PRACTICE EXAM 36: ALGEBRA II REGENTS SIMULATION

PART I — Multiple Choice (24 questions × 2 credits = 48 credits)

1. Express the sum (4 − 3i) + (−2 + 7i) in a + bi form.

A. 2 − 4i

B. 6 + 4i

C. 6 − 10i

D. 2 + 4i

2. Simplify the expression $(27x^6)^{(1/3)}$.

A. $9x^2$

B. $3x^3$

C. $3x^2$

D. $27x^2$

3. What are the solutions to the equation $x^2 - 4x + 7 = 0$?

A. $x = 2 \pm i\sqrt{3}$

B. $x = -2 \pm i\sqrt{3}$

C. $x = 2 \pm 2i\sqrt{3}$

D. $x = 4 \pm i\sqrt{3}$

4. Factored completely, the expression $2x^2 + 7x + 3$ is equivalent to

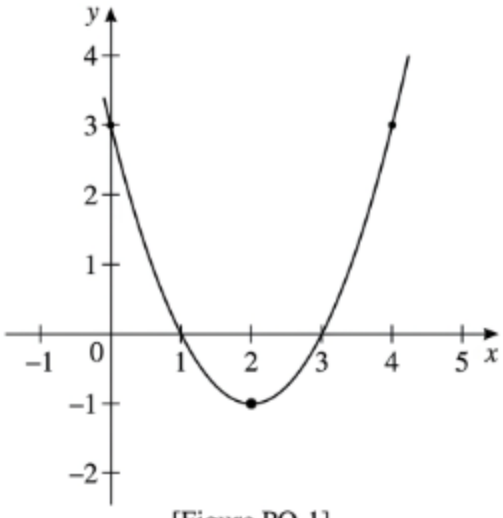
A. $(2x + 3)(x + 1)$

B. $(2x + 1)(x + 3)$

C. $(2x - 1)(x - 3)$

D. $(x + 1)(x + 3)$

5. The graph of a quadratic function is shown below. What is the equation of its axis of symmetry?



[Figure PQ-1]

A. $x = -2$

B. $y = -1$

C. $y = 2$

D. $x = 2$

6. What is the solution to the equation $\ln(x) = 0$?

A. $x = 1$

B. $x = 0$

C. $x = e$

D. $x = -1$

7. What is the common ratio of the geometric sequence 80, 20, 5, …?

A. 4

B. −4

C. 1/4

D. −1/4

8. For what value of k is $(x - 2)$ a factor of $f(x) = x^3 - 3x^2 + kx - 4$?

A. $k = 2$

B. $k = 4$

C. $k = 6$

D. $k = -4$

9. If $f(x) = 2x + 1$ and $g(x) = x^2$, what is the value of $g(f(3))$?

A. 19

B. 49

C. 37

D. 13

10. What is the solution set of the equation $\sqrt{(3x - 2)} = x - 2$?

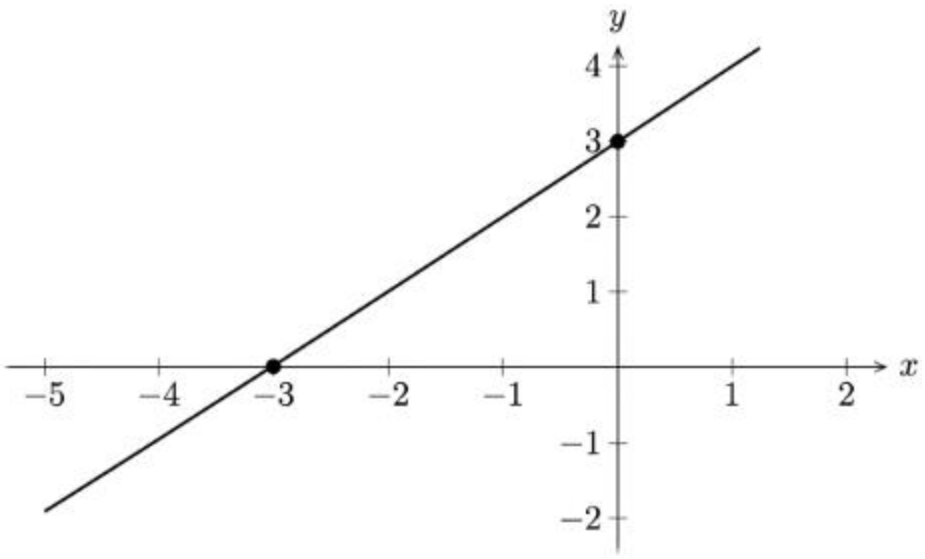
A. {1, 6}

B. {1}

C. { }

D. {6}

11. Using the graph of the function f(x) shown below, what is a zero of the function?



A. −3

B. 3

C. 0

D. −6

12. If \$500 is invested at an annual interest rate of 8% compounded annually, what is the value of the investment after 2 years?

A. \$580.00

B. \$540.00

C. \$583.20

D. \$1080.00

13. What is the average rate of change of f(x) = √x over the interval [4, 16]?

A. 2

B. 12

C. 6

D. 1/6

14. Which statement best describes the nature of the solutions of $x^2 - 10x + 25 = 0$?

A. Two distinct irrational solutions

B. One repeated rational solution

C. Two complex (non-real) solutions

D. Two distinct rational solutions

15. What is the exact value of $\sin(\pi/3)$?

A. $\sqrt{3}/2$

B. 1/2

C. $\sqrt{2}/2$

D. $-\sqrt{3}/2$

16. In the right triangle below, what is the measure of angle A to the nearest degree?

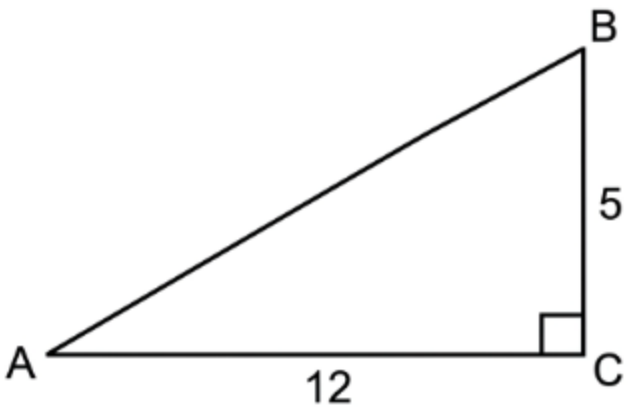


Figure PQ-3

A. 67°

B. 45°

C. 23°

D. 30°

17. What is the solution to the equation $2^x = 20$, rounded to the nearest hundredth?

A. x = 10

B. x = 5.00

C. x = 3.00

D. x = 4.32

18. For the function $y = 4 \sin(2x) - 3$, what is the equation of the midline?

A. $y = -3$

B. $y = 4$

C. $y = 1$

D. $y = 2$

19. What is the sum of the first 8 terms of an arithmetic sequence whose first term is 6 and whose common difference is -2?

A. 0

B. -16

C. -8

D. 8

20. A standard six-sided die is rolled. Given that the result is an even number, what is the probability that it is greater than 3?

A. 1/2

B. 2/3

C. 1/3

D. 1/6

21. A data set is normally distributed with a mean of 70 and a standard deviation of 8. What is the z-score of a value of 86?

A. 16

B. 0.5

C. 1

D. 2

22. Given that $\sin \theta = -3/5$ and θ is in Quadrant III, what is the value of $\cos \theta$?

A. $-4/5$

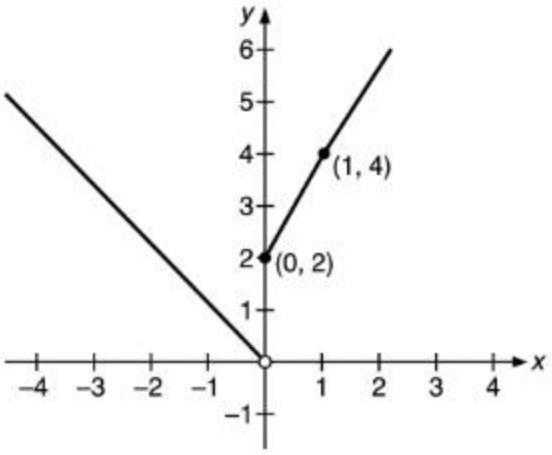
B. $4/5$

C. $-3/4$

D. $3/4$

23. The graph below shows a piecewise function f(x). What is the value of f(1)?

Figure PQ-4



A. 1

B. 2

C. 4

D. 0

24. Written in vertex form, the expression $x^2 - 6x + 2$ is equivalent to

A. $(x - 3)^2 + 2$

B. $(x - 3)^2 - 7$

C. $(x + 3)^2 - 7$

D. $(x - 6)^2 - 34$

PART II — Short Constructed Response (8 questions × 2 credits = 16 credits)

Show all work. A correct answer with no supporting work will receive only 1 credit.

25. Solve the equation $x^2 - 4x + 13 = 0$ algebraically. Express your solutions in a + bi form.

26. Solve algebraically for x: 4 / x + 4 / (x − 2) = 3. State any restrictions on the variable and identify any extraneous solutions that must be rejected.

27. Given $f(x) = 2x^3 + x^2 - 8x - 4$, determine whether (x + 1) is a factor of f(x). Justify your answer using the Remainder Theorem or the Factor Theorem.

28. Write an equation, in the form $g(x) = a \cdot f(x - h) + k$, for the function obtained by vertically stretching $f(x) = |x|$ by a factor of 2 and then translating the result 3 units to the left and 1 unit down.

29. Solve algebraically for x: $16^x = 64$.

30. Express $3 / (x^2 + x) + 2 / (x + 1)$ as a single rational expression in simplest form. State any restrictions on the variable.

31. A sequence is defined recursively by $a_1 = 64$ and $a_n = a_{n-1} / 4$ for $n \geq 2$. Find the value of a_4.

32. Given that $\tan \theta = 7/24$ and θ terminates in Quadrant III, find the exact value of $\cos \theta$. Show the algebraic work that justifies your answer.

PART III — Extended Constructed Response (3 questions × 4 credits = 12 credits)

Show all work. Partial credit is awarded according to the scoring rubric.

33. A bacteria culture grows according to the function $N(t) = 200(1.5)^t$, where $N(t)$ is the number of bacteria and t is the time in hours.

(a) State the initial number of bacteria and the hourly percent rate of growth.

(b) Algebraically determine the number of hours it will take for the culture to grow to 1000 bacteria. Round your answer to the nearest tenth of an hour.

34. The table below shows the height $h(t)$, in feet, of a ball at time t seconds after being thrown upward.

t (seconds)	0	1	2	3	4
h(t) (feet)	3	18	23	18	3

(a) Using regression, write a quadratic function $h(t) = at^2 + bt + c$ that best models the data. Round each coefficient to the nearest tenth.

(b) Use the regression model from part (a) to determine the maximum height reached by the ball and the time at which the maximum height occurs. Round each answer to the nearest tenth.

35. A circle and a line are graphed below. The circle is defined by $x^2 + y^2 = 14$, and the line is defined by $y = x - 1$.

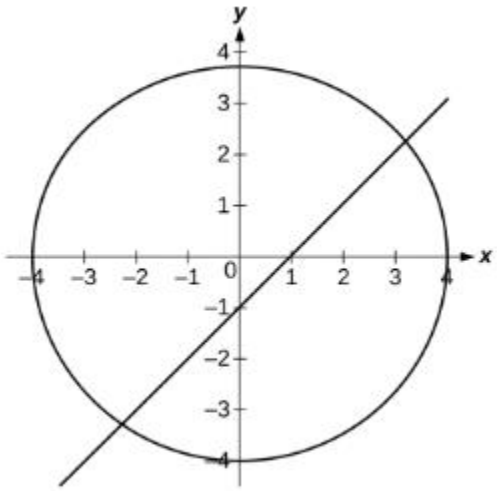


Figure PQ-5

(a) Algebraically determine all points of intersection between the circle and the line. Express coordinates in exact form (radicals permitted).

(b) Verify your answer by substituting one of the intersection points back into both original equations. Show the substitution clearly.

PART IV — Long Constructed Response (1 question × 6 credits = 6 credits)

Show all work. This problem requires multiple steps and integrates concepts from several chapters.

36. In a certain city, the number of hours of daylight varies sinusoidally throughout the year. The longest day of the year has 14 hours of daylight, and the shortest day has 10 hours of daylight. One full cycle takes 12 months. At time t = 0 (the start of the year), the city experiences its shortest day, with the minimum amount of daylight.

(a) Write a function D(t) that models the number of hours of daylight as a function of time t in months.

(b) State the amplitude, period, and midline of the function D(t), and explain what each represents in the context of the daylight hours.

(c) Algebraically determine all times during the first 12 months at which the city has 12.5 hours of daylight. Round each answer to the nearest hundredth of a month. Show all algebraic work.

ANSWER KEY WITH EXPLANATIONS – PRACTICE EXAM 36

1. D — Combine real parts and imaginary parts separately: $(4 - 2) + (-3 + 7)i = 2 + 4i$. Adding complex numbers treats the real and imaginary components independently. The result is $2 + 4i$.

2. C — Apply the exponent to each factor: $(27x^6)^{(1/3)} = 27^{(1/3)} \cdot x^{(6/3)} = 3x^2$. The cube root of 27 is 3 and the exponent 6 divides by 3 to give 2. The result is $3x^2$.

3. A — Quadratic formula: $x = [4 \pm \sqrt{(16 - 28)}]/2 = [4 \pm \sqrt{(-12)}]/2 = [4 \pm 2i\sqrt{3}]/2 = 2 \pm i\sqrt{3}$. The radical $\sqrt{(-12)}$ simplifies to $2i\sqrt{3}$, which reduces with the denominator. The negative discriminant gives complex conjugates.

4. B — Factoring gives $(2x + 1)(x + 3)$, since the outer and inner products 6x and x combine to the middle term 7x. The constant terms 1 and 3 multiply to 3. Checking the expansion confirms the factorization.

5. D — The axis of symmetry is the vertical line through the vertex. With the vertex at $(2, -1)$, the axis is $x = 2$. The axis is always a vertical line of the form $x = h$.

6. A — A natural log of 0 means e raised to 0: $x = e^0 = 1$. Any nonzero base raised to the zero power equals 1. The solution is $x = 1$.

7. C — The common ratio is the quotient of consecutive terms: 20/80 = 1/4, confirmed by 5/20 = 1/4. Each term is one-fourth of the previous one. The ratio is 1/4.

8. B — By the Factor Theorem, $(x - 2)$ is a factor when $f(2) = 0$: $8 - 12 + 2k - 4 = 0$, so $2k - 8 = 0$ and $k = 4$. Setting the function value at $x = 2$ to zero solves for k. The required value is 4.

9. B — Evaluate the inner function first: $f(3) = 2(3) + 1 = 7$, then $g(7) = 7^2 = 49$. Composition works from the inside out. Squaring 7 gives 49.

10. D — Squaring gives $3x - 2 = x^2 - 4x + 4$, so $x^2 - 7x + 6 = 0$ and $(x - 1)(x - 6) = 0$. The candidate $x = 1$ fails because $\sqrt{1} = 1 \neq -1$, leaving $x = 6$ as the only valid solution. The check removes the extraneous root.

11. A — A zero of a function is where its graph crosses the x-axis. The line crosses at $(-3, 0)$, so -3 is a zero. Reading the x-intercept directly gives the zero.

12. C — Apply the compound interest formula: $A = 500(1.08)^2 = 500(1.1664) = 583.20$. Squaring the growth factor accounts for two years of compounding. The investment is worth $583.20.

13. D — Average rate of change is $[f(16) - f(4)]/(16 - 4) = (4 - 2)/12 = 2/12 = 1/6$. This is the slope of the secant line over the interval. The square-root values 4 and 2 drive the result.

14. B — The discriminant is $b^2 - 4ac = 100 - 100 = 0$, indicating one repeated real solution. Since $x^2 - 10x + 25 = (x - 5)^2$, the repeated root 5 is rational. A zero discriminant always yields a single repeated solution.

15. A — The sine of $\pi/3$ (60°) is a standard unit-circle value equal to $\sqrt{3}/2$. The 30-60-90 reference triangle gives this ratio. The value is $\sqrt{3}/2$.

16. C — The angle A has opposite side 5 and adjacent side 12, so $\tan A = 5/12$ and $A = \arctan(5/12) \approx 22.6°$, which rounds to 23°. The tangent ratio relates the two legs. The inverse tangent gives the angle.

17. D — Take the log of both sides: $x = \log_2(20) = \ln(20)/\ln(2) \approx 4.32$. The change-of-base formula converts to natural logs. The result rounds to 4.32.

18. A — In $y = a \sin(bx) + d$, the midline is the vertical shift d, which is -3. The midline is the horizontal line about which the function oscillates. The amplitude of 4 does not affect its location.

19. C — Using $S = n/2(2a_1 + (n - 1)d) = 8/2(12 + 7(-2)) = 4(12 - 14) = 4(-2) = -8$. The negative common difference pulls the sum below zero. The sum of the first 8 terms is -8.

20. B — The even outcomes are 2, 4, and 6; of these, 4 and 6 are greater than 3. The conditional probability is 2/3. Restricting the sample space to even results gives the denominator of 3.

21. D — The z-score measures standard deviations from the mean: $z = (86 - 70)/8 = 16/8 = 2$. The value lies two standard deviations above the mean. The z-score is 2.

22. A — In Quadrant III both sine and cosine are negative, and the 3-4-5 triangle gives the cosine ratio. Therefore $\cos\theta = -4/5$. The quadrant fixes the negative sign on cosine.

23. C — For $x \geq 0$, the function follows the segment $f(x) = 2x + 2$, so $f(1) = 2(1) + 2 = 4$. Since 1 is in the domain of the upper piece, that rule applies. The value is 4.

24. B — Complete the square: $x^2 - 6x + 2 = (x^2 - 6x + 9) - 9 + 2 = (x - 3)^2 - 7$. Half of −6 squared is 9, added and subtracted to preserve value. The vertex form reveals the vertex at $(3, -7)$.

Part II (Short Constructed Response)

25. x = 2 ± 3i — Quadratic formula: $x = [4 \pm \sqrt{(16 - 52)}]/2 = [4 \pm \sqrt{(-36)}]/2 = [4 \pm 6i]/2 = 2 \pm 3i$. The negative discriminant produces complex conjugates. Dividing each term by 2 gives the simplified a + bi form.

26. x = 4 and x = 2/3; no extraneous solutions — Multiplying through by $x(x - 2)$ gives $4(x - 2) + 4x = 3x(x - 2)$, which simplifies to $3x^2 - 14x + 8 = 0$ and factors to $(3x - 2)(x - 4) = 0$. The restrictions are $x \neq 0$ and $x \neq 2$, and neither solution violates them, so both are valid.

27. No, (x + 1) is not a factor — By the Remainder Theorem, $f(-1) = -2 + 1 + 8 - 4 = 3$. A nonzero remainder means $(x + 1)$ does not divide $f(x)$ evenly. Only a remainder of zero would confirm a factor.

28. g(x) = 2|x + 3| − 1 — A vertical stretch by 2 multiplies the function by 2, giving $2|x|$; shifting 3 units left replaces x with $(x + 3)$; shifting 1 unit down subtracts 1. Combining these produces $2|x + 3| - 1$. The horizontal shift moves opposite the sign inside.

29. x = 3/2 — Write both sides with base 2: $16^x = 2^{(4x)}$ and $64 = 2^6$, so $4x = 6$. Solving gives $x = 3/2$. Equal bases allow the exponents to be set equal.

30. (2x + 3) / [x(x + 1)], x ≠ 0, x ≠ −1 — Factor the first denominator as $x(x + 1)$, then use it as the common denominator: $3/[x(x + 1)] + 2x/[x(x + 1)] = (3 + 2x)/[x(x + 1)]$. The numerator combines to $2x + 3$. Restrictions come from the factored denominator.

31. $a_4 = 1$ — Apply the recursion: $a_2 = 64/4 = 16$, $a_3 = 16/4 = 4$, $a_4 = 4/4 = 1$. Each term is one-fourth of the previous one. Building up term by term reaches 1.

32. cos θ = −24/25 — From $\tan\theta = 7/24$, the reference triangle has legs 7 and 24 with hypotenuse 25. In Quadrant III cosine is negative, so $\cos\theta = -24/25$. The quadrant fixes the negative sign.

Part III (Extended Constructed Response)

33. (a) 200, 50% growth; (b) ≈ 4.0 hours — The coefficient 200 is the initial count, and the base $1.5 = 1 + 0.5$ indicates 50% hourly growth. For part (b), $1000 = 200(1.5)^t$ gives $1.5^t = 5$, so $t = \ln(5)/\ln(1.5) \approx 4.0$ hours. Logarithms isolate the exponent.

34. (a) $h(t) = -5.0t^2 + 20.0t + 3.0$; (b) 23.0 ft at t = 2.0 s — Quadratic regression on the data returns a = −5.0, b = 20.0, c = 3.0. The vertex occurs at $t = -b/(2a) = -20/(-10) = 2.0$ seconds, and $h(2) = -20 + 40 + 3 = 23.0$ feet. The negative leading coefficient confirms a maximum.

35. (a) $((1 + 3\sqrt{3})/2, (-1 + 3\sqrt{3})/2)$ and $((1 - 3\sqrt{3})/2, (-1 - 3\sqrt{3})/2)$ — Substituting $y = x - 1$ into the circle gives $x^2 + (x - 1)^2 = 14$, which simplifies to $2x^2 - 2x - 13 = 0$, so $x = (1 \pm 3\sqrt{3})/2$. The y-values follow from $y = x - 1$. **(b)** Checking $((1 + 3\sqrt{3})/2, (-1 + 3\sqrt{3})/2)$: $x^2 + y^2 = (28 + 6\sqrt{3})/4 + (28 - 6\sqrt{3})/4 = 56/4 = 14$, and $y = x - 1 = (1 + 3\sqrt{3})/2 - 1 = (-1 + 3\sqrt{3})/2$, so both equations hold.

Part IV (Long Constructed Response)

36. (a) $D(t) = -2\cos(\pi t/6) + 12$ — The amplitude is $(14 - 10)/2 = 2$ and the midline is $(14 + 10)/2 = 12$. The period of 12 months gives $b = 2\pi/12 = \pi/6$, and starting at the minimum at t = 0 calls for a negative cosine. This yields $D(t) = -2\cos(\pi t/6) + 12$.

(b) Amplitude 2, period 12 months, midline D = 12 — The amplitude of 2 hours is the distance from the midline to the longest or shortest day. The period of 12 months is the time for one complete yearly cycle. The midline D = 12 hours is the average daylight around which the value oscillates.

(c) t ≈ 3.48 months and t ≈ 8.52 months — Setting $12.5 = -2\cos(\pi t/6) + 12$ gives $\cos(\pi t/6) = -0.25$. Over the first 12 months, $\pi t/6 = 1.8235$ and 4.4597 radians, so t = 3.48 and 8.52 months. These are the two times the rising and falling daylight passes through 12.5 hours.