

# PRACTICE EXAM 39: ALGEBRA II

## REGENTS SIMULATION

---

### PART I — Multiple Choice (24 questions $\times$ 2 credits = 48 credits)

1. What is the value of  $i^{10} + i^5$  expressed in  $a + bi$  form?

A.  $1 + i$

B.  $-1 + i$

C.  $-1 - i$

D.  $1 - i$

2. Simplify the expression  $(x^{1/2})^4$ .

A.  $x^{1/8}$

B.  $x^{1/2}$

C.  $x^2$

D.  $x^8$

3. What are the solutions to the equation  $3x^2 + 12 = 0$ ?

A.  $x = \pm 4i$

B.  $x = \pm 2$

C.  $x = \pm\sqrt{2}i$

D.  $x = \pm 2i$

4. Factored completely, the expression  $27x^3 + 8$  is equivalent to

A.  $(3x + 2)(9x^2 - 6x + 4)$

B.  $(3x + 2)(9x^2 + 6x + 4)$

C.  $(3x - 2)(9x^2 + 6x + 4)$

D.  $(3x + 2)(3x^2 - 4)$

5. What is the value of  $\log_8(2)$ ?

A. 3

B. 4

C.  $1/3$

D.  $1/4$

6. In a geometric sequence, the second term is 12 and the fourth term is 48. What is the common ratio, given that it is positive?

A. 4

B. 2

C. 3

D. 6

7. When  $p(x) = 2x^3 - x^2 + 3x - 1$  is divided by  $(x - 1)$ , the remainder is

A. 3

B. 1

C. 5

D. -1

8. If  $f(x) = \sqrt{x + 1}$  and  $g(x) = x^2 - 1$ , what is the value of  $f(g(3))$ ?

A. 8

B.  $\sqrt{8}$

C. 9

D. 3

9. What is the solution set of the equation  $x = \sqrt{x + 6}$ ?

A.  $\{-2, 3\}$

B.  $\{3\}$

C.  $\{-2\}$

D.  $\{ \}$

10. What is the average rate of change of  $f(x) = -x^2 + 5x$  over the interval  $[1, 4]$ ?

A. 5

B. 3

C. 0

D. -1

11. If \$2000 is invested at an annual interest rate of 6% compounded annually, what is the value of the investment after 3 years?

A. \$2382.03

B. \$2360.00

C. \$2120.00

D. \$3816.00

12. What is the solution to the equation  $2 \log(x) = \log(36)$ ?

A.  $x = 18$

B.  $x = 72$

C.  $x = 3$

D.  $x = 6$

13. Which statement best describes the nature of the solutions of  $9x^2 - 12x + 4 = 0$ ?

- A. Two distinct rational solutions
- B. Two complex (non-real) solutions
- C. One repeated rational solution
- D. Two distinct irrational solutions

14. What is the exact value of  $\sin(2\pi/3)$ ?

- A.  $1/2$
- B.  $\sqrt{3}/2$
- C.  $-\sqrt{3}/2$
- D.  $-1/2$

15. For the function  $y = -2 \sin(x) - 5$ , what is the equation of the midline?

- A.  $y = -2$
- B.  $y = 2$
- C.  $y = 5$
- D.  $y = -5$

16. What is the sum of the first 12 terms of an arithmetic sequence whose first term is 3 and whose common difference is 5?

- A. 366
- B. 372

C. 330

D. 183

17. Which function is the inverse of  $f(x) = 2^x$ ?

A.  $f^{-1}(x) = x^2$

B.  $f^{-1}(x) = (1/2)^x$

C.  $f^{-1}(x) = \log_2(x)$

D.  $f^{-1}(x) = 2^{-x}$

18. A single card is drawn from a standard 52-card deck. Given that the card is a face card, what is the probability that it is a king?

A.  $1/13$

B.  $1/3$

C.  $1/4$

D.  $4/13$

19. A data set is normally distributed with a mean of 25 and a standard deviation of 4. What is the z-score of a value of 17?

A. 2

B. -8

C. 8

D.  $-2$

20. Given that  $\sin \theta = 7/25$  and  $\theta$  is in Quadrant II, what is the value of  $\cos \theta$ ?

A.  $-24/25$

B.  $24/25$

C.  $-7/24$

D.  $7/24$

21. Two data sets have the same mean. Set A has a standard deviation of 3, and Set B has a standard deviation of 8. Which statement is true?

A. Set A's data is more spread out than Set B's

B. Set B's data is more spread out than Set A's

C. The two data sets have identical spread

D. Set A has a higher mean than Set B

22. Written in vertex form, the expression  $3x^2 - 12x + 7$  is equivalent to

A.  $3(x - 2)^2 + 7$

B.  $3(x + 2)^2 - 5$

C.  $(x - 2)^2 - 5$

D.  $3(x - 2)^2 - 5$

23. What is the sum of the infinite geometric series  $9 + 3 + 1 + 1/3 + \dots$ ?

- A. 12
- B. 18
- C. 13.5
- D. 27

24. What is the solution to the equation  $6^x = 100$ , rounded to the nearest hundredth?

- A.  $x = 2.57$
- B.  $x = 16.67$
- C.  $x = 3.00$
- D.  $x = 1.78$

**PART II — Short Constructed Response (8 questions  $\times$  2 credits = 16 credits)**

**Show all work. A correct answer with no supporting work will receive only 1 credit.**

25. Solve the equation  $x^2 + 4x + 7 = 0$  algebraically. Express your solutions in  $a + bi$  form.

26. Solve algebraically for  $x$ :  $2 / (x - 1) + 3 / (x + 1) = 5 / (x^2 - 1)$ . State any restrictions on the variable and identify any extraneous solutions that must be rejected.

27. Given  $f(x) = x^3 - 6x^2 + 11x - 6$ , determine whether  $(x - 1)$  is a factor of  $f(x)$ . Justify your answer using the Remainder Theorem or the Factor Theorem.

28. Write an equation, in the form  $g(x) = a \cdot f(x) + k$ , for the function obtained by vertically stretching  $f(x) = \sqrt{x}$  by a factor of 2, reflecting the result across the  $x$ -axis, and then translating it 3 units up.

29. Solve algebraically for  $x$ :  $25^x = 125$ .

30. Express  $4 / (x + 3) + x / (x^2 - 9)$  as a single rational expression in simplest form. State any restrictions on the variable.

31. A sequence is defined recursively by  $a_1 = 1$  and  $a_n = (a_{n-1})^2 + 1$  for  $n \geq 2$ . Find the value of  $a_4$ .

32. Given that  $\tan \theta = 15/8$  and  $\theta$  terminates in Quadrant III, find the exact value of  $\sin \theta$ . Show the algebraic work that justifies your answer.

**PART III — Extended Constructed Response (3 questions  $\times$  4 credits = 12 credits)**

**Show all work. Partial credit is awarded according to the scoring rubric.**

33. The amount of a drug in a patient's bloodstream is modeled by the function  $D(t) = 500(0.8)^t$ , where  $D(t)$  is the amount in milligrams and  $t$  is the time in hours.

(a) State the initial amount of the drug and the hourly percent rate of decay.

(b) Algebraically determine the number of hours it will take for the amount of the drug to fall to 200 milligrams. Round your answer to the nearest tenth of an hour.

34. The table below shows a company's production cost  $C(x)$ , in thousands of dollars, as a function of the number of units  $x$  produced (in hundreds).

$x$ (hundreds)   0   1   2   3   4
--- --- --- --- ---
$C(x)$ (thousands)   20   13   8   5   4

(a) Using regression, write a quadratic function  $C(x) = ax^2 + bx + c$  that best models the data. Round each coefficient to the nearest tenth.

(b) Use the regression model from part (a) to determine the minimum production cost and the number of units at which the minimum cost occurs. Round each answer to the nearest tenth.

35. A circle is defined by the equation  $x^2 + y^2 = 30$ , and a line is defined by the equation  $y = x + 2$ .

(a) Algebraically determine all points of intersection between the circle and the line. Express coordinates in exact form (radicals permitted).

(b) Verify your answer by substituting one of the intersection points back into both original equations. Show the substitution clearly.

**PART IV — Long Constructed Response (1 question  $\times$  6 credits = 6 credits)**

**Show all work. This problem requires multiple steps and integrates concepts from several chapters.**

36. A Ferris wheel has a radius of 25 meters, and its center is 28 meters above the ground. The wheel completes one full revolution every 5 minutes. A rider boards the Ferris wheel at the lowest point at time  $t = 0$ .

(a) Write a function  $h(t)$  that models the rider's height in meters above the ground as a function of time  $t$  in minutes.

(b) State the amplitude, period, and midline of the function  $h(t)$ , and explain what each represents in the context of the Ferris wheel.

(c) Algebraically determine all times during the first 10 minutes of operation at which the rider is at a height of 40 meters above the ground. Round each answer to the nearest hundredth of a minute. Show all algebraic work.

## ANSWER KEY WITH EXPLANATIONS – PRACTICE EXAM 39

**1. B** — Powers of  $i$  cycle every four:  $i^{10} = i^2 = -1$  and  $i^5 = i^1 = i$ , so the sum is  $-1 + i$ . Reducing each exponent by its remainder mod 4 gives the values. The result is  $-1 + i$ .

**2. C** — A power raised to a power multiplies exponents:  $(x^{1/2})^4 = x^{(1/2 \cdot 4)} = x^2$ . Multiplying  $1/2$  by 4 gives 2. The result is  $x^2$ .

**3. D** — Isolate the square:  $3x^2 = -12$ , so  $x^2 = -4$  and  $x = \pm 2i$ . The negative value under the root introduces the imaginary unit. Both roots are pure imaginary.

**4. A** — This is a sum of cubes,  $(3x)^3 + 2^3 = (3x + 2)(9x^2 - 6x + 4)$ . The middle term of the quadratic factor is  $-ab = -6x$ , making it negative. The sign pattern distinguishes it from a difference of cubes.

**5. C** — Since  $8 = 2^3$ , the cube root of 8 is 2, so  $\log_8(2) = 1/3$ . The exponent that takes 8 to 2 is  $1/3$ . The value is  $1/3$ .

- 6. B** — Dividing the fourth term by the second gives  $r^2 = 48/12 = 4$ , so  $r = 2$  (taking the positive root). The terms span two ratio steps, hence the square. The common ratio is 2.
- 7. A** — By the Remainder Theorem, the remainder is  $p(1) = 2 - 1 + 3 - 1 = 3$ . Substituting the zero of the divisor avoids long division. The remainder is 3.
- 8. D** — Evaluate the inner function first:  $g(3) = 9 - 1 = 8$ , then  $f(8) = \sqrt{8 + 1} = \sqrt{9} = 3$ . Composition works from the inside out. The square root of 9 is 3.
- 9. B** — Squaring gives  $x^2 = x + 6$ , so  $x^2 - x - 6 = 0$  and  $(x - 3)(x + 2) = 0$ . The candidate  $x = -2$  fails because  $\sqrt{4} = 2 \neq -2$ , leaving  $x = 3$  as the only valid solution. The check removes the extraneous root.
- 10. C** — Average rate of change is  $[f(4) - f(1)]/(4 - 1)$ . With  $f(4) = -16 + 20 = 4$  and  $f(1) = -1 + 5 = 4$ , this is  $(4 - 4)/3 = 0$ . The equal endpoint values give a rate of change of zero.
- 11. A** — Apply the compound interest formula:  $A = 2000(1.06)^3 = 2000(1.191016) \approx 2382.03$ . Cubing the growth factor accounts for three years of compounding. The investment is worth \$2382.03.
- 12. D** — Apply the power rule:  $\log(x^2) = \log(36)$ , so  $x^2 = 36$  and  $x = 6$ , taking the positive value required by the domain. Equal logarithms require equal arguments. The solution is 6.
- 13. C** — The discriminant is  $b^2 - 4ac = 144 - 144 = 0$ , indicating one repeated real solution. Since  $9x^2 - 12x + 4 = (3x - 2)^2$ , the repeated root  $2/3$  is rational. A zero discriminant always yields a single repeated solution.
- 14. B** — The angle  $2\pi/3$  lies in Quadrant II with reference angle  $\pi/3$ , where sine is positive. Since  $\sin(\pi/3) = \sqrt{3}/2$ ,  $\sin(2\pi/3) = \sqrt{3}/2$ . The quadrant keeps the value positive.
- 15. D** — In  $y = a \sin(x) + d$ , the midline is the vertical shift  $d$ , which is  $-5$ . The midline is the horizontal line about which the function oscillates. The amplitude of 2 does not affect its location.
- 16. A** — Using  $S = n/2(2a_1 + (n - 1)d) = 12/2(6 + 55) = 6(61) = 366$ . The formula sums the arithmetic series directly. The sum of the first 12 terms is 366.
- 17. C** — The inverse of an exponential function with base 2 is the logarithm with base 2:  $f^{-1}(x) = \log_2(x)$ . Logarithms undo exponentials. The inverse is  $\log_2(x)$ .
- 18. B** — There are 12 face cards (jacks, queens, kings), of which 4 are kings, giving  $4/12 = 1/3$ . Restricting the sample space to face cards sets the denominator at 12. The conditional probability is  $1/3$ .
- 19. D** — The z-score measures standard deviations from the mean:  $z = (17 - 25)/4 = -8/4 = -2$ . The negative sign indicates the value is below the mean. The z-score is  $-2$ .
- 20. A** — In Quadrant II cosine is negative, and the 7-24-25 triangle gives the cosine ratio. Therefore  $\cos \theta = -24/25$ . The quadrant fixes the negative sign on cosine.

**21. B** — Standard deviation measures spread, and a larger value means more dispersion. Since Set B's standard deviation of 8 exceeds Set A's 3, Set B's data is more spread out. The mean being equal does not affect the comparison of spread.

**22. D** — Factor out 3 and complete the square:  $3(x^2 - 4x) + 7 = 3(x^2 - 4x + 4) - 12 + 7 = 3(x - 2)^2 - 5$ . The subtracted 12 accounts for 3 times the added 4. The vertex form reveals the vertex at  $(2, -5)$ .

**23. C** — The common ratio is  $1/3$ , and an infinite geometric series with  $|r| < 1$  sums to  $a_1/(1 - r) = 9/(1 - 1/3) = 9/(2/3) = 13.5$ . The ratio ensures convergence. The series sums to 13.5.

**24. A** — Take the log of both sides:  $x = \log_6(100) = \ln(100)/\ln(6) \approx 2.57$ . The change-of-base formula converts to natural logs. The result rounds to 2.57.

## Part II (Short Constructed Response)

**25.  $x = -2 \pm i\sqrt{3}$**  — Quadratic formula:  $x = [-4 \pm \sqrt{(16 - 28)}]/2 = [-4 \pm \sqrt{(-12)}]/2 = [-4 \pm 2i\sqrt{3}]/2 = -2 \pm i\sqrt{3}$ . The radical  $\sqrt{(-12)}$  simplifies to  $2i\sqrt{3}$ , which reduces with the denominator. The result is a pair of complex conjugates.

**26.  $x = 6/5$ ; no extraneous solutions** — Multiplying through by  $(x - 1)(x + 1)$  gives  $2(x + 1) + 3(x - 1) = 5$ , which simplifies to  $5x - 1 = 5$ , so  $x = 6/5$ . The restrictions are  $x \neq 1$  and  $x \neq -1$ , and  $x = 6/5$  violates neither, so it is valid.

**27. Yes,  $(x - 1)$  is a factor** — By the Factor Theorem,  $f(1) = 1 - 6 + 11 - 6 = 0$ . A remainder of zero confirms that  $(x - 1)$  divides  $f(x)$  evenly. A nonzero value would have meant it is not a factor.

**28.  $g(x) = -2\sqrt{x} + 3$**  — A vertical stretch by 2 gives  $2\sqrt{x}$ ; reflecting across the x-axis negates it to  $-2\sqrt{x}$ ; translating 3 units up adds 3. Combining these produces  $-2\sqrt{x} + 3$ . The reflection and stretch both act on the leading coefficient.

**29.  $x = 3/2$**  — Write both sides with base 5:  $25^x = 5^{(2x)}$  and  $125 = 5^3$ , so  $2x = 3$ . Solving gives  $x = 3/2$ . Equal bases allow the exponents to be set equal.

**30.  $(5x - 12) / [(x - 3)(x + 3)]$ ,  $x \neq 3$ ,  $x \neq -3$**  — Over the common denominator  $(x - 3)(x + 3)$ :  $4(x - 3) + x = 4x - 12 + x = 5x - 12$ . The numerator combines to  $5x - 12$ . Restrictions come from the factored denominator.

**31.  $a_4 = 26$**  — Apply the recursion:  $a_2 = 1^2 + 1 = 2$ ,  $a_3 = 2^2 + 1 = 5$ ,  $a_4 = 5^2 + 1 = 26$ . Each term squares the previous term and adds 1. Building up term by term reaches 26.

**32.  $\sin \theta = -15/17$**  — With  $\tan \theta = 15/8$ , the reference triangle has legs 15 and 8 with hypotenuse 17. In Quadrant III sine is negative, so  $\sin \theta = -15/17$ . The quadrant fixes the negative sign.

## Part III (Extended Constructed Response)

**33. (a) 500 mg, 20% decay; (b)  $\approx 4.1$  hours** — The coefficient 500 is the initial amount, and the base  $0.8 = 1 - 0.2$  indicates 20% hourly decay. For part (b),  $200 = 500(0.8)^t$  gives  $0.8^t = 0.4$ , so  $t = \ln(0.4)/\ln(0.8) \approx 4.1$  hours. Logarithms isolate the exponent.

**34. (a)  $C(x) = 1.0x^2 - 8.0x + 20.0$ ; (b) \$4 thousand at  $x = 4$  hundred units** — Quadratic regression on the data returns  $a = 1.0$ ,  $b = -8.0$ ,  $c = 20.0$ . The vertex occurs at  $x = -b/(2a) = 8/2 = 4$ , and  $C(4) = 16 - 32 + 20 = 4$  thousand dollars. The positive leading coefficient confirms a minimum.

**35. (a)  $(-1 + \sqrt{14}, 1 + \sqrt{14})$  and  $(-1 - \sqrt{14}, 1 - \sqrt{14})$**  — Substituting  $y = x + 2$  into the circle gives  $x^2 + (x + 2)^2 = 30$ , which simplifies to  $2x^2 + 4x - 26 = 0$ , then  $x^2 + 2x - 13 = 0$ , so  $x = -1 \pm \sqrt{14}$ . The  $y$ -values follow from  $y = x + 2$ . **(b)** Checking  $(-1 + \sqrt{14}, 1 + \sqrt{14})$ :  $x^2 + y^2 = (15 - 2\sqrt{14}) + (15 + 2\sqrt{14}) = 30$ , and  $y = x + 2 = (-1 + \sqrt{14}) + 2 = 1 + \sqrt{14}$ , so both equations hold.

## Part IV (Long Constructed Response)

**36. (a)  $h(t) = -25 \cos(2\pi t/5) + 28$**  — The amplitude is the 25-meter radius and the midline is the 28-meter center height. The period of 5 minutes gives  $b = 2\pi/5$ , and boarding at the lowest point at  $t = 0$  calls for a negative cosine. This yields  $h(t) = -25 \cos(2\pi t/5) + 28$ .

**(b) Amplitude 25, period 5 min, midline  $h = 28$**  — The amplitude of 25 meters equals the wheel's radius, the distance from the center to the rim. The period of 5 minutes is the time for one full revolution. The midline  $h = 28$  meters is the center height, about which the rider's height oscillates.

**(c)  $t \approx 1.65, 3.35, 6.65, \text{ and } 8.35$  minutes** — Setting  $40 = -25 \cos(2\pi t/5) + 28$  gives  $\cos(2\pi t/5) = -0.48$ . Over the first 10 minutes,  $2\pi t/5 = 2.0710, 4.2122, 8.3542, \text{ and } 10.4954$  radians, so  $t = 1.65, 3.35, 6.65, \text{ and } 8.35$  minutes. Each pair corresponds to the rider passing through 40 meters while ascending and descending on each revolution.