PRACTICE EXAM 33: ALGEBRA II REGENTS SIMULATION

PART I — Multiple Choice (24 questions × 2 credits = 48 credits)

1. Express $(2 + i)^2$ in a + bi form.

A. 4 + i

B. 5 + 4i

C. 3 + 4i

D. 3 − 4i

2. Simplify the expression $(x^6)^{(1/2)}$.

A. $x^{(1/3)}$

B. x^8

C. x^{12}

D. x^3

3. What are the solutions to the equation $x^2 + 9 = 0$?

A. $x = \pm 3i$

B. $x = \pm 9i$

C. $x = \pm 3$

D. $x = \pm\sqrt{3}\, i$

4. Factored completely, the expression $8x^3 + 1$ is equivalent to

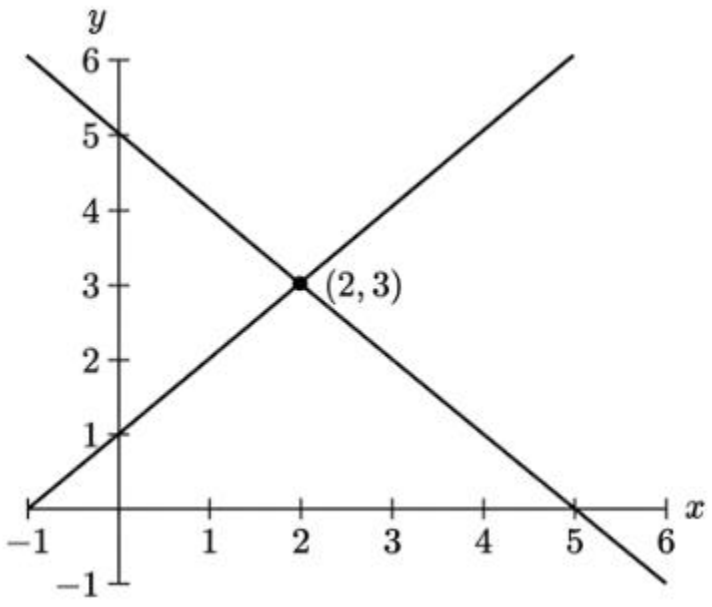
A. $(2x + 1)(4x^2 + 2x + 1)$

B. $(2x + 1)(4x^2 - 2x + 1)$

C. $(2x - 1)(4x^2 + 2x + 1)$

D. $(2x + 1)(2x^2 - 1)$

5. The graph below shows a system of two linear equations. What is the solution to the system?



A. (3, 2)

B. (1, 2)

C. (0, 5)

D. (2, 3)

6. What is the value of $\log_5(125)$?

A. 3

B. 25

C. 5

D. 15

7. What is the sum of the infinite geometric series $16 + 8 + 4 + 2 + \ldots$?

A. 16

B. 24

C. 32

D. The series does not converge

8. When $p(x) = x^3 - 2x + 5$ is divided by $(x - 1)$, the remainder is

A. 5

B. 4

C. −2

D. 8

9. If $f(x) = 3x$ and $g(x) = x^2 - 1$, what is the value of $g(f(2))$?

A. 35

B. 11

C. 17

D. 23

10. What is the solution to the equation $\sqrt{(2x - 1)} = 3$?

A. $x = 3$

B. $x = 4$

C. $x = 5$

D. $x = 10$

11. What is the average rate of change of $f(x) = 3^x$ over the interval [1, 3]?

A. 3

B. 9

C. 24

D. 12

12. Which function models a quantity that begins at 50 and triples each year?

A. $f(x) = 50(0.3)^x$

B. $f(x) = 50(3)^x$

C. $f(x) = 3(50)^x$

D. $f(x) = 50 + 3x$

13. Which equation best represents the absolute value function graphed below?

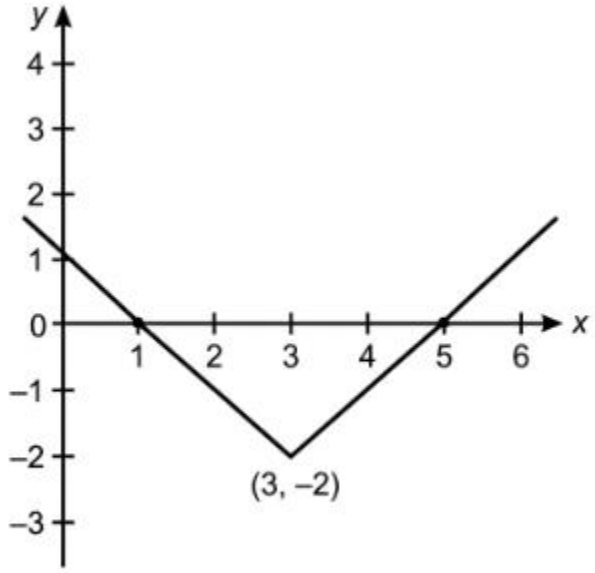


Figure PQ-2

A. $y = |x + 3| - 2$

B. $y = |x - 3| + 2$

C. $y = |x - 3| - 2$

D. $y = |x + 3| + 2$

14. Which statement best describes the nature of the solutions of $x^2 - 8x + 16 = 0$?

A. One repeated rational solution

B. Two distinct rational solutions

C. Two distinct irrational solutions

D. Two complex (non-real) solutions

15. What is the exact value of $\sin(7\pi/6)$?

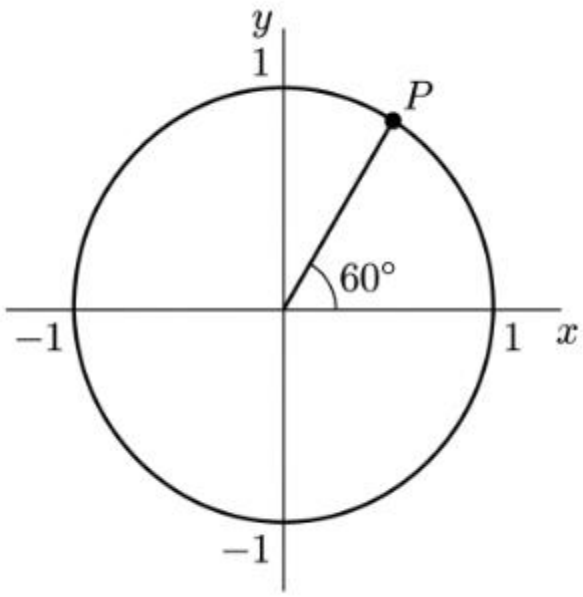
A. 1/2

B. $\sqrt{3}/2$

C. $-\sqrt{3}/2$

D. $-1/2$

16. Point P lies on the unit circle at a terminal angle of 60° as shown below. What are the coordinates of point P?



A. (√3/2, 1/2)

B. (1/2, √3/2)

C. (1/2, −√3/2)

D. (−1/2, √3/2)

17. What is the solution to the equation $\log_4(x) = 2$?

A. x = 16

B. x = 8

C. x = 6

D. x = 2

18. For the function $y = -3\cos(x) + 4$, what is the value of the midline?

A. $y = -3$

B. $y = 3$

C. $y = 7$

D. $y = 4$

19. What is the sum of the first 10 positive even integers?

A. 90

B. 100

C. 110

D. 120

20. A spinner is divided into 8 equal sectors numbered 1 through 8. What is the probability of landing on a prime number?

A. 1/2

B. 3/8

C. 5/8

D. 1/4

21. A data set is normally distributed with a mean of 200 and a standard deviation of 20. Approximately what percent of the data is below 180?

A. 34%

B. 16%

C. 84%

D. 68%

22. A correlation coefficient of $r = -0.92$ between two variables indicates

A. A weak positive linear relationship

B. A strong positive linear relationship

C. A strong negative linear relationship

D. No linear relationship at all

23. Given that $\cos \theta = 7/25$ and θ is in Quadrant IV, what is the value of $\sin \theta$?

A. 24/25

B. 7/24

C. −7/25

D. −24/25

24. Written in vertex form, the expression $x^2 - 8x + 10$ is equivalent to

A. $(x - 4)^2 + 10$

B. $(x - 4)^2 - 6$

C. $(x + 4)^2 - 6$

D. $(x - 8)^2 - 54$

PART II — Short Constructed Response (8 questions × 2 credits = 16 credits)

Show all work. A correct answer with no supporting work will receive only 1 credit.

25. Solve the equation $x^2 - 6x + 10 = 0$ algebraically. Express your solutions in a + bi form.

26. Solve algebraically for x: x / (x + 1) = 3 / (x + 5). State any restrictions on the variable and identify any extraneous solutions that must be rejected.

27. Given $f(x) = x^3 + 4x^2 - 7x + 2$, determine whether $(x - 1)$ is a factor of f(x). Justify your answer using the Remainder Theorem or the Factor Theorem.

28. Write an equation, in the form $g(x) = a \cdot f(x - h) + k$, for the function obtained by vertically compressing $f(x) = x^2$ by a factor of 1/2 and then translating the result 4 units left and 1 unit down.

29. Solve algebraically for x: 3^(x + 2) = 27^(x − 1).

30. Express 3 / (x − 4) + 2 / (x + 1) as a single rational expression in simplest form. State any restrictions on the variable.

31. A sequence is defined recursively by $a_1 = 1$ and $a_n = 3a_{n-1} + 2$ for $n \geq 2$. Find the value of a_4.

32. Given that tan θ = 5/12 and θ terminates in Quadrant III, find the exact value of sin θ. Show the algebraic work that justifies your answer.

PART III — Extended Constructed Response (3 questions × 4 credits = 12 credits)

Show all work. Partial credit is awarded according to the scoring rubric.

33. The amount of a medication in a patient's bloodstream is modeled by the function M(t) = 80(0.5)^(t/4), where M(t) is the amount in milligrams and t is the time in hours.

(a) State the initial amount of medication and the number of hours that represents the half-life of the medication.

(b) Algebraically determine the number of hours it will take for the amount of medication to decrease to 15 milligrams. Round your answer to the nearest tenth of an hour.

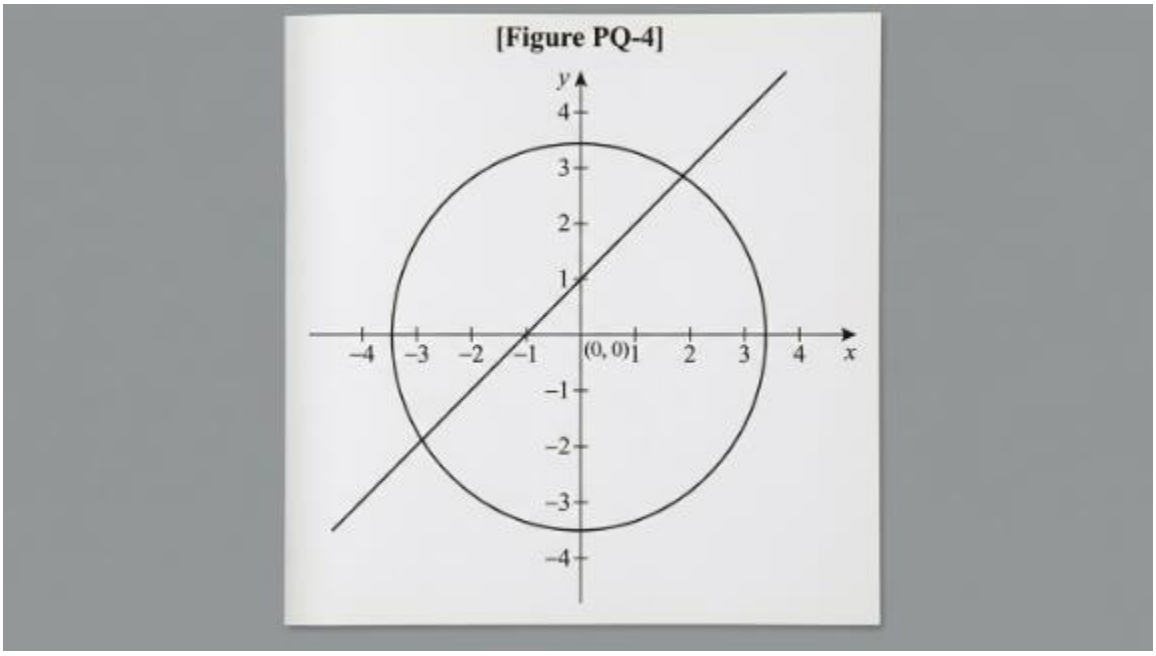
34. The table below shows a company's daily profit P(x), in thousands of dollars, as a function of the selling price x, in dollars, of its product.

x (dollars)	1	3	5	7	9
P(x) (thousands)	10	34	42	34	10

(a) Using regression, write a quadratic function $P(x) = ax^2 + bx + c$ that best models the data. Round each coefficient to the nearest tenth.

(b) Use the regression model from part (a) to determine the maximum profit and the selling price at which the maximum profit occurs. Round each answer to the nearest tenth.

35. A circle and a line are graphed below. The circle is defined by $x^2 + y^2 = 12$, and the line is defined by $y = x + 1$.



(a) Algebraically determine all points of intersection between the circle and the line. Express coordinates in exact form (radicals permitted).

(b) Verify your answer by substituting one of the intersection points back into both original equations. Show the substitution clearly.

PART IV — Long Constructed Response (1 question × 6 credits = 6 credits)

Show all work. This problem requires multiple steps and integrates concepts from several chapters.

36. The temperature inside a climate-controlled greenhouse varies sinusoidally over each 24-hour day. The maximum temperature is 86°F and the minimum temperature is 66°F. At time $t = 0$ (midnight), the temperature is at its minimum.

(a) Write a function T(t) that models the greenhouse temperature in degrees Fahrenheit as a function of time t in hours.

(b) State the amplitude, period, and midline of the function T(t), and explain what each represents in the context of the greenhouse temperature.

(c) Algebraically determine all times during the first 24 hours at which the temperature is 80°F. Round each answer to the nearest hundredth of an hour. Show all algebraic work.

ANSWER KEY WITH EXPLANATIONS – PRACTICE EXAM 33

1. C — Square the binomial and apply $i^2 = -1$: $(2 + i)^2 = 4 + 4i + i^2 = 4 + 4i - 1 = 3 + 4i$. The middle term doubles the product $2 \cdot i$, and the i^2 term reduces the real part. The result is $3 + 4i$.

2. D — A power raised to a power multiplies the exponents: $(x^6)^{(1/2)} = x^{(6 \cdot 1/2)} = x^3$. Multiplying 6 by 1/2 gives 3. This is equivalent to taking the square root of x^6.

3. A — Isolate the square: $x^2 = -9$, so $x = \pm\sqrt{(-9)} = \pm 3i$. The negative under the radical introduces the imaginary unit i. Both roots are pure imaginary numbers.

4. B — This is a sum of cubes, $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$, with $a = 2x$ and $b = 1$: $(2x + 1)(4x^2 - 2x + 1)$. The middle term of the quadratic factor is $-ab$, making it negative. The sign pattern distinguishes it from a difference of cubes.

5. D — The solution to a system of linear equations is the point where the lines cross. The graph shows the lines intersecting at (2, 3), so that ordered pair satisfies both equations. Reading the intersection point gives the solution.

6. A — A logarithm asks for the exponent on the base: $5^3 = 125$, so $\log_5(125) = 3$. Rewriting in exponential form makes the value clear. The base 5 raised to 3 equals 125.

7. C — An infinite geometric series with $|r| < 1$ sums to $a_1/(1 - r) = 16/(1 - 1/2) = 32$. The ratio 1/2 ensures convergence. The series approaches 32 as more terms are added.

8. B — By the Remainder Theorem, dividing by $(x - 1)$ gives the remainder $p(1) = 1 - 2 + 5 = 4$. Substituting the zero of the divisor avoids long division. The remainder is 4.

9. A — Evaluate the inner function first: $f(2) = 3(2) = 6$, then $g(6) = 6^2 - 1 = 35$. Composition works from the inside out. Substituting 6 into g gives 35.

10. C — Square both sides: $2x - 1 = 9$, so $2x = 10$ and $x = 5$. Checking, $\sqrt{(2 \cdot 5 - 1)} = \sqrt{9} = 3$, which is valid. The single solution survives the check.

11. D — Average rate of change is $[f(3) - f(1)]/(3 - 1) = (27 - 3)/2 = 24/2 = 12$. This is the slope of the secant line over the interval. The exponential values 27 and 3 drive the result.

12. B — A quantity that triples each year uses base 3 with the initial value 50 as the coefficient, giving $f(x) = 50(3)^x$. The base is the growth factor and the coefficient is the starting amount. Multiplying by 3 each year produces exponential growth.

13. C — The vertex form $y = |x - h| + k$ uses the vertex $(3, -2)$, giving $y = |x - 3| - 2$. The opposite sign on h places the vertex at $x = 3$, and $k = -2$ lowers it. The x-intercepts at 1 and 5 confirm the fit.

14. A — The discriminant is $b^2 - 4ac = 64 - 64 = 0$, indicating one repeated real solution. Since $x^2 - 8x + 16 = (x - 4)^2$, the repeated root 4 is rational. A zero discriminant always yields a single repeated solution.

15. D — The angle $7\pi/6$ lies in Quadrant III with reference angle $\pi/6$, where sine is negative. Since $\sin(\pi/6) = 1/2$, $\sin(7\pi/6) = -1/2$. The quadrant determines the negative sign.

16. B — On the unit circle, the coordinates of a terminal point are $(\cos \theta, \sin \theta)$. At 60°, this is $(\cos 60°, \sin 60°) = (1/2, \sqrt{3}/2)$. Both coordinates are positive in the first quadrant.

17. A — Rewrite in exponential form: $x = 4^2 = 16$. Converting the logarithmic equation isolates x. Checking, $\log_4(16) = 2$ confirms the solution.

18. D — In $y = a \cos(x) + d$, the midline is the vertical shift d, which is 4. The midline is the horizontal line about which the function oscillates. The amplitude of 3 does not affect the midline location.

19. C — The first 10 positive even integers, 2 through 20, sum to 110. Using $n/2(\text{first} + \text{last}) = 10/2(2 + 20) = 5(22) = 110$ confirms it. The arithmetic series formula matches the direct addition.

20. A — The prime numbers from 1 to 8 are 2, 3, 5, and 7, giving 4 favorable outcomes out of 8. The probability is $4/8 = 1/2$. Counting the primes correctly is the key step.

21. B — A value of 180 is one standard deviation below the mean of 200. Since 50% lies below the mean and 34% lies between −1 SD and the mean, about 16% lies below −1 SD. The empirical rule gives the tail percentage.

22. C — A correlation coefficient of −0.92 is close to −1, indicating a strong negative linear relationship. The negative sign means the variables move in opposite directions. The magnitude near 1 signals a strong association.

23. D — In Quadrant IV sine is negative while cosine is positive, and the 7-24-25 triangle gives the sine ratio. Therefore $\sin \theta = -24/25$. The quadrant fixes the negative sign on sine.

24. B — Complete the square: $x^2 - 8x + 10 = (x^2 - 8x + 16) - 16 + 10 = (x - 4)^2 - 6$. Half of −8 squared is 16, added and subtracted to preserve value. The vertex form reveals the vertex at (4, −6).

Part II (Short Constructed Response)

25. x = 3 ± i — Quadratic formula: $x = [6 \pm \sqrt{(36 - 40)}]/2 = [6 \pm \sqrt{(-4)}]/2 = [6 \pm 2i]/2 = 3 \pm i$. The negative discriminant produces complex conjugates. Dividing each term by 2 gives the simplified a + bi form.

26. x = −3 and x = 1; no extraneous solutions — Cross-multiplying gives $x(x + 5) = 3(x + 1)$, which becomes $x^2 + 2x - 3 = 0$ and factors to $(x + 3)(x - 1) = 0$. The restrictions are $x \neq -1$ and $x \neq -5$, and neither solution violates them, so both are valid.

27. Yes, (x − 1) is a factor — By the Factor Theorem, $f(1) = 1 + 4 - 7 + 2 = 0$. A remainder of zero confirms that (x − 1) divides f(x) evenly. A nonzero value would have meant it is not a factor.

28. g(x) = (1/2)(x + 4)² − 1 — A vertical compression by 1/2 multiplies the function by 1/2; shifting 4 units left replaces x with (x + 4); shifting 1 unit down subtracts 1. Combining these gives $(1/2)(x + 4)^2 - 1$. The horizontal shift moves opposite the sign inside the parentheses.

29. x = 5/2 — Write the right side as a power of 3: 27^(x − 1) = 3^(3x − 3), so 3^(x + 2) = 3^(3x − 3) requires $x + 2 = 3x - 3$. Solving gives $5 = 2x$, so $x = 5/2$. Equal bases allow the exponents to be set equal.

30. (5x − 5) / [(x − 4)(x + 1)], x ≠ 4, x ≠ −1 — Over the common denominator $(x - 4)(x + 1)$: $3(x + 1) + 2(x - 4) = 3x + 3 + 2x - 8 = 5x - 5$. The numerator combines to 5x − 5, which factors as $5(x - 1)$. Restrictions come from the original denominators.

31. $a_4 = 53$ — Apply the recursion: $a_2 = 3(1) + 2 = 5$, $a_3 = 3(5) + 2 = 17$, $a_4 = 3(17) + 2 = 53$. Each term triples the previous term and adds 2. Building up term by term reaches 53.

32. sin θ = −5/13 — From $\tan \theta = 5/12$, the reference triangle has legs 5 and 12 with hypotenuse 13. In Quadrant III sine is negative, so $\sin \theta = -5/13$. The quadrant fixes the negative sign.

Part III (Extended Constructed Response)

33. (a) 80 mg, half-life 4 hours; (b) ≈ 9.7 hours — The coefficient 80 is the initial amount, and the exponent t/4 with base 0.5 means the amount halves every 4 hours. For part (b), 15 = 80(0.5)^(t/4) gives $0.5^{(t/4)} = 0.1875$, so $t/4 = \log_{0.5}(0.1875) \approx 2.42$ and $t \approx 9.7$ hours. Logarithms isolate the exponent.

34. (a) P(x) = −2.0x² + 20.0x − 8.0; (b) $42 thousand at x = $5 — Quadratic regression on the data returns a = −2.0, b = 20.0, c = −8.0. The vertex occurs at $x = -b/(2a) = -20/(-4) = 5$ dollars, and $P(5) = -50 + 100 - 8 = 42$ thousand dollars. The negative leading coefficient confirms a maximum.

35. (a) ((−1 + √23)/2, (1 + √23)/2) and ((−1 − √23)/2, (1 − √23)/2) — Substituting $y = x + 1$ into the circle gives $x^2 + (x + 1)^2 = 12$, which simplifies to $2x^2 + 2x - 11 = 0$, so $x = (-1 \pm \sqrt{23})/2$. The y-values follow

from $y = x + 1$. **(b)** Checking $\left(\frac{-1 + \sqrt{23}}{2}, \frac{1 + \sqrt{23}}{2}\right)$: $x^2 + y^2 = \frac{24 - 2\sqrt{23}}{4} + \frac{24 + 2\sqrt{23}}{4} = \frac{48}{4} = 12$, and $y = x + 1 = \frac{-1 + \sqrt{23}}{2} + 1 = \frac{1 + \sqrt{23}}{2}$, so both equations hold.

Part IV (Long Constructed Response)

36. (a) $T(t) = -10\cos(\pi t/12) + 76$ — The amplitude is $(86 - 66)/2 = 10$ and the midline is $(86 + 66)/2 = 76$. The period of 24 hours gives $b = 2\pi/24 = \pi/12$, and starting at the minimum at $t = 0$ calls for a negative cosine. This yields $T(t) = -10\cos(\pi t/12) + 76$.

(b) Amplitude 10, period 24 hr, midline T = 76 — The amplitude of 10°F is the distance from the midline to the daily high or low. The period of 24 hours is the time for one complete temperature cycle. The midline T = 76°F is the average temperature around which the value oscillates.

(c) $t \approx 7.57$ hours and $t \approx 16.43$ hours — Setting $80 = -10\cos(\pi t/12) + 76$ gives $\cos(\pi t/12) = -0.4$. Over the first 24 hours, $\pi t/12 = 1.9823$ and 4.3009 radians, so $t = 7.57$ and 16.43 hours. These are the two times the rising and falling temperature passes through 80°F.