PRACTICE EXAM 37: ALGEBRA II REGENTS SIMULATION

PART I — Multiple Choice (24 questions × 2 credits = 48 credits)

1. What is the product $(3 + 4i)(3 - 4i)$?

A. $9 - 16i$

B. $9 + 16i$

C. 25

D. −7

2. What is the value of 16^(−1/2)?

A. −4

B. 1/4

C. −1/4

D. 4

3. What are the solutions to the equation $2x^2 + 2x + 5 = 0$?

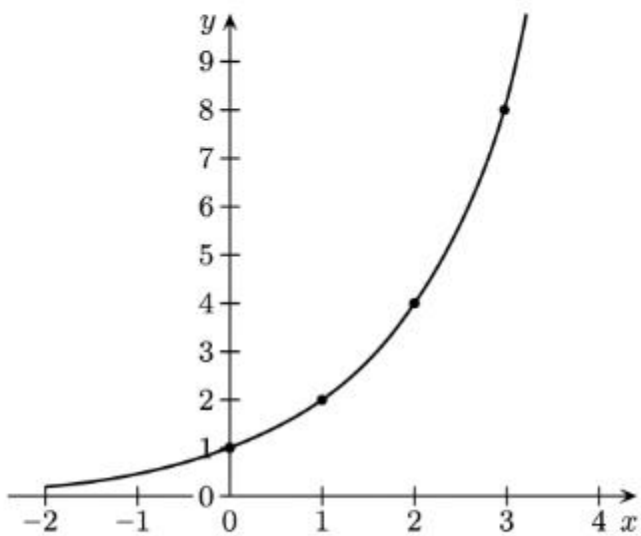
A. $x = (-1 \pm 3i)/2$

B. x = (1 ± 3i)/2

C. x = −1 ± 3i

D. x = (−1 ± 6i)/2

4. Which type of function is represented by the graph below?



A. Linear

B. Quadratic

C. Logarithmic

D. Exponential

5. Factored completely, the expression $9x^2 - 49$ is equivalent to

A. $(9x - 7)(x + 7)$

B. $(3x - 7)^2$

C. $(3x - 7)(3x + 7)$

D. $3(x - 7)(x + 7)$

6. What is the value of $\log_2(1)$?

A. 0

B. 1

C. 2

D. −1

7. What is the 5th term of a geometric sequence whose first term is 2 and whose common ratio is 3?

A. 54

B. 162

C. 486

D. 108

8. When $p(x) = x^4 - 3x^2 + 2$ is divided by $(x - 1)$, the remainder is

A. 6

B. 4

C. −2

D. 0

9. Which function has an inverse that is also a function?

A. $f(x) = x^2$

B. $f(x) = |x|$

C. $f(x) = x^2 - 4$

D. $f(x) = x^3$

10. What is the solution to the equation $\sqrt{x} + 3 = 7$?

A. $x = 4$

B. $x = 10$

C. $x = 16$

D. $x = 100$

11. The graph of a quadratic function is shown below. What is the minimum value of the function?

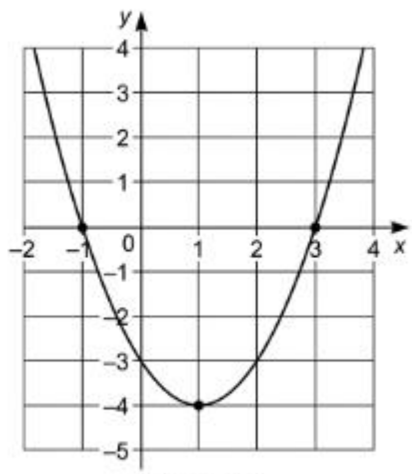


Figure PQ-2

A. −4

B. 1

C. 4

D. −1

12. A radioactive substance is modeled by f(t) = 64(1/2)^(t/5), where t is the time in years. What is the value of f(15)?

A. 16

B. 8

C. 4

D. 32

13. The table below shows values of the function f(x).

x	1	2	3	4	5

f(x)	3	12	27	48	75

What is the average rate of change of $f(x)$ over the interval [2, 5]?

A. 63

B. 12

C. 21

D. 24

14. Which statement best describes the nature of the solutions of $2x^2 - 3x - 4 = 0$?

A. Two complex (non-real) solutions

B. One repeated rational solution

C. Two distinct rational solutions

D. Two distinct irrational solutions

15. What is the exact value of $\tan(\pi/3)$?

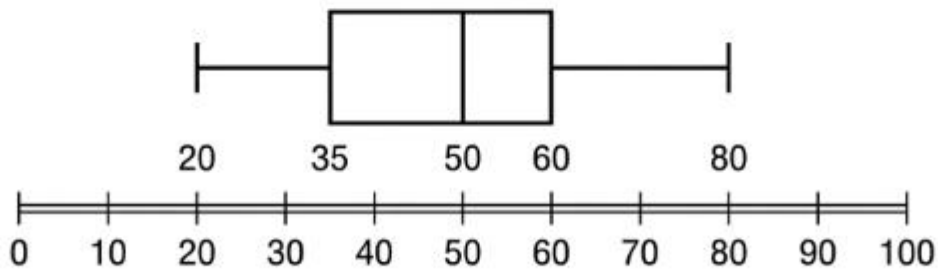
A. $\sqrt{3}$

B. $\sqrt{3}/3$

C. 1

D. 1/2

16. The box-and-whisker plot below summarizes a set of data. What is the median of the data?



A. 35

B. 50

C. 60

D. 25

17. What is the solution to the equation $\log_5(2x - 1) = 2$?

A. x = 5

B. x = 12.5

C. x = 3

D. x = 13

18. For the function y = 2 sin(3x) + 1, what is the amplitude?

A. 1

B. 3

C. 2

D. $2\pi/3$

19. Which expression represents the nth term of an arithmetic sequence whose first term is −5 and whose common difference is 3?

A. $a_n = 3n - 8$

B. $a_n = -5 + 3n$

C. $a_n = 3n - 5$

D. $a_n = -5n + 3$

20. A fair coin is flipped and a fair six-sided die is rolled. What is the probability of getting heads and a 4?

A. 1/8

B. 1/12

C. 1/6

D. 2/8

21. A data set is normally distributed with a mean of 50 and a standard deviation of 10. Approximately what percent of the data falls between 30 and 50?

A. 95%

B. 68%

C. 34%

D. 47.5%

22. The graph below shows function f(x) (solid) and function g(x) (dashed). Which transformation maps f(x) onto g(x)?

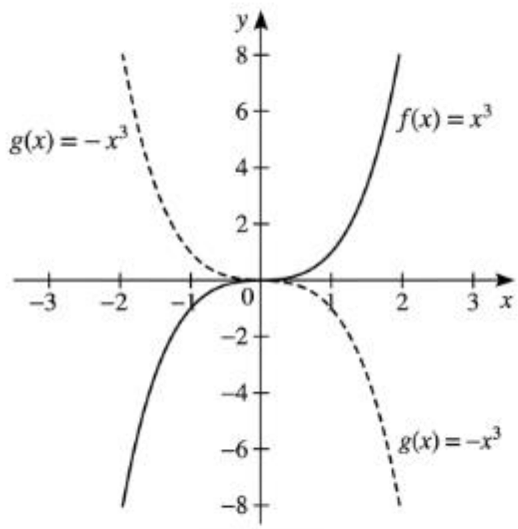


Figure PQ-4

A. Reflection across the y-axis

B. Vertical shift down 3

C. Reflection across the x-axis

D. Horizontal shift right 1

23. Given that cos θ = 9/41 and θ is in Quadrant I, what is the value of sin θ?

A. 41/40

B. 40/41

C. 9/40

D. −40/41

24. Written in vertex form, the expression $x^2 + 4x - 1$ is equivalent to

A. $(x + 2)^2 - 5$

B. $(x + 2)^2 - 1$

C. $(x - 2)^2 - 5$

D. $(x + 4)^2 - 17$

PART II — Short Constructed Response (8 questions × 2 credits = 16 credits)

Show all work. A correct answer with no supporting work will receive only 1 credit.

25. Solve the equation $x^2 + 2x + 5 = 0$ algebraically. Express your solutions in $a + bi$ form.

26. Solve algebraically for x: $(x + 1) / x = (x + 6) / (2x)$. State any restrictions on the variable and identify any extraneous solutions that must be rejected.

27. Given $f(x) = x^3 - 4x^2 + x + 6$, determine whether $(x - 2)$ is a factor of $f(x)$. Justify your answer using the Remainder Theorem or the Factor Theorem.

28. Write an equation, in the form $g(x) = a \cdot f(x) + k$, for the function obtained by reflecting $f(x) = \sqrt[3]{x}$ across the x-axis and then translating the result 2 units up.

29. Solve algebraically for x: $27^x = 9^{(x + 1)}$.

30. Express $x / (x - 5) - 2 / (x + 5)$ as a single rational expression in simplest form. State any restrictions on the variable.

31. A sequence is defined recursively by $a_1 = 1$ and $a_n = a_{n-1} + 2n$ for $n \geq 2$. Find the value of a_4.

32. Given that $\sin \theta = -12/13$ and θ terminates in Quadrant III, find the exact value of $\tan \theta$. Show the algebraic work that justifies your answer.

PART III — Extended Constructed Response (3 questions × 4 credits = 12 credits)

Show all work. Partial credit is awarded according to the scoring rubric.

33. The value of an investment is modeled by the function $A(t) = 1500(1.045)^t$, where $A(t)$ is the value in dollars and t is the time in years.

(a) State the initial value of the investment and the annual percent rate of growth.

(b) Algebraically determine the number of years it will take for the investment to grow to \$2500. Round your answer to the nearest tenth of a year.

34. The table below shows the height h(x), in feet, of a parabolic arch at horizontal distance x feet from the left support.

x (feet)	0	1	2	3	4

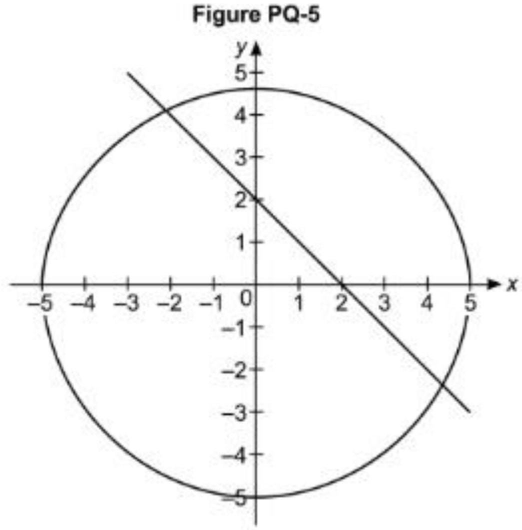
h(x) (feet)	1	8	13	16	17

(a) Using regression, write a quadratic function $h(x) = ax^2 + bx + c$ that best models the data. Round each coefficient to the nearest tenth.

(b) Use the regression model from part (a) to determine the maximum height of the arch and the horizontal distance at which the maximum height occurs. Round each answer to the nearest tenth.

35. A circle and a line are graphed below. The circle is defined by $x^2 + y^2 = 22$, and the line is defined by $y = -x + 2$.

Figure PQ-5



(a) Algebraically determine all points of intersection between the circle and the line. Express coordinates in exact form (radicals permitted).

(b) Verify your answer by substituting one of the intersection points back into both original equations. Show the substitution clearly.

PART IV — Long Constructed Response (1 question × 6 credits = 6 credits)

Show all work. This problem requires multiple steps and integrates concepts from several chapters.

36. The pendulum of a grandfather clock swings back and forth. Its horizontal displacement from the center position oscillates between 20 cm in the positive direction and 20 cm in the negative direction, completing one full swing cycle every 2 seconds. At time $t = 0$, the pendulum is at the center position (displacement 0 cm) and moving in the positive direction.

(a) Write a function $x(t)$ that models the horizontal displacement of the pendulum in centimeters as a function of time t in seconds.

(b) State the amplitude, period, and midline of the function $x(t)$, and explain what each represents in the context of the swinging pendulum.

(c) Algebraically determine all times during the first 2 seconds at which the pendulum's displacement is 8 cm in the positive direction. Round each answer to the nearest hundredth of a second. Show all algebraic work.

ANSWER KEY WITH EXPLANATIONS – PRACTICE EXAM 37

1. C — Multiplying conjugates gives a real result: $(3 + 4i)(3 - 4i) = 9 - 16i^2 = 9 + 16 = 25$. The product of complex conjugates eliminates the imaginary part. The answer is the real number 25.

2. B — A negative fractional exponent means the reciprocal of the root: $16^{(-1/2)} = 1/\sqrt{16} = 1/4$. The 1/2 takes the square root and the negative sign inverts. The result is 1/4.

3. A — Quadratic formula: $x = [-2 \pm \sqrt{(4 - 40)}]/4 = [-2 \pm \sqrt{(-36)}]/4 = [-2 \pm 6i]/4 = (-1 \pm 3i)/2$. The negative discriminant produces the imaginary part. Dividing each term by 4 gives the simplified form.

4. D — The curve has a horizontal asymptote at $y = 0$ and doubles in value at each step, the signature of exponential growth. Linear and quadratic functions have no horizontal asymptote. The constant multiplicative pattern identifies it as exponential.

5. C — This is a difference of squares: $9x^2 - 49 = (3x)^2 - 7^2 = (3x - 7)(3x + 7)$. The two factors differ only in sign. Recognizing both terms as perfect squares is the key step.

6. A — A logarithm of 1 is always 0 because any base raised to 0 equals 1: $2^0 = 1$. The exponent that produces 1 is 0. The value is 0.

7. B — The nth term is $a_n = a_1 \cdot r^{(n-1)}$, so $a_5 = 2 \cdot 3^4 = 2 \cdot 81 = 162$. The exponent $n - 1 = 4$ applies the ratio four times. The fifth term is 162.

8. D — By the Remainder Theorem, the remainder is $p(1) = 1 - 3 + 2 = 0$. A remainder of zero means $(x - 1)$ divides the polynomial evenly. Substituting the zero of the divisor gives 0.

9. D — Only $f(x) = x^3$ is one-to-one, passing the horizontal line test, so its inverse is also a function. The squaring and absolute value functions map two inputs to the same output, failing the test. A one-to-one function guarantees an inverse function.

10. C — Subtract 3, then square: $\sqrt{x} = 4$, so $x = 16$. Isolating the radical before squaring is essential. Checking, $\sqrt{16} + 3 = 7$ confirms the solution.

11. A — The minimum value of an upward-opening parabola is the y-coordinate of its vertex, which is -4. The vertex $(1, -4)$ is the lowest point. Reading the y-value of the vertex gives the minimum.

12. B — Substitute $t = 15$: $f(15) = 64(1/2)^{(15/5)} = 64(1/2)^3 = 64/8 = 8$. The exponent $15/5 = 3$ represents three half-life periods. The amount remaining is 8.

13. C — Average rate of change is $[f(5) - f(2)]/(5 - 2) = (75 - 12)/3 = 63/3 = 21$. This is the slope of the secant line over the interval. The table values 75 and 12 drive the result.

14. D — The discriminant is $b^2 - 4ac = 9 + 32 = 41$, which is positive but not a perfect square. This produces two distinct irrational real solutions. The nature of the discriminant classifies the roots.

15. A — Tangent is sine over cosine, and at $\pi/3$ this is $(\sqrt{3}/2)/(1/2) = \sqrt{3}$. The 30-60-90 reference triangle gives these values. The tangent of 60° is $\sqrt{3}$.

16. B — The median is marked by the vertical line inside the box, located at 50. It divides the data into lower and upper halves. The middle line of the box always shows the median.

17. D — Rewrite in exponential form: $2x - 1 = 5^2 = 25$, so $2x = 26$ and $x = 13$. Converting the logarithmic equation isolates x. Checking shows the argument stays positive.

18. C — In $y = a \sin(bx) + d$, the amplitude is $|a|$, which is 2. The amplitude is the coefficient of the sine term. The period and vertical shift do not affect it.

19. A — The nth term is $a_n = a_1 + (n - 1)d = -5 + 3(n - 1) = 3n - 8$. Distributing and combining constants simplifies the expression. The closed form is $3n - 8$.

20. B — The events are independent, so multiply their probabilities: $1/2 \cdot 1/6 = 1/12$. The coin and die outcomes do not affect each other. The combined probability is 1/12.

21. D — The value 30 is two standard deviations below the mean of 50. The region from −2 SD to the mean covers half of the 95% within ±2 SD, which is 47.5%. The empirical rule gives this area.

22. C — Each point maps from (x, y) to $(x, -y)$, which negates the y-coordinate while keeping x the same. That transformation is a reflection across the x-axis. The mirror symmetry about the horizontal axis confirms it.

23. B — With $\cos\theta = 9/41$ in Quadrant I, the 9-40-41 triangle gives $\sin\theta = 40/41$. All ratios are positive in Quadrant I. The missing leg of 40 produces the sine value.

24. A — Complete the square: $x^2 + 4x - 1 = (x^2 + 4x + 4) - 4 - 1 = (x + 2)^2 - 5$. Half of 4 squared is 4, added and subtracted to preserve value. The vertex form reveals the vertex at $(-2, -5)$.

Part II (Short Constructed Response)

25. $x = -1 \pm 2i$ — Quadratic formula: $x = [-2 \pm \sqrt{(4 - 20)}]/2 = [-2 \pm \sqrt{(-16)}]/2 = [-2 \pm 4i]/2 = -1 \pm 2i$. The negative discriminant produces complex conjugates. Dividing each term by 2 gives the simplified $a + bi$ form.

26. $x = 4$; $x = 0$ is rejected — Cross-multiplying gives $2x(x + 1) = x(x + 6)$, which becomes $2x^2 + 2x = x^2 + 6x$ and simplifies to $x^2 - 4x = 0$, so $x(x - 4) = 0$. The restriction $x \neq 0$ makes $x = 0$ extraneous, leaving $x = 4$ as the only valid solution.

27. Yes, $(x - 2)$ is a factor — By the Factor Theorem, $f(2) = 8 - 16 + 2 + 6 = 0$. A remainder of zero confirms that $(x - 2)$ divides $f(x)$ evenly. A nonzero value would have meant it is not a factor.

28. $g(x) = -\sqrt[3]{x} + 2$ — Reflecting across the x-axis negates the function to $-\sqrt[3]{x}$; shifting 2 units up adds 2. Combining these produces $-\sqrt[3]{x} + 2$. The reflection acts on the entire function before the vertical shift.

29. $x = 2$ — Write both sides with base 3: $27^x = 3^{(3x)}$ and $9^{(x + 1)} = 3^{(2x + 2)}$, so $3x = 2x + 2$. Solving gives $x = 2$. Equal bases allow the exponents to be set equal.

30. $(x^2 + 3x + 10) / [(x - 5)(x + 5)]$, $x \neq 5$, $x \neq -5$ — Over the common denominator $(x - 5)(x + 5)$: $x(x + 5) - 2(x - 5) = x^2 + 5x - 2x + 10 = x^2 + 3x + 10$. The numerator combines to $x^2 + 3x + 10$, which does not factor. Restrictions come from the original denominators.

31. $a_4 = 19$ — Apply the recursion: $a_2 = 1 + 2(2) = 5$, $a_3 = 5 + 2(3) = 11$, $a_4 = 11 + 2(4) = 19$. Each term adds twice the index to the previous term. Building up term by term reaches 19.

32. $\tan\theta = 12/5$ — With $\sin\theta = -12/13$ in Quadrant III, the 5-12-13 triangle gives $\cos\theta = -5/13$ because cosine is negative there. Then $\tan\theta = \sin/\cos = (-12/13)/(-5/13) = 12/5$. The two negatives make the tangent positive.

Part III (Extended Constructed Response)

33. (a) \$1500, 4.5% growth; (b) ≈ 11.6 years — The coefficient 1500 is the initial value, and the base 1.045 = 1 + 0.045 indicates 4.5% annual growth. For part (b), $2500 = 1500(1.045)^t$ gives $1.045^t = 5/3$, so $t = \ln(5/3)/\ln(1.045) \approx 11.6$ years. Logarithms isolate the exponent.

34. (a) $h(x) = -1.0x^2 + 8.0x + 1.0$; (b) 17.0 ft at x = 4.0 ft — Quadratic regression on the data returns a = −1.0, b = 8.0, c = 1.0. The vertex occurs at $x = -b/(2a) = -8/(-2) = 4.0$ feet, and $h(4) = -16 + 32 + 1 = 17.0$ feet. The negative leading coefficient confirms a maximum.

35. (a) $(1 + \sqrt{10}, 1 - \sqrt{10})$ and $(1 - \sqrt{10}, 1 + \sqrt{10})$ — Substituting $y = -x + 2$ into the circle gives $x^2 + (-x + 2)^2 = 22$, which simplifies to $2x^2 - 4x - 18 = 0$, then $x^2 - 2x - 9 = 0$, so $x = 1 \pm \sqrt{10}$. The y-values follow from $y = -x + 2$. **(b)** Checking $(1 + \sqrt{10}, 1 - \sqrt{10})$: $x^2 + y^2 = (11 + 2\sqrt{10}) + (11 - 2\sqrt{10}) = 22$, and $y = -x + 2 = -(1 + \sqrt{10}) + 2 = 1 - \sqrt{10}$, so both equations hold.

Part IV (Long Constructed Response)

36. (a) $x(t) = 20 \sin(\pi t)$ — The amplitude is 20 cm and the midline is 0 (the center position). The period of 2 seconds gives $b = 2\pi/2 = \pi$, and starting at the center while moving in the positive direction calls for a positive sine function. This yields $x(t) = 20 \sin(\pi t)$.

(b) Amplitude 20, period 2 s, midline x = 0 — The amplitude of 20 cm is the maximum displacement of the pendulum from the center in either direction. The period of 2 seconds is the time for one complete swing cycle. The midline x = 0 is the center position about which the pendulum oscillates.

(c) t ≈ 0.13 seconds and t ≈ 0.87 seconds — Setting $8 = 20 \sin(\pi t)$ gives $\sin(\pi t) = 0.4$. Over the first 2 seconds, $\pi t = 0.4115$ and 2.7301 radians, so t = 0.13 and 0.87 seconds. These are the two times the pendulum passes through 8 cm in the positive direction while swinging outward and returning.