift right 2, reflect across the x-axis, shift up 5

D. Shift left 2, reflect across the y-axis, shift up 5

PART II — Short Constructed Response (8 questions × 2 credits = 16 credits)

Show all work. A correct answer with no supporting work will receive only 1 credit.

25. Solve the equation $x^2 - 4x + 20 = 0$ algebraically. Express your solutions in $a + bi$ form.

26. Solve algebraically for x: $x/(x - 3) + 2 = 6/(x - 3)$. State any extraneous solutions and indicate which (if any) are rejected.

27. Given $p(x) = x^3 + 2x^2 - 5x - 6$, determine whether $(x + 1)$ is a factor of $p(x)$. Justify your answer using the Factor Theorem.

28. Write an equation, in the form $g(x) = a \cdot f(x - h) + k$, for the function obtained by vertically stretching $f(x) = x^2$ by a factor of 3 and then translating the resulting graph 2 units down.

29. Solve algebraically for x: $\log_4(x + 6) - \log_4(x) = 1$.

30. Express $x/(x^2 - 4) - 3/(x + 2)$ as a single rational expression in simplest form. State any restrictions on the variable.

31. Find the sum of the first 8 terms of the geometric sequence with first term $a_1 = 2$ and common ratio $r = 3$. Show the algebraic work used to arrive at your answer.

32. Given that $\tan \theta = 5/12$ and θ terminates in Quadrant III, find the exact value of $\sin \theta$. Show the algebraic work that justifies your answer.

PART III — Extended Constructed Response (3 questions × 4 credits = 12 credits)

Show all work. Partial credit is awarded according to the scoring rubric.

33. A culture of bacteria triples every 4 hours. The initial population of the culture is 250 bacteria.

(a) Write an exponential function $P(t)$ that models the population of the bacteria after t hours.

(b) Algebraically determine the number of hours needed for the population to reach 10,000 bacteria. Round your answer to the nearest tenth of an hour.

34. The data table below shows the average closing price of a certain stock, in dollars, over the first six months of a calendar year.

Month (t)	1	2	3	4	5	6
Price P(t)	45	48	53	58	65	72

(a) Using regression, write an exponential function $P(t) = a \cdot b^t$ that best models the data. Round each coefficient to the nearest hundredth.

(b) Use the regression model from part (a) to predict the average closing price of the stock during month 10. Round your answer to the nearest dollar.

35. Algebraically solve for all values of x in the interval $0 \leq x \leq 2\pi$:

$$2 \sin^2 x - \sin x - 1 = 0$$

Express each solution in exact form (radians).

PART IV — Long Constructed Response (1 question × 6 credits = 6 credits)

Show all work. This problem requires multiple steps and integrates concepts from several chapters.

36. The water depth at a harbor varies with the tide. The maximum depth of the water is 14 meters, which occurs at high tide. The minimum depth of the water is 4 meters, which occurs at low tide. One complete cycle from one high tide to the next high tide takes 12 hours. At time $t = 0$ hours, the water is at high tide.

(a) Write a cosine function $d(t)$ that models the water depth, in meters, as a function of t, the number of hours since the first high tide.

(b) State the amplitude, period, and midline of the function $d(t)$, and explain what each represents in the context of the tidal cycle at the harbor.

(c) Algebraically determine all times during the first 24 hours of operation at which the water depth is exactly 11 meters. Round each answer to the nearest hundredth of an hour. Show all algebraic work.

ANSWER KEY WITH EXPLANATIONS – EXAM 40

1. D — The Remainder Theorem states that the remainder when $p(x)$ is divided by $(x - c)$ equals $p(c)$. Evaluating $p(-2) = 2(-8) + 5(4) - 4(-2) + 3 = -16 + 20 + 8 + 3 = 15$. This eliminates the need for long division and confirms the remainder is 15.

2. B — Using FOIL on $(3 - 2i)(3 - 2i)$ gives $9 - 6i - 6i + 4i^2 = 9 - 12i - 4 = 5 - 12i$. The key step is recognizing that $i^2 = -1$, which converts the $+4i^2$ term into -4. The result must be written in $a + bi$ form with $a = 5$ and $b = -12$.

3. A — Cross-multiplying gives $2(x + 10) = 5(x + 1)$, which expands to $2x + 20 = 5x + 5$. Solving yields $15 = 3x$, so $x = 5$. Substituting back confirms neither denominator becomes zero, so no extraneous solution exists.

4. C — Combine logarithms using the product rule: $\log_3[x(x - 6)] = 3$, giving $x^2 - 6x = 27$. Solving $x^2 - 6x - 27 = 0$ by factoring yields $(x - 9)(x + 3) = 0$. The solution $x = -3$ is rejected because log of a negative number is undefined, leaving $x = 9$.

5. A — The angle $7\pi/6$ lies in Quadrant III, where sine is negative, with a reference angle of $\pi/6$. Since $\sin(\pi/6) = 1/2$, applying the Quadrant III sign convention yields $\sin(7\pi/6) = -1/2$. This is a standard unit-circle value students must recognize directly.

6. D — The difference of cubes formula $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$ applies with $a = x$ and $b = 2$. Substituting gives $(x - 2)(x^2 + 2x + 4)$. The quadratic factor $x^2 + 2x + 4$ has a negative discriminant and is irreducible over the reals.

7. C — Apply the arithmetic series formula $S_n = (n/2)[2a_1 + (n - 1)d]$ with $n = 10$, $a_1 = 3$, and $d = 4$. This gives $S_{10} = 5[6 + 36] = 5(42) = 210$. Recognizing the constant difference of 4 between terms confirms the sequence is arithmetic.

8. B — For an exponential decay function $f(t) = a \cdot b^t$, the annual rate of decay equals $1 - b$ expressed as a percent. Here $b = 0.92$, so the rate is $1 - 0.92 = 0.08$ or 8%. The base of 0.92 means the asset retains 92% of its value each year, losing 8%.

9. B — The discriminant $b^2 - 4ac = (-12)^2 - 4(9)(4) = 144 - 144 = 0$. A discriminant of zero indicates exactly one repeated real solution, and since all coefficients are rational and $\sqrt{0} = 0$, that solution ($x = 2/3$) is rational. This produces one repeated rational root.

10. A — First evaluate the inner function $g(2) = 2(2) - 3 = 1$. Then substitute into f, giving $f(1) = (1)^2 + 1 = 2$. Composition requires working from the inside out, which is the standard order of operations for $f(g(x))$.

11. D — To find the inverse, swap x and y in $y = (x - 4)/2$ to get $x = (y - 4)/2$. Solving for y gives $2x = y - 4$, so $y = 2x + 4$. The inverse undoes the operations of the original function in reverse order.

12. C — Rewriting in exponential form: $2x + 3 = 5^2 = 25$. Subtract 3 from both sides to get $2x = 22$, then divide by 2 to obtain $x = 11$. Substituting back confirms $2(11) + 3 = 25$, and $\log_5(25) = 2$ verifies the solution.

13. A — In the function form $a \cos(bx) + d$, the amplitude equals $|a|$, the absolute value of the coefficient in front of the cosine. Here $a = -4$, so amplitude $= |-4| = 4$. Amplitude is always a non-negative quantity representing the vertical distance from the midline to the maximum.

14. C — By the Remainder Theorem, the remainder is $p(1) = (1)^4 - 2(1)^3 + 1 - 5 = 1 - 2 + 1 - 5 = -5$. This shortcut eliminates the need for synthetic or long division. The negative remainder also confirms that $(x - 1)$ is not a factor of $p(x)$.

15. D — Using the empirical rule for normal distributions, approximately 95% of data lies within 2 standard deviations of the mean. The score 130 is exactly 2 standard deviations above 100, so the area above it is half of the remaining 5%, which equals 2.5%. This is a direct application of the 68-95-99.7 rule.

16. B — The first term is $a_1 = 3$, and the common ratio $r = 12/3 = 4$. The explicit formula for a geometric sequence is $a_n = a_1 \cdot r^{(n - 1)}$, giving $a_n = 3 \cdot 4^{(n - 1)}$. Verification: $a_3 = 3 \cdot 4^2 = 48$, matching the given third term.

17. A — Using the quadratic formula: $x = (6 \pm \sqrt{36 - 100})/2 = (6 \pm \sqrt{-64})/2 = (6 \pm 8i)/2$. Simplifying yields $x = 3 \pm 4i$. The negative discriminant of -64 confirms two complex conjugate solutions.

18. D — Factor numerator and denominator: $(x + 2)(x + 3)$ over $(x + 2)(x - 2)$. Canceling the common factor $(x + 2)$ leaves $(x + 3)/(x - 2)$. Restrictions $x \neq -2$ and $x \neq 2$ come from the original denominator and must be stated to preserve the domain.

19. B — Rewrite both sides with base 2: $8 = 2^3$ and $32 = 2^5$, so the equation becomes $2^{(6x)} = 2^5$. Equating exponents gives $6x = 5$, so $x = 5/6$. The one-to-one property of exponential functions allows direct comparison of exponents once bases match.

20. C — Use the Pythagorean identity $\sin^2\theta + \cos^2\theta = 1$, giving $\sin^2\theta = 1 - 9/25 = 16/25$, so $\sin\theta = \pm 4/5$. In Quadrant III, sine is negative, so $\sin\theta = -4/5$. The quadrant determines the sign of each trigonometric function.

21. D — With replacement, the two draws are independent events. P(red) = 4/12 = 1/3 and P(green) = 5/12. Multiplying gives P(red then green) = (1/3)(5/12) = 5/36. Replacement is essential because it keeps the total number of marbles constant at 12.

22. B — A simple random sample requires every member of the population to have an equal and independent chance of being selected. Numbering each student and using a random number generator satisfies both conditions exactly. The other methods introduce convenience, stratification, or selection bias that violate the simple random sample definition.

23. A — A constant change in y for each unit change in x is the defining feature of a linear relationship with constant slope. Linear regression models data of the form $y = mx + b$, where m represents that constant rate of change. Other regression types (quadratic, exponential, logarithmic) all produce non-constant rates of change.

24. C — The transformation $g(x) = -|x - 2| + 5$ contains three distinct operations: $(x - 2)$ shifts the graph right 2 units, the leading negative sign reflects it across the x-axis, and +5 shifts it up 5 units. These transformations are applied in the order shown, producing a downward-opening V with vertex at (2, 5).