PRACTICE EXAM 34: ALGEBRA II REGENTS SIMULATION

PART I — Multiple Choice (24 questions × 2 credits = 48 credits)

1. Express 1 / (2 − i) in a + bi form.

A. 2 − i

B. 2/5 + (1/5)i

C. 2/5 − (1/5)i

D. 1/2 + i

2. Simplify the expression $\sqrt[5]{(x^2)} \cdot \sqrt[5]{(x^3)}$.

A. x

B. x^(6/25)

C. x^(1/5)

D. x^5

3. What are the solutions to the equation $x^2 - 2x + 5 = 0$?

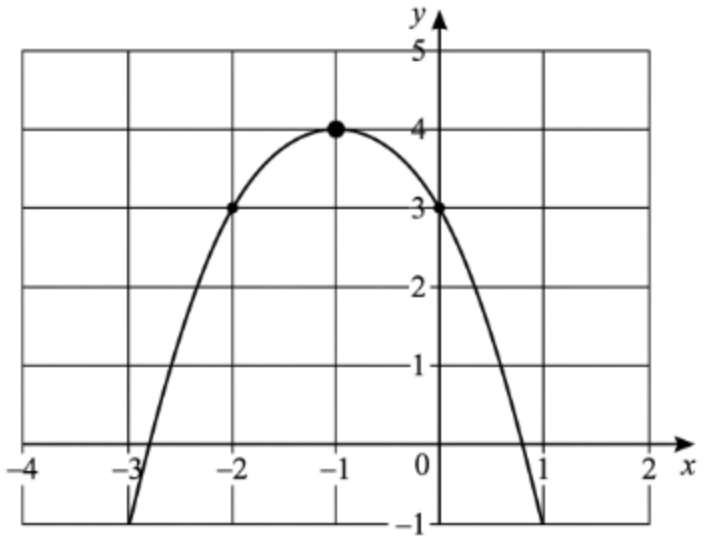
A. x = −1 ± 2i

B. $x = 2 \pm 4i$

C. $x = 1 \pm 4i$

D. $x = 1 \pm 2i$

4. The graph of a quadratic function is shown below. What are the coordinates of its vertex?



A. (1, 4)

B. (−1, −4)

C. (−1, 4)

D. (4, −1)

5. Factored completely, the expression $x^4 - 81$ is equivalent to

A. $(x^2 - 9)(x^2 + 9)$

B. $(x - 3)^4$

C. $(x - 3)(x + 3)(x^2 + 9)$

D. $(x - 3)^2(x + 3)^2$

6. What is the value of $\log_3(1/9)$?

A. -2

B. 2

C. $-1/2$

D. 3

7. What is the geometric mean of 4 and 16?

A. 10

B. 8

C. 6

D. 12

8. Which binomial is a factor of $f(x) = x^3 + 2x^2 - 5x - 6$?

A. $(x + 2)$

B. $(x - 1)$

C. $(x - 3)$

D. $(x + 3)$

9. If $f(x) = \sqrt{x}$ and $g(x) = x + 7$, what is the value of $f(g(9))$?

A. 4

B. 16

C. 5

D. 2

10. What is the solution set of the equation $\sqrt{(x + 4)} = x - 2$?

A. {0, 5}

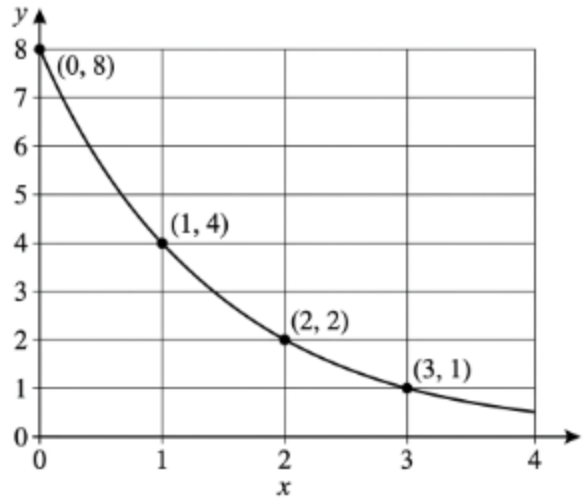
B. {5}

C. {0}

D. { }

11. Using the graph of the exponential function f(x) shown below, what is the value of f(2)?

[Figure PQ-2: Decreasing Exponential Curve]



A. 8

B. 4

C. 2

D. 1

12. What is the solution to the equation $5^x = 40$, rounded to the nearest hundredth?

A. x = 8

B. x = 1.60

C. x = 3.69

D. x = 2.29

13. The table below shows values of the function f(x).

| x | 0 | 1 | 2 | 3 | 4 |

|---|---|---|---|---|---|

| f(x) | 2 | 5 | 10 | 17 | 26 |

What is the average rate of change of f(x) over the interval [1, 4]?

A. 7

B. 21

C. 8

D. 5

14. Which statement best describes the nature of the solutions of $3x^2 - 5x + 1 = 0$?

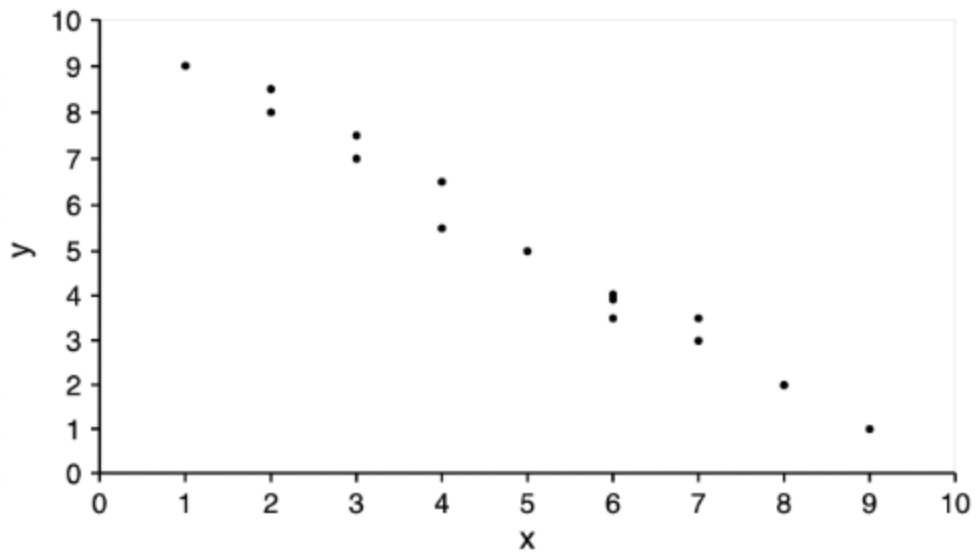
A. Two complex (non-real) solutions

B. Two distinct rational solutions

C. Two distinct irrational solutions

D. One repeated rational solution

15. The scatter plot below shows the relationship between two variables. Which best describes the correlation?



A. Strong positive correlation

B. Strong negative correlation

C. No correlation

D. Weak positive correlation

16. What is the exact value of $\tan(5\pi/6)$?

A. $\sqrt{3}$

B. $-\sqrt{3}$

C. $\sqrt{3}/3$

D. $-\sqrt{3}/3$

17. What is the solution to the equation $\log_2(x) - \log_2(3) = 4$?

A. $x = 16$

B. $x = 24$

C. $x = 48$

D. $x = 8$

18. For the function $y = 5 \sin(4x)$, what is the period?

A. $\pi/2$

B. π

C. 4

D. 2π

19. In an arithmetic sequence, the third term is 11 and the seventh term is 27. What is the first term?

A. 7

B. 5

C. 1

D. 3

20. For two events, $P(A) = 0.6$, $P(B) = 0.5$, and $P(A \text{ and } B) = 0.3$. Which statement correctly describes events A and B?

A. Not independent, because $P(A) \neq P(B)$

B. Independent, because $P(A) \cdot P(B) = P(A \text{ and } B)$

C. Not independent, because $P(A \text{ and } B) \neq 0$

D. Independent, because P(A) + P(B) = 1.1

21. A data set is normally distributed with a mean of 500 and a standard deviation of 100. Which value is closest to the 97.5th percentile?

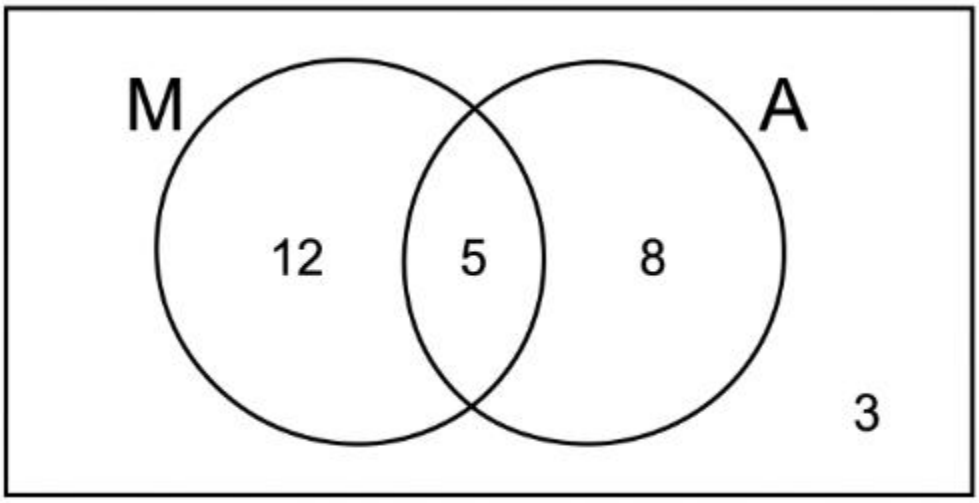
A. 700

B. 600

C. 800

D. 650

22. The Venn diagram below shows the number of students enrolled in Music (M) and Art (A). How many students are enrolled in Music only?



[Figure PQ-4: Clean black-line technical Venn diagram on a white background.]

A. 17

B. 5

C. 12

D. 8

23. Given that $\cos \theta = -4/5$ and θ is in Quadrant II, what is the value of $\sin \theta$?

A. $-3/5$

B. $4/3$

C. $-4/3$

D. $3/5$

24. Written in vertex form, the expression $2x^2 + 8x + 5$ is equivalent to

A. $2(x + 2)^2 + 5$

B. $2(x + 2)^2 - 3$

C. $2(x - 2)^2 - 3$

D. $(x + 2)^2 - 3$

PART II — Short Constructed Response (8 questions × 2 credits = 16 credits)

Show all work. A correct answer with no supporting work will receive only 1 credit.

25. Solve the equation $x^2 - 2x + 26 = 0$ algebraically. Express your solutions in a + bi form.

26. Solve algebraically for x: $(x + 4) / (x - 2) = (x + 1) / (x - 3)$. State any restrictions on the variable and identify any extraneous solutions that must be rejected.

27. Given $f(x) = x^3 + x^2 - 4x - 4$, determine whether $(x + 2)$ is a factor of $f(x)$. Justify your answer using the Remainder Theorem or the Factor Theorem.

28. Write an equation, in the form $g(x) = a \cdot f(x - h) + k$, for the function obtained by vertically stretching $f(x) = \sqrt{x}$ by a factor of 3 and then translating the result 1 unit to the right.

29. Solve algebraically for x: $8^x = 4^(x + 1)$.

30. Express $2x / (x^2 - 9) + 3 / (x + 3)$ as a single rational expression in simplest form. State any restrictions on the variable.

31. A sequence is defined recursively by $a_1 = 2$, $a_2 = 3$, and $a_n = a_{n-1} + a_{n-2}$ for $n \geq 3$. Find the value of a_5.

32. Given that $\sin \theta = 4/5$ and θ terminates in Quadrant II, find the exact value of $\tan \theta$. Show the algebraic work that justifies your answer.

PART III — Extended Constructed Response (3 questions × 4 credits = 12 credits)

Show all work. Partial credit is awarded according to the scoring rubric.

33. The value of an investment is modeled by the function $A(t) = 2000(1.06)^t$, where $A(t)$ is the value in dollars and t is the time in years.

(a) State the initial value of the investment and the annual percent rate of growth.

(b) Algebraically determine the number of years it will take for the investment to double in value. Round your answer to the nearest tenth of a year.

34. The table below shows the height h(x), in meters, of a stream of water from a fountain at horizontal distance x meters from the nozzle.

| x (meters) | 0 | 1 | 2 | 3 | 4 |

|---|---|---|---|---|---|

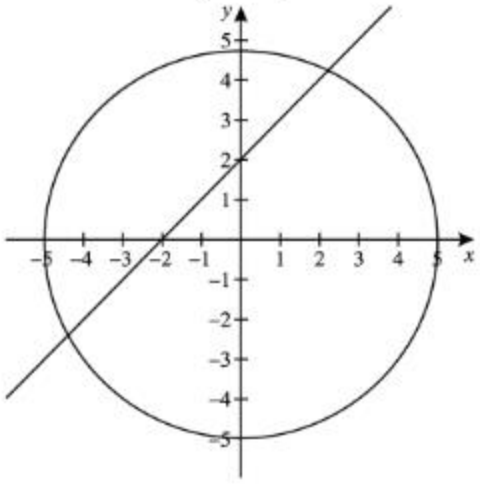
| h(x) (meters) | 2 | 7 | 10 | 11 | 10 |

(a) Using regression, write a quadratic function $h(x) = ax^2 + bx + c$ that best models the data. Round each coefficient to the nearest tenth.

(b) Use the regression model from part (a) to determine the maximum height of the water stream and the horizontal distance at which the maximum height occurs. Round each answer to the nearest tenth.

35. A circle and a line are graphed below. The circle is defined by $x^2 + y^2 = 18$, and the line is defined by $y = x + 2$.

Figure PQ-5



(a) Algebraically determine all points of intersection between the circle and the line. Express coordinates in exact form (radicals permitted).

(b) Verify your answer by substituting one of the intersection points back into both original equations. Show the substitution clearly.

PART IV — Long Constructed Response (1 question × 6 credits = 6 credits)

Show all work. This problem requires multiple steps and integrates concepts from several chapters.

36. A water wheel on a mill has a radius of 5 meters, and its center axle is 2 meters above the surface of the water. The wheel completes one full rotation every 8 seconds. A marked bucket on the rim of the wheel begins at the highest point of its rotation at time $t = 0$.

(a) Write a function $h(t)$ that models the height of the marked bucket in meters relative to the water surface as a function of time t in seconds.

(b) State the amplitude, period, and midline of the function h(t), and explain what each represents in the context of the water wheel.

(c) Algebraically determine all times during the first 8 seconds at which the marked bucket is exactly at the water surface (height 0 meters). Round each answer to the nearest hundredth of a second. Show all algebraic work.

ANSWER KEY WITH EXPLANATIONS — PRACTICE EXAM 34

1. B — Multiply numerator and denominator by the conjugate 2 + i: (2 + i)/[(2 − i)(2 + i)] = (2 + i)/(4 + 1) = (2 + i)/5 = 2/5 + (1/5)i. The conjugate clears the imaginary part from the denominator. Splitting into real and imaginary parts gives the a + bi form.

2. A — Multiplying like bases adds exponents: x^(2/5) · x^(3/5) = x^(5/5) = x. The fractional exponents sum to 1, leaving x to the first power. The fifth roots combine into a single whole power.

3. D — Quadratic formula: x = [2 ± √(4 − 20)]/2 = [2 ± √(−16)]/2 = [2 ± 4i]/2 = 1 ± 2i. The negative discriminant produces the imaginary part. Dividing both terms by 2 gives the simplified form.

4. C — The vertex is the turning point of the parabola, shown at (−1, 4) on the graph. For a downward-opening parabola, this point is the maximum. Reading the coordinates directly gives the vertex.

5. C — Factor as a difference of squares twice: $x^4 - 81 = (x^2 - 9)(x^2 + 9) = (x - 3)(x + 3)(x^2 + 9)$. The factor $x^2 + 9$ is a sum of squares and does not factor over the reals. Complete factoring requires breaking down $x^2 - 9$.

6. A — Rewrite the argument as a power of 3: $1/9 = 3^{-2}$, so $\log_3(1/9) = -2$. The negative exponent reflects the reciprocal. The base 3 raised to −2 equals 1/9.

7. B — The geometric mean of two numbers is the square root of their product: √(4 · 16) = √64 = 8. This is the middle term of a geometric sequence between 4 and 16. The product under the radical gives 64.

8. D — By the Factor Theorem, f(−3) = −27 + 18 + 15 − 6 = 0, so (x + 3) is a factor. The other binomials produce nonzero remainders when their roots are tested. A zero result confirms the factor.

9. A — Evaluate the inner function first: g(9) = 9 + 7 = 16, then f(16) = √16 = 4. Composition works from the inside out. Taking the square root of 16 gives 4.

10. B — Squaring gives $x + 4 = x^2 - 4x + 4$, so $x^2 - 5x = 0$ and x(x − 5) = 0. The candidate x = 0 fails because $\sqrt{4} = 2 \neq -2$, leaving x = 5 as the only valid solution. The check removes the extraneous root.

11. C — The graph passes through the point (2, 2), so f(2) = 2. Reading the y-value at x = 2 gives the function value. The decay curve confirms the output at that input.

12. D — Take the log of both sides: $x = \log_5(40) = \ln(40)/\ln(5) \approx 2.29$. The change-of-base formula converts to natural logs. The result rounds to 2.29.

13. A — Average rate of change is $[f(4) - f(1)]/(4 - 1) = (26 - 5)/3 = 21/3 = 7$. This is the slope of the secant line over the interval. The table values 26 and 5 drive the result.

14. C — The discriminant is $b^2 - 4ac = 25 - 12 = 13$, which is positive but not a perfect square. This produces two distinct irrational real solutions. The nature of the discriminant classifies the roots.

15. B — The points descend steadily from upper-left to lower-right, indicating that as x increases, y decreases. A tight downward pattern signals a strong negative correlation. The clear linear trend rules out weak or no correlation.

16. D — The angle $5\pi/6$ lies in Quadrant II with reference angle $\pi/6$, where tangent is negative. Since $\tan(\pi/6) = \sqrt{3}/3$, $\tan(5\pi/6) = -\sqrt{3}/3$. The quadrant determines the negative sign.

17. C — Apply the quotient rule: $\log_2(x/3) = 4$, so $x/3 = 2^4 = 16$ and $x = 48$. Combining the logarithms into one allows conversion to exponential form. Multiplying by 3 gives 48.

18. A — The period of $\sin(bx)$ is $2\pi/b$; with $b = 4$, the period is $2\pi/4 = \pi/2$. The coefficient of x compresses the graph horizontally. The amplitude of 5 does not affect the period.

19. D — The common difference is $d = (27 - 11)/(7 - 3) = 16/4 = 4$. Working back from the third term, $a_1 = 11 - 2(4) = 3$. Subtracting two common differences recovers the first term.

20. B — Two events are independent when $P(A) \cdot P(B) = P(A \text{ and } B)$. Here $0.6 \cdot 0.5 = 0.3$, which equals the given P(A and B), confirming independence. The multiplication rule is the defining test.

21. A — The 97.5th percentile of a normal distribution corresponds to approximately two standard deviations above the mean. That value is $500 + 2(100) = 700$. The empirical rule places 95% within ±2 SD, leaving 2.5% above +2 SD.

22. C — The region inside circle M but outside the overlap represents Music only, which contains 12 students. The overlapping region of 5 belongs to both subjects, not Music only. Reading the non-overlapping portion gives the answer.

23. D — In Quadrant II sine is positive while cosine is negative, and the 3-4-5 triangle gives the sine ratio. Therefore $\sin\theta = 3/5$. The quadrant fixes the positive sign on sine.

24. B — Factor out 2 and complete the square: $2(x^2 + 4x) + 5 = 2(x^2 + 4x + 4) - 8 + 5 = 2(x + 2)^2 - 3$. The subtracted 8 accounts for the factored-out 2 times the added 4. The vertex form reveals the vertex at $(-2, -3)$.

Part II (Short Constructed Response)

25. x = 1 ± 5i — Quadratic formula: $x = [2 \pm \sqrt{(4 - 104)}]/2 = [2 \pm \sqrt{(-100)}]/2 = [2 \pm 10i]/2 = 1 \pm 5i$. The negative discriminant produces complex conjugates. Dividing each term by 2 gives the simplified a + bi form.

26. x = 5; no extraneous solutions — Cross-multiplying gives $(x + 4)(x - 3) = (x + 1)(x - 2)$, which expands to $x^2 + x - 12 = x^2 - x - 2$ and simplifies to $2x = 10$, so $x = 5$. The restrictions are $x \neq 2$ and $x \neq 3$, and $x = 5$ violates neither, so it is valid.

27. Yes, (x + 2) is a factor — By the Factor Theorem, $f(-2) = -8 + 4 + 8 - 4 = 0$. A remainder of zero confirms that $(x + 2)$ divides $f(x)$ evenly. A nonzero value would have meant it is not a factor.

28. g(x) = 3√(x − 1) — A vertical stretch by 3 multiplies the function by 3, giving $3\sqrt{x}$; shifting 1 unit right replaces x with $(x - 1)$. Combining these produces $3\sqrt{(x - 1)}$. The horizontal shift moves opposite the sign inside the radical.

29. x = 2 — Write both sides with base 2: $8^x = 2^{(3x)}$ and $4^{(x + 1)} = 2^{(2x + 2)}$, so $3x = 2x + 2$. Solving gives $x = 2$. Equal bases allow the exponents to be set equal.

30. (5x − 9) / [(x − 3)(x + 3)], x ≠ 3, x ≠ −3 — Over the common denominator $(x - 3)(x + 3)$: $2x/[(x - 3)(x + 3)] + 3(x - 3)/[(x - 3)(x + 3)] = [2x + 3(x - 3)]/[(x - 3)(x + 3)] = (5x - 9)/[(x - 3)(x + 3)]$. The numerator combines to $5x - 9$.

31. $a_5 = 13$ — Apply the recursion: $a_3 = 3 + 2 = 5$, $a_4 = 5 + 3 = 8$, $a_5 = 8 + 5 = 13$. Each term is the sum of the two preceding terms. Building up term by term reaches 13.

32. tan θ = −4/3 — With $\sin \theta = 4/5$ in Quadrant II, the 3-4-5 triangle gives $\cos \theta = -3/5$ because cosine is negative there. Then $\tan \theta = \sin/\cos = (4/5)/(-3/5) = -4/3$. The quadrant makes the tangent negative.

Part III (Extended Constructed Response)

33. (a) $2000, 6% growth; (b) ≈ 11.9 years — The coefficient 2000 is the initial value, and the base $1.06 = 1 + 0.06$ indicates 6% annual growth. For part (b), $4000 = 2000(1.06)^t$ gives $2 = 1.06^t$, so $t = \ln(2)/\ln(1.06) \approx 11.9$ years. Logarithms isolate the exponent.

34. (a) $h(x) = -1.0x^2 + 6.0x + 2.0$; (b) 11.0 m at x = 3.0 m — Quadratic regression on the data returns $a = -1.0$, $b = 6.0$, $c = 2.0$. The vertex occurs at $x = -b/(2a) = -6/(-2) = 3.0$ meters, and $h(3) = -9 + 18 + 2 = 11.0$ meters. The negative leading coefficient confirms a maximum.

35. (a) (−1 + 2√2, 1 + 2√2) and (−1 − 2√2, 1 − 2√2) — Substituting $y = x + 2$ into the circle gives $x^2 + (x + 2)^2 = 18$, which simplifies to $2x^2 + 4x - 14 = 0$, then $x^2 + 2x - 7 = 0$, so $x = -1 \pm 2\sqrt{2}$. The y-values follow from $y = x + 2$. **(b)** Checking $(-1 + 2\sqrt{2}, 1 + 2\sqrt{2})$: $x^2 + y^2 = (9 - 4\sqrt{2}) + (9 + 4\sqrt{2}) = 18$, and $y = x + 2 = (-1 + 2\sqrt{2}) + 2 = 1 + 2\sqrt{2}$, so both equations hold.

Part IV (Long Constructed Response)

36. (a) $h(t) = 5\cos(\pi t/4) + 2$ — The amplitude is the 5-meter radius and the midline is the 2-meter axle height. The period of 8 seconds gives $b = 2\pi/8 = \pi/4$, and starting at the highest point at $t = 0$ calls for a positive cosine. This yields $h(t) = 5\cos(\pi t/4) + 2$.

(b) Amplitude 5, period 8 s, midline $h = 2$ — The amplitude of 5 meters equals the wheel's radius, the distance from the axle to the rim. The period of 8 seconds is the time for one full rotation. The midline $h = 2$ meters is the axle height above the water, the center about which the bucket oscillates.

(c) $t \approx 2.52$ seconds and $t \approx 5.48$ seconds — Setting $0 = 5\cos(\pi t/4) + 2$ gives $\cos(\pi t/4) = -0.4$. Over the first 8 seconds, $\pi t/4 = 1.9823$ and 4.3009 radians, so $t = 2.52$ and 5.48 seconds. These are the two times the descending and rising bucket passes through the water surface.