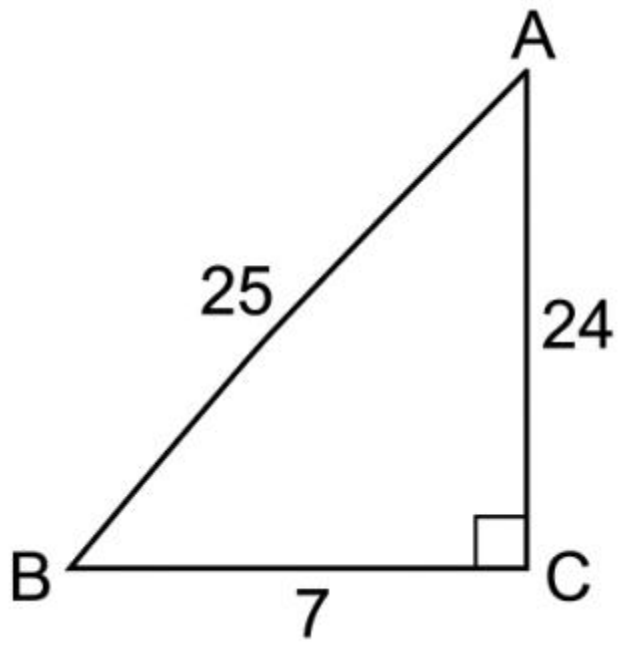
# PRACTICE EXAM 32: ALGEBRA II REGENTS SIMULATION

**PART I — Multiple Choice (24 questions × 2 credits = 48 credits)**

1. In the right triangle below, what is the value of sin A?




A. 7/25

B. 24/25

C. 7/24

D. 25/7

2. What is the value of $i^{23}$?

A. 1

B. i

C. −i

D. −1

3. What is the value of 8^(2/3)?

A. 16

B. 32/3

C. 12

D. 4

4. What are the solutions to the equation $x^2 - 4x + 1 = 0$?

A. $x = 2 \pm \sqrt{2}$

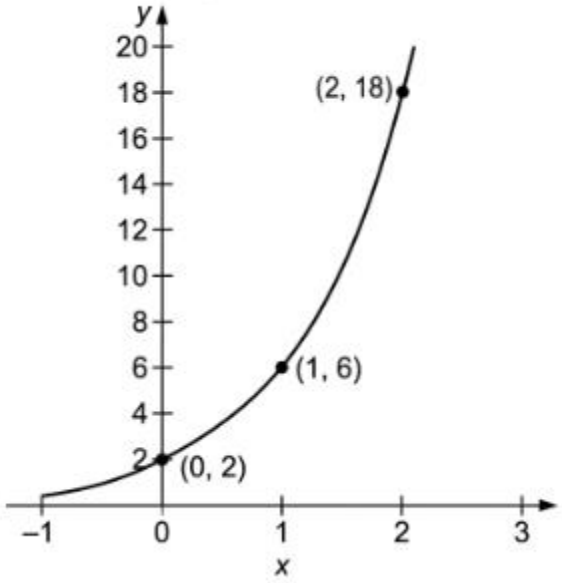
B. $x = 2 \pm \sqrt{3}$

C. $x = -2 \pm \sqrt{3}$

D. $x = 4 \pm \sqrt{3}$

5. The graph of the exponential function $f(x) = a \cdot b^x$ is shown below. What is the value of the growth factor b?

Figure PQ-2




A. 2

B. 3

C. 6

D. 1/3

6. What is the value of log(1000)?

A. 3

B. 100

C. 2

D. 30

7. What is the sum of the finite geometric series 3 + 6 + 12 + 24 + 48?

A. 48

B. 96

C. 93

D. 189

8. What is the solution set of the equation $(x - 1)(x + 2)(x - 4) = 0$?

A. $\{-1, 2, -4\}$

B. $\{1, 2, 4\}$

C. $\{-1, -2, 4\}$

D. $\{1, -2, 4\}$

9. If $f(x) = (x - 1) / 2$, what is $f^{-1}(x)$?

A. $2x + 1$

B. $2x - 1$

C. $(x + 1) / 2$

D. $(x - 1) / 2$

10. What is the solution set of the equation $\sqrt{(x + 6)} = x$?

A. $\{-2, 3\}$

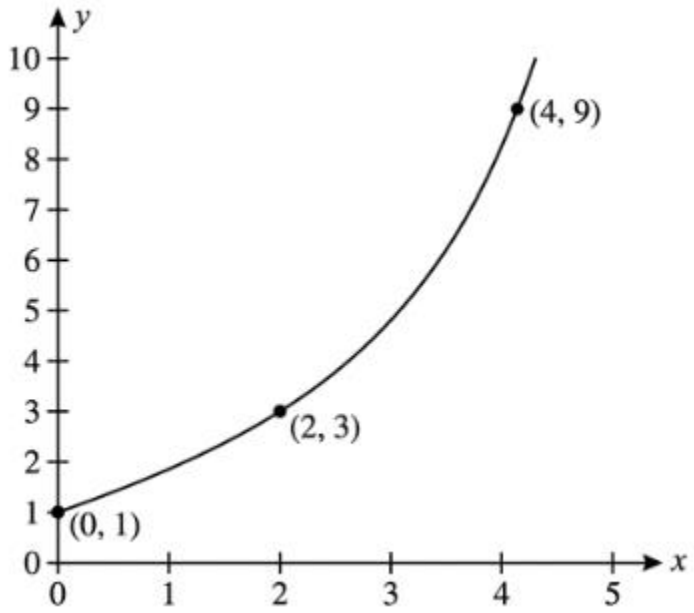
B. $\{-2\}$

C. { }

D. {3}

11. Using the graph of the function f(x) shown below, what is the average rate of change of f(x) over the interval [0, 4]?

[Figure PQ-3:]




A. 8

B. 4

C. 2

D. 1/2

12. The value of a car is modeled by $V(t) = 24000(0.85)^t$, where t is the time in years. What is the annual percent rate of decrease?

A. 85%

B. 15%

C. 8.5%

D. 1.5%

13. The graph of $f(x) = \sqrt{x}$ is transformed to produce $g(x) = \sqrt{(x - 2)} + 3$. Which best describes the transformation?

A. Right 2 and up 3

B. Left 2 and up 3

C. Right 2 and down 3

D. Left 2 and down 3

14. Which statement best describes the nature of the solutions of $2x^2 + x + 3 = 0$?

A. Two distinct rational solutions

B. One repeated rational solution

C. Two complex (non-real) solutions

D. Two distinct irrational solutions

15. What is the exact value of $\cos(2\pi/3)$?

A. 1/2

B. $\sqrt{3}/2$

C. $-\sqrt{3}/2$

D. −1/2

16. The histogram below shows the distribution of quiz scores for a class. How many students scored 80 or higher?

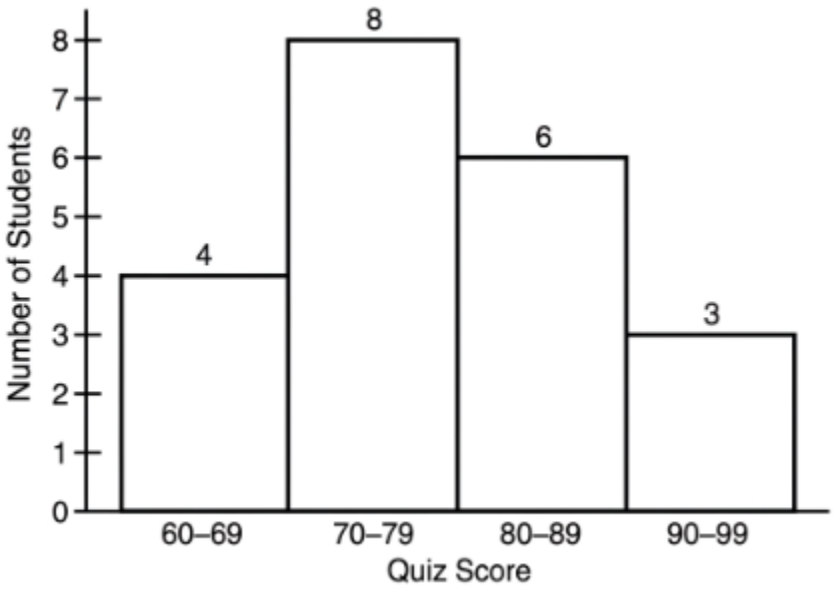



**Figure PQ-4**

A. 9

B. 14

C. 3

D. 6

17. What is the solution to the equation $2 \cdot 3^x = 54$?

A. x = 2

B. x = 3

C. x = 9

D. $x = 27$

18. For the function $y = 2 \sin(\pi x) + 1$, what is the period?

A. $\pi$

B. $2\pi$

C. 2

D. 1

19. What is the 20th term of an arithmetic sequence whose first term is 5 and whose common difference is 4?

A. 80

B. 76

C. 85

D. 81

20. Two fair coins are flipped simultaneously. What is the probability of obtaining at least one head?

A. 1/2

B. 3/4

C. 1/4

D. 1/3

21. A data set is normally distributed with a mean of 100 and a standard deviation of 15. Approximately what percent of the data falls between 85 and 130?

A. 81.5%

B. 95%

C. 68%

D. 50%

22. Which method is most appropriate for estimating the average height of all students in a large high school?

A. A controlled experiment with random assignment

B. An observational study limited to the tallest students

C. A census restricted to a single grade level only

D. A random sample survey of students from all grades

23. Given that $\sin\theta = -5/13$ and $\theta$ is in Quadrant III, what is the value of $\tan\theta$?

A. $-5/12$

B. $12/5$

C. $5/12$

D. $-12/5$

24. What is the maximum value of the function $f(x) = -2(x - 3)^2 + 7$?

A. 3

B. 7

C. −2

D. −7

**PART II — Short Constructed Response (8 questions × 2 credits = 16 credits)**

**Show all work. A correct answer with no supporting work will receive only 1 credit.**

25. Solve the equation $x^2 + 10x + 34 = 0$ algebraically. Express your solutions in a + bi form.

26. Solve algebraically for x: $1 / (x - 2) + 1 / (x + 2) = 4 / (x^2 - 4)$. State any restrictions on the variable and identify any extraneous solutions that must be rejected.

27. Given $f(x) = 2x^3 - 7x^2 + 4x + 4$, determine whether $(x - 2)$ is a factor of f(x). Justify your answer using the Remainder Theorem or the Factor Theorem.

28. Write an equation, in the form $g(x) = a \cdot f(x) + k$, for the function obtained by vertically stretching $f(x) = x^2$ by a factor of 2, reflecting the result across the x-axis, and then translating it 5 units down.

29. Solve algebraically for x: 2^(x − 1) = 1/8.

30. Express $4 / (x - 1) - 3 / x$ as a single rational expression in simplest form. State any restrictions on the variable.

31. A sequence is defined recursively by $a_1 = 1$ and $a_n = 2a_{n-1} + n$ for $n \geq 2$. Find the value of $a_4$.

32. Given that $\cos \theta = -3/5$ and $\theta$ terminates in Quadrant II, find the exact value of $\sin \theta$. Show the algebraic work that justifies your answer.

**PART III — Extended Constructed Response (3 questions × 4 credits = 12 credits)**

**Show all work. Partial credit is awarded according to the scoring rubric.**

33. A bacteria culture grows according to the function P(t) = 1200(2)^(t/3), where P(t) is the population and t is the time in hours.

(a) State the initial population of the culture and the number of hours it takes for the population to double.

(b) Algebraically determine the number of hours it will take for the population to reach 5000. Round your answer to the nearest tenth of an hour.

34. The table below shows the height h(t), in meters, of a rocket at time t seconds after launch.

| t (seconds) | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| h(t) (meters) | 5 | 26 | 41 | 50 | 53 |

(a) Using regression, write a quadratic function $h(t) = at^2 + bt + c$ that best models the data. Round each coefficient to the nearest tenth.

(b) Use the regression model from part (a) to determine the maximum height reached by the rocket and the time at which the maximum height occurs. Round each answer to the nearest tenth.

35. A circle and a line are graphed below. The circle is defined by $x^2 + y^2 = 8$, and the line is defined by $y = -x + 2$.

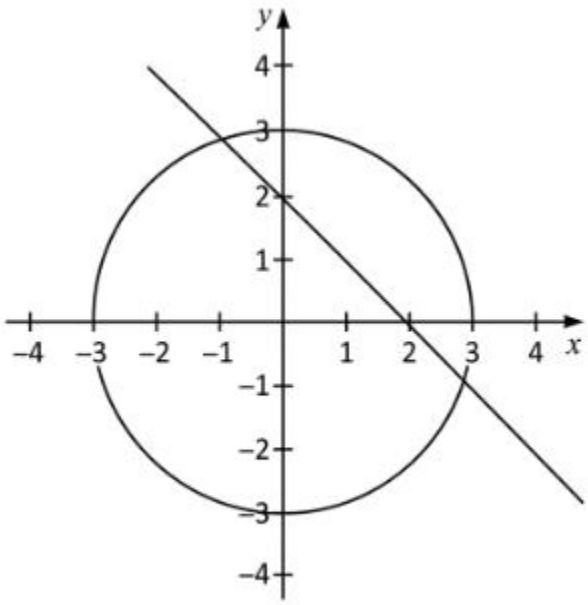


Figure PQ-5

(a) Algebraically determine all points of intersection between the circle and the line. Express coordinates in exact form (radicals permitted).

(b) Verify your answer by substituting one of the intersection points back into both original equations. Show the substitution clearly.

**PART IV — Long Constructed Response (1 question × 6 credits = 6 credits)**

**Show all work. This problem requires multiple steps and integrates concepts from several chapters.**

36. A buoy bobs up and down with the ocean waves. Its height relative to sea level oscillates between a crest of 1.5 meters above sea level and a trough of 1.5 meters below sea level, completing one full cycle every 5 seconds. At time t = 0, the buoy is at sea level (height 0 meters) and moving upward.

(a) Write a function h(t) that models the height of the buoy in meters relative to sea level as a function of time t in seconds.

(b) State the amplitude, period, and midline of the function h(t), and explain what each represents in the context of the buoy on the waves.

(c) Algebraically determine all times during the first 10 seconds at which the buoy is at a height of 1 meter above sea level. Round each answer to the nearest hundredth of a second. Show all algebraic work.

# ANSWER KEY WITH EXPLANATIONS — PRACTICE EXAM 32

**1. A** — Sine of an angle is the opposite side over the hypotenuse. For angle A, the opposite side BC = 7 and the hypotenuse AB = 25, so sin A = 7/25. Identifying which side is opposite the given angle is the key step.

**2. C** — Powers of i cycle every four: since 23 = 4(5) + 3, $i^{23} = i^3 = -i$. The remainder when the exponent is divided by 4 determines the value. A remainder of 3 corresponds to −i.

**3. D** — A rational exponent applies the root then the power: 8^(2/3) = $(\sqrt[3]{8})^2 = 2^2 = 4$. The denominator 3 takes the cube root and the numerator 2 squares the result. Evaluating the root first keeps the numbers small.

**4. B** — Quadratic formula: x = [4 ± √(16 − 4)]/2 = [4 ± √12]/2 = [4 ± 2√3]/2 = 2 ± √3. The discriminant 12 simplifies to 2√3, which reduces with the denominator. The positive non-perfect-square discriminant gives irrational roots.

**5. B** — The growth factor is the ratio of consecutive y-values: 6/2 = 3 and 18/6 = 3. A constant multiplier between equally spaced x-values identifies b. The base of the exponential function is 3.

**6. A** — A common logarithm asks for the power of 10: since $10^3 = 1000$, $\log(1000) = 3$. Rewriting the argument as a power of 10 reveals the exponent. The base-10 log of 1000 is 3.

**7. C** — Using $S = a_1(r^n - 1)/(r - 1) = 3(2^5 - 1)/(2 - 1) = 3(31) = 93$. The common ratio is 2 and there are five terms. Direct addition of the terms also gives 93.

**8. D** — By the zero-product property, each factor set to zero gives $x = 1$, $x = -2$, and $x = 4$. The roots are the opposites of the constants inside each factor. The solution set is $\{1, -2, 4\}$.

**9. A** — To invert, swap x and y in $y = (x - 1)/2$ and solve: $x = (y - 1)/2$ gives $y = 2x + 1$. The inverse undoes division by 2 and subtraction of 1 in reverse. The result is a linear function.

**10. D** — Squaring gives $x + 6 = x^2$, so $x^2 - x - 6 = 0$ and $(x - 3)(x + 2) = 0$. The candidate $x = -2$ fails because $\sqrt{4} = 2 \neq -2$, leaving $x = 3$ as the only valid solution. The check eliminates the extraneous root.

**11. C** — Average rate of change is $[f(4) - f(0)]/(4 - 0) = (9 - 1)/4 = 8/4 = 2$. This is the slope of the secant line connecting the endpoints. Reading the y-values from the graph gives the result.

**12. B** — In $a(1 - r)^t$, the base 0.85 equals $1 - 0.15$, so the rate of decrease is 15%. A base below 1 signals decay, and 1 minus the base gives the rate. The decimal 0.15 converts to 15%.

**13. A** — The $(x - 2)$ inside the radical shifts the graph right 2, and the +3 outside shifts it up 3. Horizontal shifts move opposite the sign inside, while vertical shifts follow the sign outside. The combined transformation is right 2 and up 3.

**14. C** — The discriminant is $b^2 - 4ac = 1 - 24 = -23$, which is negative. A negative discriminant produces two complex (non-real) conjugate solutions. The sign of the discriminant fully classifies the roots.

**15. D** — The angle $2\pi/3$ lies in Quadrant II with reference angle $\pi/3$, where cosine is negative. Since $\cos(\pi/3) = 1/2$, $\cos(2\pi/3) = -1/2$. The quadrant determines the negative sign.

**16. A** — Scoring 80 or higher combines the 80–89 bar (6 students) and the 90–99 bar (3 students), totaling 9. Adding the frequencies of all qualifying intervals gives the count. The two upper bars sum to 9.

**17. B** — Divide both sides by 2 to get $3^x = 27 = 3^3$, so $x = 3$. Isolating the exponential expression first allows the bases to be matched. Equal bases give equal exponents.

**18. C** — The period of $\sin(bx)$ is $2\pi/b$; with $b = \pi$, the period is $2\pi/\pi = 2$. The coefficient of x inside the sine determines the period. The amplitude and vertical shift do not affect it.

**19. D** — The nth term is $a_n = a_1 + (n - 1)d$, so $a_{20} = 5 + 19(4) = 5 + 76 = 81$. Multiplying the common difference by 19 gives the increase from the first term. The 20th term is 81.

**20. B** — The probability of at least one head is the complement of no heads: $1 - P(TT) = 1 - 1/4 = 3/4$. Subtracting the single unfavorable outcome from 1 is the efficient approach. Three of the four equally likely outcomes contain a head.

**21. A** — The value 85 is one standard deviation below the mean and 130 is two above. The region from −1 SD to the mean is 34%, and from the mean to +2 SD is 47.5%, totaling 81.5%. Adding the two empirical-rule areas gives the percent.

**22. D** — A random sample survey of students from all grades produces a representative, unbiased estimate of the average height. Random selection across the whole population avoids bias. Limiting to one grade or to the tallest students would distort the estimate.

**23. C** — In Quadrant III both sine and cosine are negative, so cos θ = −12/13 from the 5-12-13 triangle. Then tan θ = sin/cos = (−5/13)/(−12/13) = 5/12. The two negatives make the tangent positive.

**24. B** — In vertex form $a(x - h)^2 + k$, the value k = 7 is the maximum because the negative leading coefficient opens the parabola downward. The vertex (3, 7) is the highest point. The maximum value is 7.

# Part II (Short Constructed Response)

**25. x = −5 ± 3i** — Quadratic formula: x = [−10 ± √(100 − 136)]/2 = [−10 ± √(−36)]/2 = [−10 ± 6i]/2 = −5 ± 3i. The negative discriminant produces complex conjugates. Dividing each term by 2 gives the simplified a + bi form.

**26. No solution; x = 2 is rejected** — Multiplying through by (x − 2)(x + 2) gives (x + 2) + (x − 2) = 4, so 2x = 4 and x = 2. Since x = 2 makes the original denominators zero, it is extraneous and rejected, leaving the equation with no valid solution.

**27. Yes, (x − 2) is a factor** — By the Factor Theorem, f(2) = 2(8) − 7(4) + 4(2) + 4 = 16 − 28 + 8 + 4 = 0. A remainder of zero confirms that (x − 2) divides f(x) evenly. A nonzero value would have meant it is not a factor.

**28. $g(x) = -2x^2 - 5$** — A vertical stretch by 2 gives $2x^2$; reflecting across the x-axis negates it to $-2x^2$; translating 5 units down subtracts 5. Combining these produces $-2x^2 - 5$. The reflection and stretch both act on the leading coefficient.

**29. x = −2** — Write the right side as a power of 2: 1/8 = 2^(−3), so 2^(x − 1) = 2^(−3) requires x − 1 = −3. Solving gives x = −2. Equal bases allow the exponents to be set equal.

**30. (x + 3) / [x(x − 1)], x ≠ 0, x ≠ 1** — Over the common denominator x(x − 1): 4x/[x(x − 1)] − 3(x − 1)/[x(x − 1)] = [4x − 3(x − 1)]/[x(x − 1)] = (x + 3)/[x(x − 1)]. The numerator simplifies to x + 3.

**31. $a_4 = 26$** — Apply the recursion: $a_2 = 2(1) + 2 = 4$, $a_3 = 2(4) + 3 = 11$, $a_4 = 2(11) + 4 = 26$. Each term doubles the previous term and adds the index. Building up term by term reaches 26.

**32. $\sin\theta = 4/5$** — Using $\sin^2\theta + \cos^2\theta = 1$: $\sin^2\theta = 1 - 9/25 = 16/25$, so $\sin\theta = \pm 4/5$. In Quadrant II sine is positive, giving 4/5. The quadrant fixes the positive sign.

# Part III (Extended Constructed Response)

**33. (a) 1200, doubles every 3 hours; (b) ≈ 6.2 hours** — The coefficient 1200 is the initial population, and the exponent t/3 with base 2 means the population doubles every 3 hours. For part (b), 5000 = 1200(2)^(t/3) gives 2^(t/3) = 25/6 ≈ 4.167, so t/3 = $\log_2(4.167)$ ≈ 2.06 and t ≈ 6.2 hours. Logarithms isolate the exponent.

**34. (a) $h(t) = -3.0t^2 + 24.0t + 5.0$; (b) 53.0 m at t = 4.0 s** — Quadratic regression on the data returns a = −3.0, b = 24.0, c = 5.0. The vertex occurs at t = −b/(2a) = −24/(−6) = 4.0 seconds, and h(4) = −48 + 96 + 5 = 53.0 meters. The negative leading coefficient confirms a maximum.

**35. (a) $(1 + \sqrt{3}, 1 - \sqrt{3})$ and $(1 - \sqrt{3}, 1 + \sqrt{3})$** — Substituting y = −x + 2 into the circle gives $x^2 + (-x + 2)^2 = 8$, which simplifies to $2x^2 - 4x - 4 = 0$, then $x^2 - 2x - 2 = 0$, so $x = 1 \pm \sqrt{3}$. The y-values follow from y = −x + 2. **(b)** Checking $(1 + \sqrt{3}, 1 - \sqrt{3})$: $x^2 + y^2 = (4 + 2\sqrt{3}) + (4 - 2\sqrt{3}) = 8$, and $y = -x + 2 = -(1 + \sqrt{3}) + 2 = 1 - \sqrt{3}$, so both equations hold.

# Part IV (Long Constructed Response)

**36. (a) $h(t) = 1.5 \sin(2\pi t/5)$** — The amplitude is 1.5 meters and the midline is 0 (sea level). The period of 5 seconds gives b = 2π/5, and starting at height 0 while moving upward calls for a positive sine function. This yields $h(t) = 1.5 \sin(2\pi t/5)$.

**(b) Amplitude 1.5, period 5 s, midline h = 0** — The amplitude of 1.5 meters is the maximum displacement of the buoy above or below sea level. The period of 5 seconds is the time for one complete wave cycle. The midline h = 0 represents sea level, the position the buoy oscillates around.

**(c) t ≈ 0.58, 1.92, 5.58, and 6.92 seconds** — Setting $1 = 1.5 \sin(2\pi t/5)$ gives $\sin(2\pi t/5) = 2/3$. Over the first 10 seconds, 2πt/5 = 0.7297, 2.4119, 7.0130, and 8.6951 radians, so t = 0.58, 1.92, 5.58, and 6.92 seconds. Each pair of angles corresponds to the buoy rising through and falling back to the 1-meter height.