PRACTICE EXAM 35: ALGEBRA II REGENTS SIMULATION

PART I — Multiple Choice (24 questions × 2 credits = 48 credits)

1. What is the value of $(3i)^2$?

A. −9

B. 9

C. 9i

D. −9i

2. What is the solution to the equation $x^{(3/2)} = 27$?

A. x = 18

B. x = 729

C. x = 3

D. x = 9

3. What are the solutions to the equation $x^2 + 4x + 8 = 0$?

A. $x = 2 \pm 2i$

B. $x = -2 \pm 2i$

C. $x = -4 \pm 4i$

D. $x = -2 \pm 4i$

4. The graph of a polynomial function is shown below. How many real zeros does the function have?

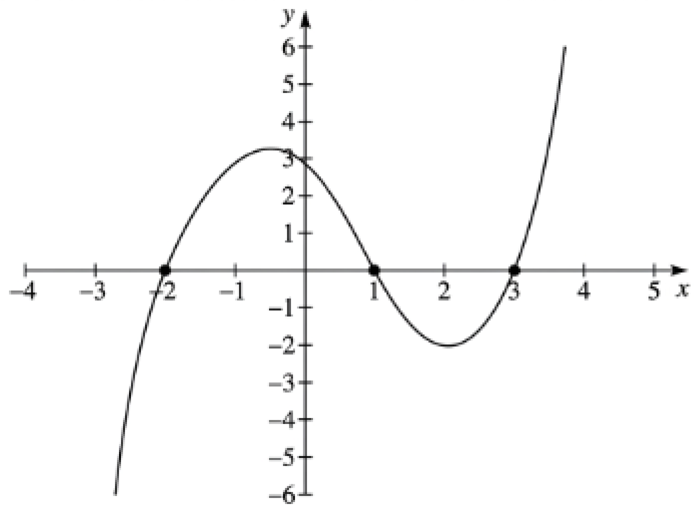


Figure PQ-1

A. 0

B. 1

C. 3

D. 2

5. Factored completely, the expression $3x^3 - 12x$ is equivalent to

A. $3x(x - 2)(x + 2)$

B. $3x(x^2 - 4)$

C. $3(x - 2)(x + 2)$

D. $x(3x - 6)(x + 2)$

6. What is the solution to the equation $\log_6(x) = 2$?

A. $x = 12$

B. $x = 36$

C. $x = 8$

D. $x = 64$

7. Which formula represents the nth term of the geometric sequence 3, 6, 12, 24, …?

A. $a_n = 3 + 2(n - 1)$

B. $a_n = 2 \cdot 3$^(n − 1)

C. $a_n = 3 \cdot 2^n$

D. $a_n = 3 \cdot 2$^(n − 1)

8. When $p(x) = 2x^3 + 3x^2 - x + 4$ is divided by $(x + 2)$, the remainder is

A. −2

B. 10

C. 2

D. 6

9. If $f(x) = x^3 + 1$, what is $f^{-1}(x)$?

A. $\sqrt[3]{x} - 1$

B. $(x - 1)^3$

C. $\sqrt[3]{(x - 1)}$

D. $1 / (x^3 + 1)$

10. What is the solution to the equation $\sqrt[3]{(x - 1)} = 2$?

A. $x = 9$

B. $x = 7$

C. $x = 5$

D. $x = 2$

11. The graph of a cosine function is shown below. What is the amplitude of the function?

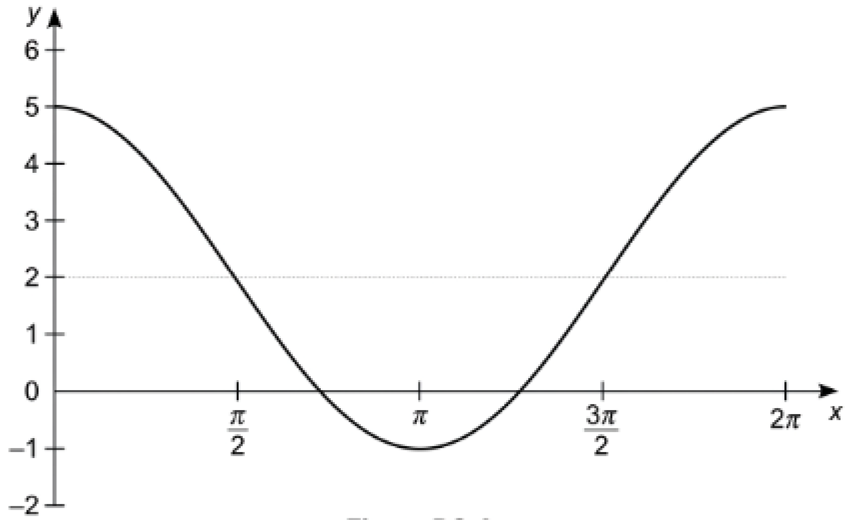


Figure PQ-2

A. 5

B. 2

C. 6

D. 3

12. A bacteria population starts at 100 and doubles every 6 hours, modeled by $P(t) = 100 \cdot 2^{(t/6)}$. What is the population after 18 hours?

A. 400

B. 800

C. 600

D. 1600

13. What is the average rate of change of $f(x) = x^2 - 2x$ over the interval $[-1, 3]$?

A. 0

B. 2

C. 4

D. −2

14. Which statement best describes the nature of the solutions of $5x^2 - 2x + 1 = 0$?

A. Two distinct rational solutions

B. One repeated rational solution

C. Two complex (non-real) solutions

D. Two distinct irrational solutions

15. What is the exact value of $\cos(3\pi/4)$?

A. $\sqrt{2}/2$

B. $-\sqrt{2}/2$

C. $\sqrt{3}/2$

D. $-1/2$

16. The expression $\log(xy^2)$ is equivalent to

A. $\log x \cdot \log y^2$

B. $2 \log(xy)$

C. $\log x + \log y$

D. log x + 2 log y

17. For the function y = 3 cos(x/2), what is the period?

A. 4π

B. π

C. 2π

D. $\pi/2$

18. In the right triangle below, what is the length of side x?

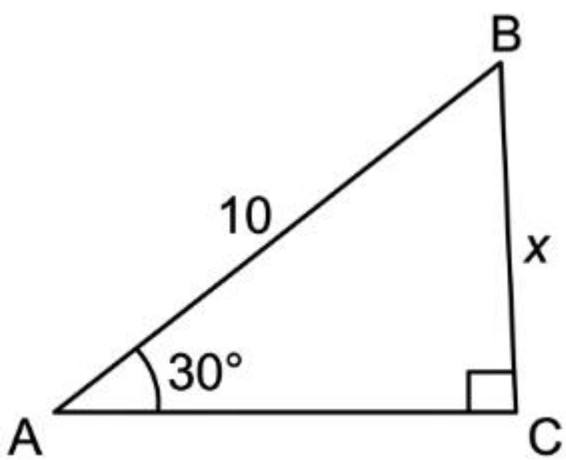


Figure PQ-3

A. $10\sqrt{3}$

B. 8.66

C. 5

D. 20

19. What is the sum of all the integers from 1 to 50, inclusive?

A. 2550

B. 1250

C. 1225

D. 1275

20. In how many ways can 2 books be selected from a shelf of 6 different books?

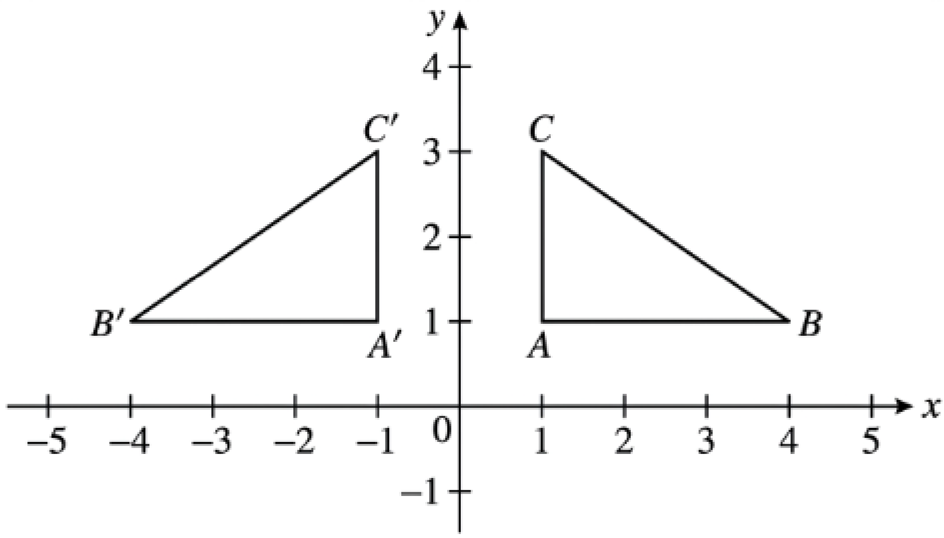
A. 30

B. 15

C. 12

D. 36

21. Triangle ABC is reflected to produce triangle A′B′C′ as shown below. Across which line was the triangle reflected?



A. The x-axis

B. The line y = x

C. The y-axis

D. The origin

22. A data set is normally distributed with a mean of 60 and a standard deviation of 6. Approximately what percent of the data falls between 54 and 66?

A. 68%

B. 95%

C. 34%

D. 50%

23. Given that tan θ = −8/15 and θ is in Quadrant IV, what is the value of cos θ?

A. −15/17

B. 15/17

C. 8/17

D. −8/17

24. Written in vertex form, the expression $x^2 + 10x + 21$ is equivalent to

A. $(x + 5)^2 + 21$

B. $(x - 5)^2 - 4$

C. $(x + 10)^2 - 79$

D. $(x + 5)^2 - 4$

PART II — Short Constructed Response (8 questions × 2 credits = 16 credits)

Show all work. A correct answer with no supporting work will receive only 1 credit.

25. Solve the equation $x^2 + 8x + 20 = 0$ algebraically. Express your solutions in $a + bi$ form.

26. Solve algebraically for x: $x / (x - 3) = 3 / (x - 3) + 2$. State any restrictions on the variable and identify any extraneous solutions that must be rejected.

27. Given $f(x) = x^3 - 2x^2 - 5x + 6$, determine whether $(x - 3)$ is a factor of $f(x)$. Justify your answer using the Remainder Theorem or the Factor Theorem.

28. Write an equation, in the form $g(x) = a \cdot f(x - h) + k$, for the function obtained by reflecting $f(x) = x^2$ across the x-axis and then translating the result 2 units to the right and 6 units up.

29. Solve algebraically for x: $9^x = 27$.

30. Express $5 / (x + 2) - 3 / (x - 1)$ as a single rational expression in simplest form. State any restrictions on the variable.

31. A sequence is defined recursively by $a_1 = 3$ and $a_n = 2a_{n-1} - 1$ for $n \geq 2$. Find the value of a_4.

32. Given that $\cos \theta = 12/13$ and θ terminates in Quadrant IV, find the exact value of $\sin \theta$. Show the algebraic work that justifies your answer.

PART III — Extended Constructed Response (3 questions × 4 credits = 12 credits)

Show all work. Partial credit is awarded according to the scoring rubric.

33. The population of a town is modeled by the function $P(t) = 25000(0.97)^t$, where P(t) is the population and t is the time in years.

(a) State the initial population of the town and the annual percent rate of decline.

(b) Algebraically determine the number of years it will take for the population to fall to 20000. Round your answer to the nearest tenth of a year.

34. The table below shows a company's monthly revenue R(x), in thousands of dollars, as a function of the number of advertisements x placed per month.

x (ads)	0	1	2	3	4

R(x) (thousands)	−5	9	19	25	27

(a) Using regression, write a quadratic function $R(x) = ax^2 + bx + c$ that best models the data. Round each coefficient to the nearest tenth.

(b) Use the regression model from part (a) to determine the maximum revenue and the number of advertisements at which the maximum revenue occurs. Round each answer to the nearest tenth.

35. A circle and a line are graphed below. The circle is defined by $x^2 + y^2 = 24$, and the line is defined by $y = -x + 4$.

Figure PQ-5

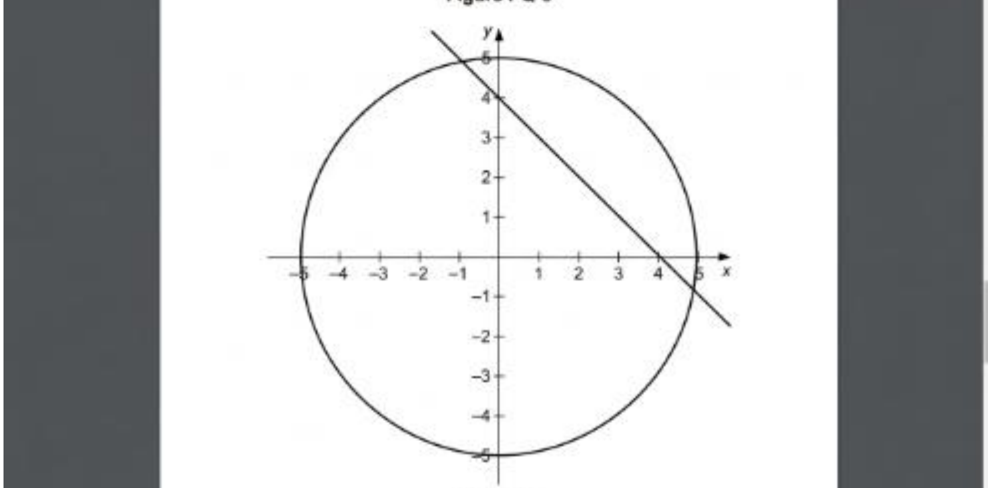


Figure PQ-5

(a) Algebraically determine all points of intersection between the circle and the line. Express coordinates in exact form (radicals permitted).

(b) Verify your answer by substituting one of the intersection points back into both original equations. Show the substitution clearly.

PART IV — Long Constructed Response (1 question × 6 credits = 6 credits)

Show all work. This problem requires multiple steps and integrates concepts from several chapters.

36. A mass attached to a spring hangs from a fixed beam and oscillates vertically. When set in motion, the mass moves between a highest point of 35 cm below the beam and a lowest point of 65 cm below the beam, completing one full oscillation every 2 seconds. At time $t = 0$, the mass is at its lowest point, 65 cm below the beam.

(a) Write a function $d(t)$ that models the distance of the mass in centimeters below the beam as a function of time t in seconds.

(b) State the amplitude, period, and midline of the function $d(t)$, and explain what each represents in the context of the oscillating mass.

(c) Algebraically determine all times during the first 4 seconds at which the mass is 55 cm below the beam. Round each answer to the nearest hundredth of a second. Show all algebraic work.

ANSWER KEY WITH EXPLANATIONS – PRACTICE EXAM 35

1. A — Square the coefficient and the imaginary unit: $(3i)^2 = 9i^2 = 9(-1) = -9$. Since $i^2 = -1$, the result is a negative real number. The imaginary part disappears upon squaring.

2. D — Raise both sides to the 2/3 power: $x = 27^{(2/3)} = (\sqrt[3]{27})^2 = 3^2 = 9$. The reciprocal exponent 2/3 undoes the 3/2 power. Taking the cube root first keeps the numbers small.

3. B — Quadratic formula: $x = [-4 \pm \sqrt{(16 - 32)}]/2 = [-4 \pm \sqrt{(-16)}]/2 = [-4 \pm 4i]/2 = -2 \pm 2i$. The negative discriminant produces the imaginary part. Dividing both terms by 2 gives the simplified form.

4. C — Real zeros are the x-values where the graph crosses the x-axis. The cubic crosses at three points, so the function has 3 real zeros. Counting the x-intercepts gives the number of real roots.

5. A — Factor out the GCF $3x$, then factor the difference of squares: $3x^3 - 12x = 3x(x^2 - 4) = 3x(x - 2)(x + 2)$. Complete factoring requires breaking down $x^2 - 4$. Stopping at $3x(x^2 - 4)$ leaves it not fully factored.

6. B — Rewrite in exponential form: $x = 6^2 = 36$. Converting the logarithmic equation isolates x. The base 6 raised to 2 equals 36.

7. D — The sequence is geometric with first term 3 and common ratio 2, so $a_n = 3 \cdot 2^{(n-1)}$. The exponent $n - 1$ ensures the first term is 3 when $n = 1$. Each term doubles the previous one.

8. C — By the Remainder Theorem, the remainder is $p(-2) = 2(-8) + 3(4) - (-2) + 4 = -16 + 12 + 2 + 4 = 2$. Substituting the zero of the divisor avoids long division. The remainder is 2.

9. C — To invert, solve $y = x^3 + 1$ for x: $x^3 = y - 1$, so $x = \sqrt[3]{y - 1}$ and $f^{-1}(x) = \sqrt[3]{x - 1}$. The inverse undoes cubing and adding 1 in reverse order. The cube root applies to the entire quantity $x - 1$.

10. A — Cube both sides: $x - 1 = 2^3 = 8$, so $x = 9$. Cubing removes the cube root cleanly. The single solution requires no extraneous-root check.

11. D — Amplitude is half the distance between the maximum and minimum: $(5 - (-1))/2 = 6/2 = 3$. It measures the vertical distance from the midline at $y = 2$ to a peak. The vertical shift does not affect the amplitude.

12. B — Substitute $t = 18$: $P(18) = 100 \cdot 2^{(18/6)} = 100 \cdot 2^3 = 100 \cdot 8 = 800$. The exponent $18/6 = 3$ represents three doubling periods. The population reaches 800 after 18 hours.

13. A — Average rate of change is $[f(3) - f(-1)]/(3 - (-1))$. With $f(3) = 9 - 6 = 3$ and $f(-1) = 1 + 2 = 3$, this is $(3 - 3)/4 = 0$. The equal endpoint values produce a rate of change of zero.

14. C — The discriminant is $b^2 - 4ac = 4 - 20 = -16$, which is negative. A negative discriminant produces two complex (non-real) conjugate solutions. The sign of the discriminant classifies the roots.

15. B — The angle $3\pi/4$ lies in Quadrant II with reference angle $\pi/4$, where cosine is negative. Since $\cos(\pi/4) = \sqrt{2}/2$, $\cos(3\pi/4) = -\sqrt{2}/2$. The quadrant determines the negative sign.

16. D — The product and power rules give $\log(xy^2) = \log x + \log y^2 = \log x + 2 \log y$. The product becomes a sum and the exponent becomes a coefficient. Both logarithm properties apply in sequence.

17. A — The period of $\cos(bx)$ is $2\pi/b$; with $b = 1/2$, the period is $2\pi/(1/2) = 4\pi$. The fractional coefficient stretches the graph horizontally. The amplitude of 3 does not affect the period.

18. C — Side x is opposite the 30° angle, so $\sin 30° = x/10$, giving $x = 10(1/2) = 5$. The sine ratio relates the opposite side to the hypotenuse. Multiplying gives a length of 5.

19. D — The sum of the first n integers is $n(n + 1)/2 = 50(51)/2 = 1275$. This formula sums the arithmetic series from 1 to 50. The product 50 times 51 divided by 2 gives 1275.

20. B — Order does not matter, so use combinations: $C(6, 2) = 6!/(2!4!) = 15$. Selecting books is a combination, not a permutation. There are 15 distinct pairs.

21. C — Each point maps from (x, y) to (−x, y), which negates the x-coordinate while keeping y the same. That transformation is a reflection across the y-axis. The mirror symmetry about the vertical axis confirms it.

22. A — The values 54 and 66 are exactly one standard deviation below and above the mean of 60. By the empirical rule, about 68% of data falls within one standard deviation. This interval captures the central 68%.

23. B — In Quadrant IV cosine is positive while tangent is negative, and the 8-15-17 triangle gives the cosine ratio. Therefore $\cos \theta = 15/17$. The quadrant fixes the positive sign on cosine.

24. D — Complete the square: $x^2 + 10x + 21 = (x^2 + 10x + 25) - 25 + 21 = (x + 5)^2 - 4$. Half of 10 squared is 25, added and subtracted to preserve value. The vertex form reveals the vertex at (−5, −4).

Part II (Short Constructed Response)

25. $x = -4 \pm 2i$ — Quadratic formula: $x = [-8 \pm \sqrt{(64 - 80)}]/2 = [-8 \pm \sqrt{(-16)}]/2 = [-8 \pm 4i]/2 = -4 \pm 2i$. The negative discriminant produces complex conjugates. Dividing each term by 2 gives the simplified a + bi form.

26. No solution; x = 3 is rejected — Multiplying through by (x − 3) gives x = 3 + 2(x − 3), which simplifies to x = 2x − 3, so x = 3. Since x = 3 makes the original denominator zero, it is extraneous and rejected, leaving the equation with no valid solution.

27. Yes, (x − 3) is a factor — By the Factor Theorem, f(3) = 27 − 18 − 15 + 6 = 0. A remainder of zero confirms that (x − 3) divides f(x) evenly. A nonzero value would have meant it is not a factor.

28. $g(x) = -(x - 2)^2 + 6$ — Reflecting across the x-axis negates the function to $-x^2$; shifting 2 units right replaces x with (x − 2); shifting 6 units up adds 6. Combining these produces $-(x - 2)^2 + 6$. The horizontal shift moves opposite the sign inside the parentheses.

29. x = 3/2 — Write both sides with base 3: $9^x = 3^{(2x)}$ and $27 = 3^3$, so 2x = 3. Solving gives x = 3/2. Equal bases allow the exponents to be set equal.

30. (2x − 11) / [(x + 2)(x − 1)], x ≠ −2, x ≠ 1 — Over the common denominator (x + 2)(x − 1): 5(x − 1) − 3(x + 2) = 5x − 5 − 3x − 6 = 2x − 11. The numerator combines to 2x − 11. Restrictions come from the original denominators.

31. $a_4 = 17$ — Apply the recursion: $a_2 = 2(3) - 1 = 5$, $a_3 = 2(5) - 1 = 9$, $a_4 = 2(9) - 1 = 17$. Each term doubles the previous term and subtracts 1. Building up term by term reaches 17.

32. $\sin \theta = -5/13$ — Using $\sin^2\theta + \cos^2\theta = 1$: $\sin^2\theta = 1 - 144/169 = 25/169$, so $\sin \theta = \pm 5/13$. In Quadrant IV sine is negative, giving −5/13. The quadrant fixes the negative sign.

Part III (Extended Constructed Response)

33. (a) 25,000, 3% decline; (b) ≈ 7.3 years — The coefficient 25000 is the initial population, and the base $0.97 = 1 - 0.03$ indicates a 3% annual decline. For part (b), $20000 = 25000(0.97)^t$ gives $0.97^t = 0.8$, so $t = \ln(0.8)/\ln(0.97) \approx 7.3$ years. Logarithms isolate the exponent.

34. (a) $R(x) = -2.0x^2 + 16.0x - 5.0$; (b) \$27 thousand at $x = 4$ ads — Quadratic regression on the data returns $a = -2.0$, $b = 16.0$, $c = -5.0$. The vertex occurs at $x = -b/(2a) = -16/(-4) = 4$ ads, and $R(4) = -32 + 64 - 5 = 27$ thousand dollars. The negative leading coefficient confirms a maximum.

35. (a) $(2 + 2\sqrt{2}, 2 - 2\sqrt{2})$ and $(2 - 2\sqrt{2}, 2 + 2\sqrt{2})$ — Substituting $y = -x + 4$ into the circle gives $x^2 + (-x + 4)^2 = 24$, which simplifies to $2x^2 - 8x - 8 = 0$, then $x^2 - 4x - 4 = 0$, so $x = 2 \pm 2\sqrt{2}$. The y-values follow from $y = -x + 4$. **(b)** Checking $(2 + 2\sqrt{2}, 2 - 2\sqrt{2})$: $x^2 + y^2 = (12 + 8\sqrt{2}) + (12 - 8\sqrt{2}) = 24$, and $y = -x + 4 = -(2 + 2\sqrt{2}) + 4 = 2 - 2\sqrt{2}$, so both equations hold.

Part IV (Long Constructed Response)

36. (a) $d(t) = 15\cos(\pi t) + 50$ — The amplitude is $(65 - 35)/2 = 15$ and the midline is $(65 + 35)/2 = 50$. The period of 2 seconds gives $b = 2\pi/2 = \pi$, and starting at the lowest point (maximum distance) at $t = 0$ calls for a positive cosine. This yields $d(t) = 15\cos(\pi t) + 50$.

(b) Amplitude 15, period 2 s, midline $d = 50$ — The amplitude of 15 cm is the distance from the rest position to the highest or lowest point of the motion. The period of 2 seconds is the time for one complete oscillation. The midline $d = 50$ cm is the rest position, the distance below the beam about which the mass oscillates.

(c) $t \approx 0.39$, 1.61, 2.39, and 3.61 seconds — Setting $55 = 15\cos(\pi t) + 50$ gives $\cos(\pi t) = 1/3$. Over the first 4 seconds, $\pi t = 1.2310$, 5.0522, 7.5142, and 11.3354 radians, so $t = 0.39$, 1.61, 2.39, and 3.61 seconds. Each pair of angles corresponds to the mass passing through the 55 cm position while descending and rising.