PRACTICE EXAM 38: ALGEBRA II REGENTS SIMULATION

PART I — Multiple Choice (24 questions × 2 credits = 48 credits)

1. Express the product $(2 - 5i)(1 + 3i)$ in $a + bi$ form.

A. $17 - i$

B. $-13 + i$

C. $2 - 15i$

D. $17 + i$

2. Which expression is equivalent to $\sqrt{(x^5)}$ written with a rational exponent?

A. $x^{(5/2)}$

B. $x^{(2/5)}$

C. x^5

D. x^{10}

3. What are the solutions to the equation $x^2 + 6x + 13 = 0$?

A. $x = 3 \pm 2i$

B. $x = -3 \pm 4i$

C. $x = -3 \pm 2i$

D. $x = -6 \pm 2i$

4. Factored completely, the expression $4x^3 - 36x$ is equivalent to

A. $4x(x^2 - 9)$

B. $4x(x - 3)(x + 3)$

C. $4(x - 3)(x + 3)$

D. $2x(2x - 6)(x + 3)$

5. What is the value of $\log_4(64)$?

A. 16

B. 3

C. 4

D. 2

6. What is the sum of the finite geometric series $2 + 6 + 18 + 54 + 162$?

A. 162

B. 324

C. 486

D. 242

7. When $p(x) = x^3 + 2x^2 - 3$ is divided by $(x + 1)$, the remainder is

A. -2

B. 0

C. -4

D. 2

8. If $f(x) = x + 2$ and $g(x) = 3x - 1$, what is the value of $f(g(4))$?

A. 11

B. 15

C. 13

D. 9

9. What is the solution to the equation $\sqrt{(2x + 3)} = 5$?

A. $x = 11$

B. $x = 14$

C. $x = 5$

D. $x = 22$

10. What is the average rate of change of $f(x) = 2x^2 + 1$ over the interval [0, 3]?

A. 18

B. 19

C. 3

D. 6

11. Which function models a quantity that begins at 400 and decreases by 20% each year?

A. $f(x) = 400(1.2)^x$

B. $f(x) = 400(0.8)^x$

C. $f(x) = 400(0.2)^x$

D. $f(x) = 400 - 0.2x$

12. What is the solution to the equation $\log_3(x) + \log_3(2) = \log_3(10)$?

A. x = 8

B. x = 20

C. x = 5

D. x = 12

13. Which statement best describes the nature of the solutions of $x^2 - 2x + 6 = 0$?

A. Two distinct rational solutions

B. One repeated rational solution

C. Two distinct irrational solutions

D. Two complex (non-real) solutions

14. What is the exact value of $\cos(\pi/4)$?

A. $\sqrt{2}/2$

B. 1/2

C. $\sqrt{3}/2$

D. $-\sqrt{2}/2$

15. For the function $y = 5 \cos(4x) + 2$, what is the period?

A. 4

B. π

C. $\pi/2$

D. 2π

16. In an arithmetic sequence, the first term is 7 and the fifth term is 23. What is the common difference?

A. 16

B. 4

C. 3

D. 5

17. If $f(x) = (x + 4) / 3$, what is $f^{-1}(x)$?

A. $3x + 4$

B. $(x - 4) / 3$

C. $3x - 4$

D. $(x + 4) / 3$

18. What is the solution to the equation $3^x = 50$, rounded to the nearest hundredth?

A. $x = 16.67$

B. $x = 5.00$

C. $x = 2.00$

D. $x = 3.56$

19. A jar contains 5 red, 3 blue, and 2 yellow marbles. If one marble is drawn at random, what is the probability that it is not blue?

A. 7/10

B. 3/10

C. 1/2

D. 2/5

20. A data set is normally distributed with a mean of 80 and a standard deviation of 5. Approximately what percent of the data is greater than 90?

A. 5%

B. 2.5%

C. 16%

D. 95%

21. Which study design provides the strongest evidence that a new teaching method causes higher test scores?

A. A survey of students about their preferred method

B. An observational study of classes already using it

C. A census of all students in the district

D. A randomized controlled experiment with random assignment

22. Given that $\tan \theta = -4/3$ and θ is in Quadrant II, what is the value of $\sin \theta$?

A. $-4/5$

B. $3/5$

C. $4/5$

D. $-3/5$

23. Written in vertex form, the expression $x^2 - 12x + 40$ is equivalent to

A. $(x - 6)^2 + 4$

B. $(x - 6)^2 - 4$

C. $(x + 6)^2 + 4$

D. $(x - 12)^2 - 104$

24. What is the sum of the infinite geometric series $24 - 12 + 6 - 3 + \ldots$?

A. 12

B. 16

C. 48

D. 8

PART II — Short Constructed Response (8 questions × 2 credits = 16 credits)

Show all work. A correct answer with no supporting work will receive only 1 credit.

25. Solve the equation $x^2 - 6x + 14 = 0$ algebraically. Express your solutions in $a + bi$ form.

26. Solve algebraically for x: $1 / (x - 1) + 1 / (x + 1) = 1$. State any restrictions on the variable and identify any extraneous solutions that must be rejected.

27. Given $f(x) = x^3 + 2x^2 - 5x - 6$, determine whether $(x + 3)$ is a factor of $f(x)$. Justify your answer using the Remainder Theorem or the Factor Theorem.

28. Write an equation, in the form $g(x) = a \cdot f(x - h) + k$, for the function obtained by vertically stretching $f(x) = x^2$ by a factor of 3 and then translating the result 1 unit to the left and 2 units up.

29. Solve algebraically for x: 4^(2x) = 32.

30. Express $2 / (x^2 - x - 6) + 1 / (x - 3)$ as a single rational expression in simplest form. State any restrictions on the variable.

31. A sequence is defined recursively by $a_1 = 5$ and $a_n = 3a_{n-1} - 4$ for $n \geq 2$. Find the value of a_4.

32. Given that $\cos \theta = -5/13$ and θ terminates in Quadrant III, find the exact value of $\tan \theta$. Show the algebraic work that justifies your answer.

PART III — Extended Constructed Response (3 questions × 4 credits = 12 credits)

Show all work. Partial credit is awarded according to the scoring rubric.

33. The value of a car is modeled by the function $V(t) = 28000(0.88)^t$, where $V(t)$ is the value in dollars and t is the time in years.

(a) State the initial value of the car and the annual percent rate of depreciation.

(b) Algebraically determine the number of years it will take for the car's value to fall to \$14000. Round your answer to the nearest tenth of a year.

34. The table below shows a company's monthly revenue R(x), in thousands of dollars, as a function of the number of new products x released.

x (products)	0	1	2	3	4
R(x) (thousands)	4	19	28	31	28

(a) Using regression, write a quadratic function $R(x) = ax^2 + bx + c$ that best models the data. Round each coefficient to the nearest tenth.

(b) Use the regression model from part (a) to determine the maximum revenue and the number of products at which the maximum revenue occurs. Round each answer to the nearest tenth.

35. A circle is defined by the equation $x^2 + y^2 = 16$, and a line is defined by the equation $y = x - 1$.

(a) Algebraically determine all points of intersection between the circle and the line. Express coordinates in exact form (radicals permitted).

(b) Verify your answer by substituting one of the intersection points back into both original equations. Show the substitution clearly.

PART IV — Long Constructed Response (1 question × 6 credits = 6 credits)

Show all work. This problem requires multiple steps and integrates concepts from several chapters.

36. A carousel horse moves up and down on its pole as the carousel turns. The horse's height above the platform oscillates between a maximum of 1.2 meters and a minimum of 0.4 meters, completing one full up-and-down cycle every 4 seconds. At time $t = 0$, the horse is at its maximum height.

(a) Write a function $h(t)$ that models the height of the horse in meters above the platform as a function of time t in seconds.

(b) State the amplitude, period, and midline of the function $h(t)$, and explain what each represents in the context of the carousel horse.

(c) Algebraically determine all times during the first 8 seconds at which the horse is at a height of 1 meter above the platform. Round each answer to the nearest hundredth of a second. Show all algebraic work.

ANSWER KEY WITH EXPLANATIONS – PRACTICE EXAM 38

1. D — Distribute and apply $i^2 = -1$: $(2 - 5i)(1 + 3i) = 2 + 6i - 5i - 15i^2 = 2 + i + 15 = 17 + i$. The $-15i^2$ term becomes +15, shifting the real part to 17. Combining the middle terms gives +i.

2. A — A radical converts to a rational exponent as the power over the index: $\sqrt{(x^5)} = x^{(5/2)}$. The index 2 becomes the denominator and the power 5 becomes the numerator. The result is $x^{(5/2)}$.

3. C — Quadratic formula: $x = [-6 \pm \sqrt{(36 - 52)}]/2 = [-6 \pm \sqrt{(-16)}]/2 = [-6 \pm 4i]/2 = -3 \pm 2i$. The negative discriminant produces the imaginary part. Dividing both terms by 2 gives the simplified form.

4. B — Factor out the GCF 4x, then factor the difference of squares: $4x^3 - 36x = 4x(x^2 - 9) = 4x(x - 3)(x + 3)$. Complete factoring requires breaking down $x^2 - 9$. Stopping at $4x(x^2 - 9)$ leaves it not fully factored.

5. B — A logarithm asks for the exponent on the base: $4^3 = 64$, so $\log_4(64) = 3$. Rewriting in exponential form makes the value clear. The base 4 raised to 3 equals 64.

6. D — Using $S = a_1(r^n - 1)/(r - 1) = 2(3^5 - 1)/(3 - 1) = 2(242)/2 = 242$. The common ratio is 3 and there are five terms. Direct addition of the terms also gives 242.

7. A — By the Remainder Theorem, the remainder is $p(-1) = -1 + 2 - 3 = -2$. Substituting the zero of the divisor avoids long division. The remainder is −2.

8. C — Evaluate the inner function first: $g(4) = 3(4) - 1 = 11$, then $f(11) = 11 + 2 = 13$. Composition works from the inside out. Adding 2 to 11 gives 13.

9. A — Square both sides: $2x + 3 = 25$, so $2x = 22$ and $x = 11$. Squaring removes the radical cleanly. Checking, $\sqrt{(22 + 3)} = \sqrt{25} = 5$ confirms the solution.

10. D — Average rate of change is $[f(3) - f(0)]/(3 - 0) = (19 - 1)/3 = 18/3 = 6$. This is the slope of the secant line over the interval. The function values 19 and 1 drive the result.

11. B — A quantity decreasing by 20% uses base $1 - 0.20 = 0.8$ with the initial value 400 as the coefficient, giving $f(x) = 400(0.8)^x$. A base below 1 signals decay. Multiplying by 0.8 each year produces the 20% decrease.

12. C — Combine the logs using the product rule: $\log_3(2x) = \log_3(10)$, so $2x = 10$ and $x = 5$. Equal logarithms with the same base require equal arguments. Dividing by 2 gives 5.

13. D — The discriminant is $b^2 - 4ac = 4 - 24 = -20$, which is negative. A negative discriminant produces two complex (non-real) conjugate solutions. The sign of the discriminant classifies the roots.

14. A — The cosine of $\pi/4$ (45°) is a standard unit-circle value equal to $\sqrt{2}/2$. The 45-45-90 reference triangle gives equal legs. The value is $\sqrt{2}/2$.

15. C — The period of $\cos(bx)$ is $2\pi/b$; with $b = 4$, the period is $2\pi/4 = \pi/2$. The coefficient of x compresses the graph horizontally. The amplitude and vertical shift do not affect the period.

16. B — The common difference is $d = (a_5 - a_1)/(5 - 1) = (23 - 7)/4 = 16/4 = 4$. Dividing the total change by the number of steps gives the difference. Each term increases by 4.

17. C — To invert, solve $y = (x + 4)/3$ for x: $3y = x + 4$, so $x = 3y - 4$ and $f^{-1}(x) = 3x - 4$. The inverse undoes division by 3 and addition of 4 in reverse order. The result is $3x - 4$.

18. D — Take the log of both sides: $x = \log_3(50) = \ln(50)/\ln(3) \approx 3.56$. The change-of-base formula converts to natural logs. The result rounds to 3.56.

19. A — The probability of not blue is the complement of blue: there are 7 non-blue marbles (5 red and 2 yellow) out of 10, giving 7/10. Subtracting the 3 blue from the total leaves 7 favorable outcomes. The probability is 7/10.

20. B — A value of 90 is two standard deviations above the mean of 80. By the empirical rule, about 5% of data lies beyond ±2 SD, so roughly 2.5% lies above +2 SD. The symmetry of the normal curve splits the outer 5% evenly.

21. D — Only a randomized controlled experiment with random assignment can establish causation, because randomization balances confounding variables across groups. Surveys and observational studies show association but not cause. Random assignment is the defining feature for causal inference.

22. C — With $\tan \theta = -4/3$ in Quadrant II, the 3-4-5 reference triangle applies, and sine is positive there. Therefore $\sin \theta = 4/5$. The quadrant fixes the positive sign on sine.

23. A — Complete the square: $x^2 - 12x + 40 = (x^2 - 12x + 36) - 36 + 40 = (x - 6)^2 + 4$. Half of −12 squared is 36, added and subtracted to preserve value. The vertex form reveals the vertex at (6, 4).

24. B — The common ratio is −1/2, and an infinite geometric series with $|r| < 1$ sums to $a_1/(1 - r) = 24/(1 - (-1/2)) = 24/(3/2) = 16$. The alternating signs come from the negative ratio. The series converges to 16.

Part II (Short Constructed Response)

25. $x = 3 \pm i\sqrt{5}$ — Quadratic formula: $x = [6 \pm \sqrt{(36 - 56)}]/2 = [6 \pm \sqrt{(-20)}]/2 = [6 \pm 2i\sqrt{5}]/2 = 3 \pm i\sqrt{5}$. The radical $\sqrt{(-20)}$ simplifies to $2i\sqrt{5}$, which reduces with the denominator. The result is a pair of complex conjugates.

26. $x = 1 \pm \sqrt{2}$; no extraneous solutions — Multiplying through by $(x - 1)(x + 1)$ gives $(x + 1) + (x - 1) = (x - 1)(x + 1)$, which becomes $2x = x^2 - 1$ and simplifies to $x^2 - 2x - 1 = 0$, so $x = 1 \pm \sqrt{2}$. The restrictions are $x \neq 1$ and $x \neq -1$, and neither solution violates them, so both are valid.

27. Yes, (x + 3) is a factor — By the Factor Theorem, $f(-3) = -27 + 18 + 15 - 6 = 0$. A remainder of zero confirms that $(x + 3)$ divides $f(x)$ evenly. A nonzero value would have meant it is not a factor.

28. $g(x) = 3(x + 1)^2 + 2$ — A vertical stretch by 3 multiplies the function by 3, giving $3x^2$; shifting 1 unit left replaces x with $(x + 1)$; shifting 2 units up adds 2. Combining these produces $3(x + 1)^2 + 2$. The horizontal shift moves opposite the sign inside.

29. $x = 5/4$ — Write both sides with base 2: $4^{(2x)} = 2^{(4x)}$ and $32 = 2^5$, so $4x = 5$. Solving gives $x = 5/4$. Equal bases allow the exponents to be set equal.

30. $(x + 4) / [(x - 3)(x + 2)]$, $x \neq 3$, $x \neq -2$ — Factor the first denominator as $(x - 3)(x + 2)$, then use it as the common denominator: $2/[(x - 3)(x + 2)] + (x + 2)/[(x - 3)(x + 2)] = (2 + x + 2)/[(x - 3)(x + 2)] = (x + 4)/[(x - 3)(x + 2)]$. The numerator combines to $x + 4$.

31. $a_4 = 83$ — Apply the recursion: $a_2 = 3(5) - 4 = 11$, $a_3 = 3(11) - 4 = 29$, $a_4 = 3(29) - 4 = 83$. Each term triples the previous term and subtracts 4. Building up term by term reaches 83.

32. $\tan \theta = 12/5$ — With $\cos \theta = -5/13$ in Quadrant III, the 5-12-13 triangle gives $\sin \theta = -12/13$ because sine is negative there. Then $\tan \theta = \sin/\cos = (-12/13)/(-5/13) = 12/5$. The two negatives make the tangent positive.

Part III (Extended Constructed Response)

33. (a) \$28,000, 12% depreciation; (b) ≈ 5.4 years — The coefficient 28000 is the initial value, and the base $0.88 = 1 - 0.12$ indicates 12% annual depreciation. For part (b), $14000 = 28000(0.88)^t$ gives $0.88^t = 0.5$, so $t = \ln(0.5)/\ln(0.88) \approx 5.4$ years. Logarithms isolate the exponent.

34. (a) $R(x) = -3.0x^2 + 18.0x + 4.0$; (b) \$31 thousand at x = 3 products — Quadratic regression on the data returns $a = -3.0$, $b = 18.0$, $c = 4.0$. The vertex occurs at $x = -b/(2a) = -18/(-6) = 3$ products, and $R(3) = -27 + 54 + 4 = 31$ thousand dollars. The negative leading coefficient confirms a maximum.

35. (a) $((1 + \sqrt{31})/2, (-1 + \sqrt{31})/2)$ and $((1 - \sqrt{31})/2, (-1 - \sqrt{31})/2)$ — Substituting $y = x - 1$ into the circle gives $x^2 + (x - 1)^2 = 16$, which simplifies to $2x^2 - 2x - 15 = 0$, so $x = (1 \pm \sqrt{31})/2$. The y-values follow from $y = x - 1$. **(b)** Checking $((1 + \sqrt{31})/2, (-1 + \sqrt{31})/2)$: $x^2 + y^2 = (32 + 2\sqrt{31})/4 + (32 - 2\sqrt{31})/4 = 64/4 = 16$, and $y = x - 1 = (1 + \sqrt{31})/2 - 1 = (-1 + \sqrt{31})/2$, so both equations hold.

Part IV (Long Constructed Response)

36. (a) $h(t) = 0.4 \cos(\pi t/2) + 0.8$ — The amplitude is $(1.2 - 0.4)/2 = 0.4$ and the midline is $(1.2 + 0.4)/2 = 0.8$. The period of 4 seconds gives $b = 2\pi/4 = \pi/2$, and starting at the maximum at $t = 0$ calls for a positive cosine. This yields $h(t) = 0.4 \cos(\pi t/2) + 0.8$.

(b) Amplitude 0.4, period 4 s, midline h = 0.8 — The amplitude of 0.4 meters is the distance from the midline to the horse's highest or lowest position. The period of 4 seconds is the time for one complete up-and-down cycle. The midline $h = 0.8$ meters is the average height about which the horse oscillates.

(c) t ≈ 0.67, 3.33, 4.67, and 7.33 seconds — Setting $1 = 0.4 \cos(\pi t/2) + 0.8$ gives $\cos(\pi t/2) = 0.5$. Over the first 8 seconds, $\pi t/2 = \pi/3, 5\pi/3, 7\pi/3$, and $11\pi/3$, so $t = 2/3, 10/3, 14/3$, and $22/3$. These round to 0.67, 3.33, 4.67, and 7.33 seconds.