PRACTICE EXAM 31: ALGEBRA II REGENTS SIMULATION

PART I — Multiple Choice (24 questions × 2 credits = 48 credits)

1. The graph of the function y = f(x) is shown below. For what values of x is f(x) = 0?

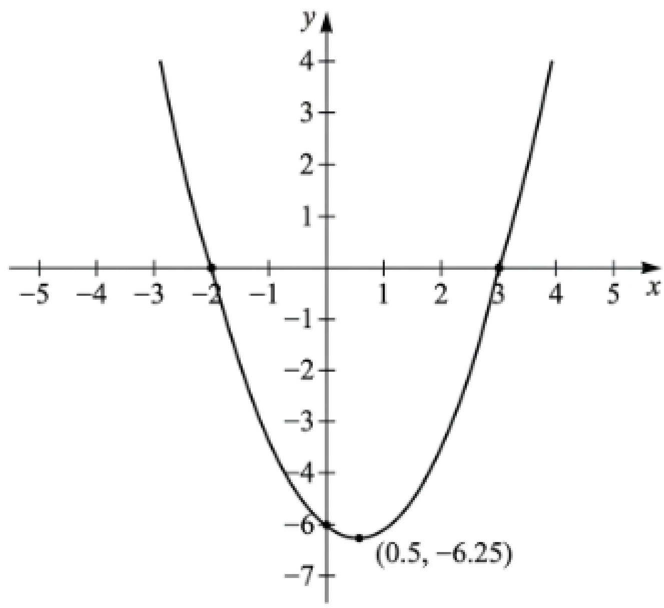


Figure PQ-1

A. {−3, 2}

B. {0, 1}

C. {2, −2}

D. {−2, 3}

2. Express the difference (5 − 2i) − (3 + 4i) in a + bi form.

A. $2 + 6i$

B. $2 - 6i$

C. $8 - 6i$

D. $8 + 6i$

3. The expression $\sqrt[4]{x^3}$ written with a rational exponent is

A. $x^{(4/3)}$

B. $x^{(12)}$

C. $x^{(3/4)}$

D. $x^{(1/12)}$

4. What are the solutions to the equation $x^2 - 5x - 14 = 0$?

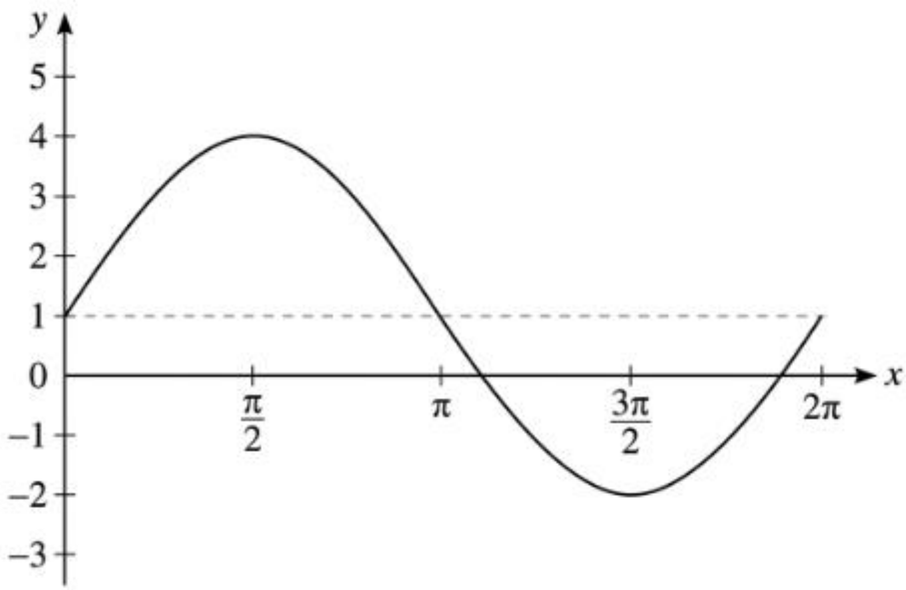
A. $x = 7$ or $x = -2$

B. $x = -7$ or $x = 2$

C. $x = 7$ or $x = 2$

D. $x = -7$ or $x = -2$

5. The graph of a sinusoidal function is shown below. What is the amplitude of the function?



[Figure PQ-2]

A. 3

B. 2

C. 5

D. 1

6. What is the value of $\log_2(32)$?

A. 2

B. 16

C. 6

D. 5

7. The 4th term of a geometric sequence is 24, and the common ratio is 2. What is the first term?

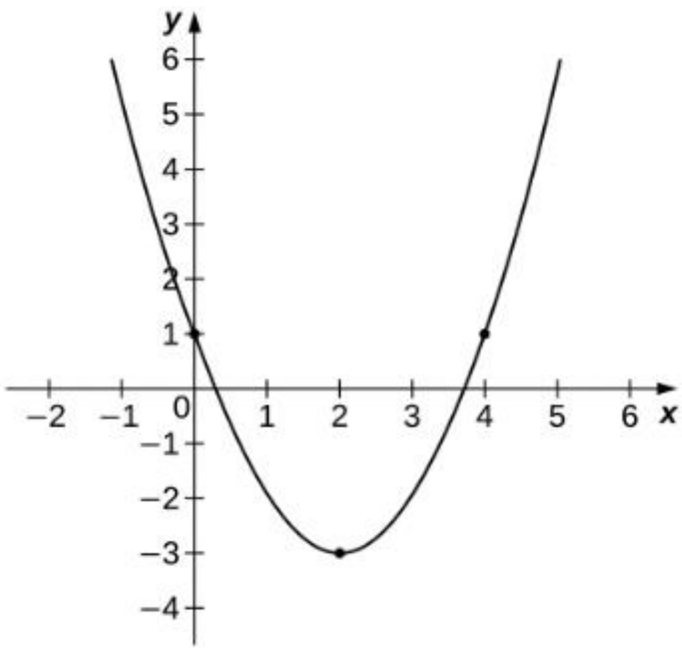
A. 6

B. 3

C. 12

D. 1.5

8. Which equation best represents the parabola graphed below?



A. $y = (x + 2)^2 - 3$

B. $y = (x - 2)^2 + 3$

C. $y = (x - 2)^2 - 3$

D. $y = (x + 2)^2 + 3$

9. If $f(x) = x^2$ and $g(x) = x + 3$, what is the value of $f(g(2))$?

A. 7

B. 13

C. 9

D. 25

10. What is the solution to the equation $\sqrt{(x + 5)} = 3$?

A. x = 2

B. x = 9

C. x = 4

D. x = 14

11. What is the average rate of change of $f(x) = 2^x$ over the interval [0, 3]?

A. 7/3

B. 3

C. 8/3

D. 7

12. A quantity decays according to $f(x) = 250(0.92)^x$. What is the percent rate of decay?

A. 92%

B. 8%

C. 9.2%

D. 0.8%

13. In the figure below, the dashed parabola is the image of the solid parabola $f(x) = x^2$ under a single transformation. Which transformation was applied?

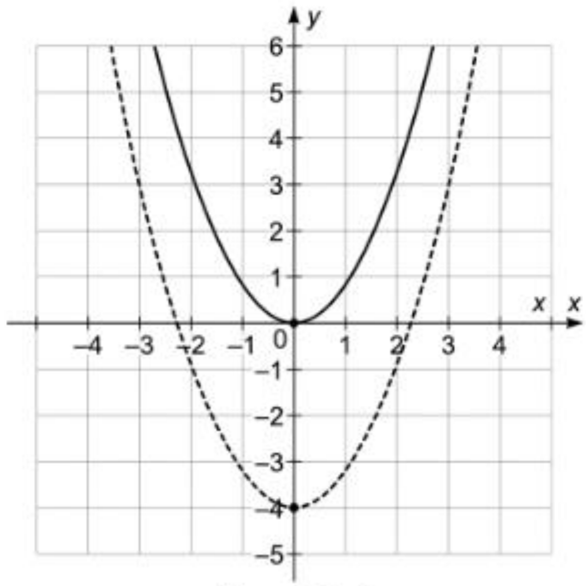


Figure PQ-4

A. Up 4 units

B. Left 4 units

C. Right 4 units

D. Down 4 units

14. Which statement best describes the nature of the solutions of $4x^2 + 4x + 1 = 0$?

A. One repeated rational solution

B. Two distinct rational solutions

C. Two distinct irrational solutions

D. Two complex (non-real) solutions

15. The expression $(x^2 + 2x - 15) / (x^2 - 9)$, in fully simplified form with restrictions, is

A. $(x - 5)/(x - 3)$, $x \neq 3$, $x \neq -3$

B. $(x + 5)/(x - 3)$, $x \neq 3$, $x \neq -3$

C. $(x + 5)/(x + 3)$, $x \neq 3$, $x \neq -3$

D. $(x - 5)/(x + 3)$, $x \neq 3$, $x \neq -3$

16. What is the exact value of $\tan(\pi/4)$?

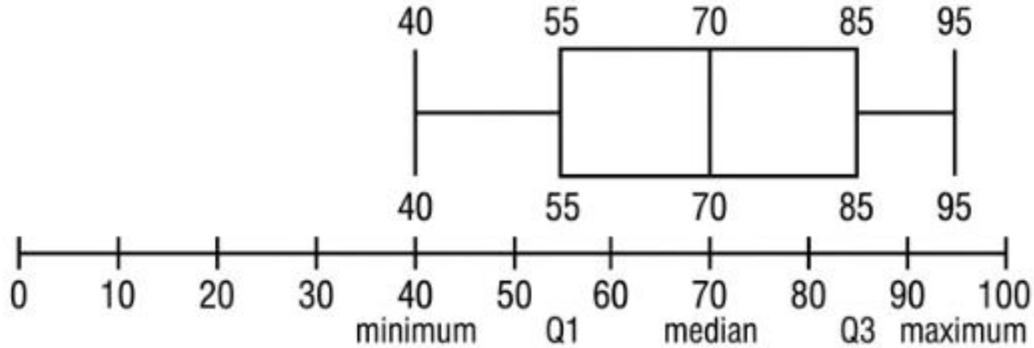
A. $\sqrt{2}/2$

B. 1

C. $\sqrt{3}$

D. 0

17. The box-and-whisker plot below summarizes a set of test scores. What is the interquartile range?



A. 55

B. 30

C. 15

D. 40

18. What is the solution to the equation $\log(x) + \log(x - 3) = 1$?

A. $x = -2$

B. $x = 10$

C. $x = 13$

D. $x = 5$

19. What is the sum of the first five terms of an arithmetic sequence whose first term is 2 and whose common difference is 3?

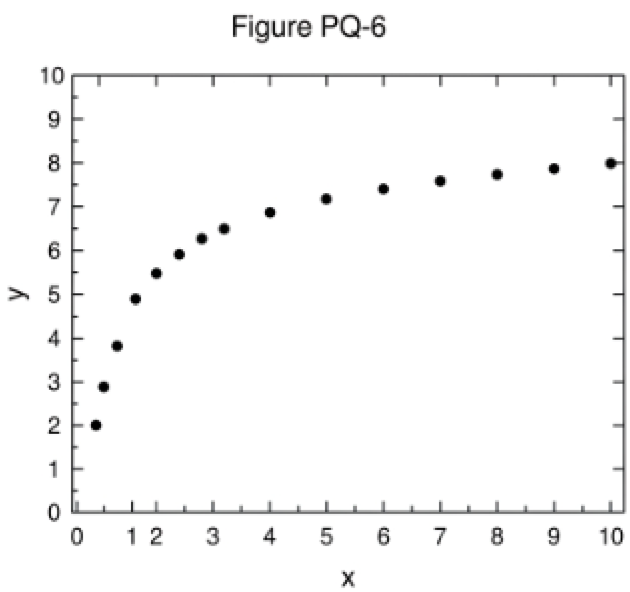
A. 40

B. 35

C. 14

D. 45

20. The scatter plot below shows the relationship between two variables. Which type of regression model best fits the data?



A. Linear

B. Exponential

C. Logarithmic

D. Quadratic

21. A bag contains 3 red marbles, 4 blue marbles, and 5 green marbles. If one marble is drawn at random, what is the probability that it is green?

A. 1/4

B. 1/3

C. 7/12

D. 5/12

22. A data set is normally distributed with a mean of 50 and a standard deviation of 5. Approximately what percent of the data is greater than 60?

A. 5%

B. 2.5%

C. 16%

D. 95%

23. Given that cos $\theta = 12/13$ and θ is in Quadrant I, what is the value of tan θ?

A. 5/12

B. 12/5

C. 13/12

D. 5/13

24. Written in vertex form, the expression $x^2 + 6x + 5$ is equivalent to

A. $(x + 3)^2 + 5$

B. $(x + 6)^2 - 31$

C. $(x + 3)^2 - 4$

D. $(x - 3)^2 - 4$

PART II — Short Constructed Response (8 questions × 2 credits = 16 credits)

Show all work. A correct answer with no supporting work will receive only 1 credit.

25. Solve the equation $x^2 - 2x + 10 = 0$ algebraically. Express your solutions in $a + bi$ form.

26. Solve algebraically for x: $2 / (x - 3) = x / (x + 3)$. State any restrictions on the variable and identify any extraneous solutions that must be rejected.

27. Given $f(x) = x^3 + 2x^2 - x - 2$, determine whether $(x + 1)$ is a factor of f(x). Justify your answer using the Remainder Theorem or the Factor Theorem.

28. Write an equation, in the form $g(x) = a \cdot f(x - h) + k$, for the function obtained by reflecting $f(x) = x^3$ across the x-axis and then translating the resulting graph 1 unit to the right and 4 units up.

29. Solve algebraically for x: 4^(x + 2) = 8^x.

30. Express $x / (x^2 - 1) - 1 / (x + 1)$ as a single rational expression in simplest form. State any restrictions on the variable.

31. A sequence is defined recursively by $a_1 = 2$ and $a_n = (a_{n-1})^2 - 1$ for $n \geq 2$. Find the value of a_4.

32. Given that $\tan \theta = -3/4$ and θ terminates in Quadrant II, find the exact value of $\sin \theta$. Show the algebraic work that justifies your answer.

PART III — Extended Constructed Response (3 questions × 4 credits = 12 credits)

Show all work. Partial credit is awarded according to the scoring rubric.

33. The value of an investment is modeled by the function $A(t) = 5000(1.03)^t$, where $A(t)$ is the value in dollars and t is the time in years.

(a) State the initial value of the investment and the annual percent rate of growth.

(b) Algebraically determine the number of years it will take for the investment to reach a value of \$8000. Round your answer to the nearest tenth of a year.

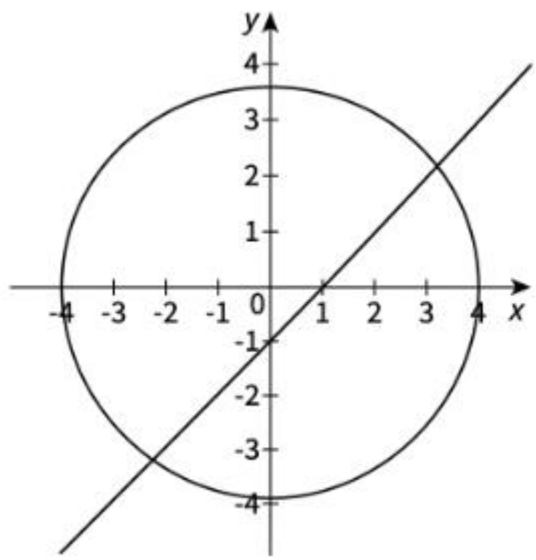
34. The table below shows the height $h(t)$, in feet, of a ball thrown upward from a platform at time t seconds after release.

t (seconds)	0	1	2	3	4
h(t) (feet)	5	53	69	53	5

(a) Using regression, write a quadratic function $h(t) = at^2 + bt + c$ that best models the data. Round each coefficient to the nearest tenth.

(b) Use the regression model from part (a) to determine the maximum height reached by the ball and the time at which the maximum height occurs. Round each answer to the nearest tenth.

35. A circle and a line are graphed below. The circle is defined by $x^2 + y^2 = 9$, and the line is defined by $y = x - 1$.



[Figure PQ-7]

(a) Algebraically determine all points of intersection between the circle and the line. Express coordinates in exact form (radicals permitted).

(b) Verify your answer by substituting one of the intersection points back into both original equations. Show the substitution clearly.

PART IV — Long Constructed Response (1 question × 6 credits = 6 credits)

Show all work. This problem requires multiple steps and integrates concepts from several chapters.

36. A wind turbine has blades that are 30 meters long, and the center hub of the turbine is 50 meters above the ground. The turbine completes one full rotation every 4 seconds. A marked point on the tip of one blade begins at the lowest point of its rotation at time $t = 0$.

(a) Write a function $h(t)$ that models the height of the marked point in meters above the ground as a function of time t in seconds.

(b) State the amplitude, period, and midline of the function $h(t)$, and explain what each represents in the context of the wind turbine.

(c) Algebraically determine all times during the first 8 seconds of operation at which the marked point is at a height of 65 meters above the ground. Round each answer to the nearest hundredth of a second. Show all algebraic work.

ANSWER KEY WITH EXPLANATIONS – PRACTICE EXAM 31

1. D — The zeros of a function are the x-values where the graph crosses the x-axis. The parabola intersects the x-axis at $x = -2$ and $x = 3$, so $f(x) = 0$ at $\{-2, 3\}$. Reading intercepts directly from the graph gives the solution set.

2. B — Distribute the subtraction and combine like terms: $(5 - 2i) - (3 + 4i) = 5 - 3 - 2i - 4i = 2 - 6i$. The real parts subtract to 2 and the imaginary parts subtract to $-6i$. Tracking the sign on each part is the key step.

3. C — A radical converts to a rational exponent as the power over the index: $\sqrt[4]{x^3} = x^{3/4}$. The index 4 becomes the denominator and the power 3 becomes the numerator. This is the standard radical-to-exponent conversion.

4. A — Factor the trinomial: $x^2 - 5x - 14 = (x - 7)(x + 2) = 0$, giving $x = 7$ or $x = -2$. The factor pair -7 and $+2$ multiplies to -14 and adds to -5. Setting each factor to zero yields the solutions.

5. A — Amplitude is half the distance between the maximum and minimum: $(4 - (-2))/2 = 6/2 = 3$. It measures the vertical distance from the midline ($y = 1$) to a peak. The amplitude is unaffected by the vertical shift.

6. D — A logarithm asks what exponent produces the argument: $2^5 = 32$, so $\log_2(32) = 5$. Rewriting in exponential form makes the value clear. The base raised to the answer must equal 32.

7. B — In a geometric sequence, $a_n = a_1 \cdot r^{(n-1)}$, so $a_4 = a_1 \cdot 2^3 = 8a_1 = 24$, giving $a_1 = 3$. Dividing the known term by r^3 recovers the first term. The exponent $n - 1 = 3$ is essential.

8. C — Vertex form $y = (x - h)^2 + k$ uses the vertex $(2, -3)$, giving $y = (x - 2)^2 - 3$. Checking the point $(0, 1)$: $(0 - 2)^2 - 3 = 4 - 3 = 1$, which matches. The opposite sign on h places the vertex at $x = 2$.

9. D — Evaluate the inner function first: $g(2) = 2 + 3 = 5$, then $f(5) = 5^2 = 25$. Composition works from the inside out. Substituting the output of g into f gives 25.

10. C — Square both sides: $x + 5 = 9$, so $x = 4$. Checking, $\sqrt{(4 + 5)} = \sqrt{9} = 3$, which is valid. Squaring removes the radical and the check confirms no extraneous root.

11. A — Average rate of change is $[f(3) - f(0)]/(3 - 0)$ with $f(3) = 8$ and $f(0) = 1$, giving $(8 - 1)/3 = 7/3$. This is the slope of the secant line over the interval. The exponential growth produces a fractional rate.

12. B — In the form $a(1 - r)^x$, the base 0.92 equals $1 - 0.08$, so the decay rate is 8%. A base below 1 indicates decay, and 1 minus the base gives the rate. The decimal 0.08 converts to 8%.

13. D — The vertex moved from $(0, 0)$ to $(0, -4)$ with no horizontal change, indicating a downward shift of 4 units. A change only in the y-coordinate of the vertex signals a vertical translation. The image lies 4 units below the original.

14. A — The discriminant is $b^2 - 4ac = 16 - 16 = 0$, indicating one repeated real solution. Since $4x^2 + 4x + 1 = (2x + 1)^2$, the repeated root $-1/2$ is rational. A zero discriminant always yields a single repeated solution.

15. C — Factor and cancel: $(x + 5)(x - 3) / [(x - 3)(x + 3)] = (x + 5)/(x + 3)$, with $x \neq 3$ and $x \neq -3$. The common factor $(x - 3)$ divides out. Restrictions come from the original denominator before canceling.

16. B — Tangent is sine over cosine, and at $\pi/4$ both equal $\sqrt{2}/2$, so $\tan(\pi/4) = (\sqrt{2}/2)/(\sqrt{2}/2) = 1$. The equal legs of the 45-45-90 reference triangle make the ratio 1. This is a standard unit-circle value.

17. B — The interquartile range is $Q3 - Q1 = 85 - 55 = 30$. It measures the spread of the middle 50% of the data. The whiskers and median do not affect the IQR.

18. D — Combine logs: $\log[x(x - 3)] = 1$ means $x(x - 3) = 10$, so $x^2 - 3x - 10 = 0$ and $(x - 5)(x + 2) = 0$. The candidate $x = -2$ is rejected because it makes a logarithm argument negative, leaving $x = 5$. Domain restrictions eliminate the extraneous root.

19. A — The five terms are 2, 5, 8, 11, 14, which sum to 40. Using $S = n/2(2a_1 + (n - 1)d) = 5/2(4 + 12) = 40$ confirms it. The arithmetic series formula matches the direct addition.

20. C — Data that rises steeply and then levels off increases at a decreasing rate, the signature of a logarithmic model. Linear data rises at a constant rate and exponential data accelerates upward. The flattening shape rules out the other models.

21. D — There are 5 green marbles out of 12 total, so the probability is 5/12. Probability is favorable outcomes over total equally likely outcomes. The total count of 3 + 4 + 5 = 12 forms the denominator.

22. B — A value of 60 lies two standard deviations above the mean of 50. By the empirical rule, about 5% of data falls beyond ±2 standard deviations, so roughly 2.5% lies above +2 SD. The symmetry of the normal curve splits the outer 5% evenly.

23. A — With $\cos \theta = 12/13$ in Quadrant I, the 5-12-13 triangle gives $\sin \theta = 5/13$, so $\tan \theta = \sin/\cos = (5/13)/(12/13) = 5/12$. All ratios are positive in Quadrant I. The 13s cancel, leaving 5/12.

24. C — Complete the square: $x^2 + 6x + 5 = (x^2 + 6x + 9) - 9 + 5 = (x + 3)^2 - 4$. Half of 6 squared is 9, added and subtracted to preserve value. The vertex form reveals the vertex at $(-3, -4)$.

Part II (Short Constructed Response)

25. $x = 1 \pm 3i$ — Quadratic formula: $x = [2 \pm \sqrt{(4 - 40)}]/2 = [2 \pm \sqrt{(-36)}]/2 = [2 \pm 6i]/2 = 1 \pm 3i$. The negative discriminant produces complex conjugate roots. Dividing each term by 2 gives the simplified a + bi form.

26. $x = 6$ and $x = -1$; no extraneous solutions — Cross-multiplying gives $2(x + 3) = x(x - 3)$, which becomes $x^2 - 5x - 6 = 0$ and factors to $(x - 6)(x + 1) = 0$. The restrictions are $x \neq 3$ and $x \neq -3$, and neither solution violates them, so both are valid.

27. Yes, $(x + 1)$ is a factor — By the Factor Theorem, $f(-1) = (-1) + 2 + 1 - 2 = 0$. A remainder of zero confirms that $(x + 1)$ divides $f(x)$ evenly. A nonzero value would have meant it is not a factor.

28. $g(x) = -(x - 1)^3 + 4$ — Reflecting across the x-axis negates the function to $-x^3$; shifting 1 unit right replaces x with $(x - 1)$; shifting 4 units up adds 4. Combining gives $-(x - 1)^3 + 4$. The horizontal shift moves opposite the sign inside the parentheses.

29. $x = 4$ — Rewrite with base 2: $4^{(x + 2)} = 2^{(2x + 4)}$ and $8^x = 2^{(3x)}$, so $2x + 4 = 3x$. Solving gives $x = 4$. Equal bases allow the exponents to be set equal.

30. $1 / [(x - 1)(x + 1)]$, $x \neq 1$, $x \neq -1$ — Over the common denominator $(x - 1)(x + 1)$: $x/[(x - 1)(x + 1)] - (x - 1)/[(x - 1)(x + 1)] = [x - (x - 1)]/[(x - 1)(x + 1)] = 1/[(x - 1)(x + 1)]$. The numerator simplifies to 1.

31. $a_4 = 63$ — Apply the recursion: $a_2 = 2^2 - 1 = 3$, $a_3 = 3^2 - 1 = 8$, $a_4 = 8^2 - 1 = 63$. Each term squares the previous term and subtracts 1. Building up term by term reaches 63.

32. $\sin \theta = 3/5$ — From $\tan \theta = -3/4$, the reference triangle has legs 3 and 4 with hypotenuse 5. In Quadrant II sine is positive while cosine is negative, so $\sin \theta = 3/5$. The quadrant fixes the positive sign on sine.

Part III (Extended Constructed Response)

33. (a) \$5000, 3% growth; (b) ≈ 15.9 years — The coefficient 5000 is the initial value, and the base 1.03 = 1 + 0.03 indicates 3% annual growth. For part (b), $8000 = 5000(1.03)^t$ gives $1.6 = 1.03^t$, so $t = \ln(1.6)/\ln(1.03) \approx 15.9$ years. Logarithms isolate the exponent.

34. (a) $h(t) = -16.0t^2 + 64.0t + 5.0$; (b) 69.0 ft at t = 2.0 s — Quadratic regression on the data returns $a = -16.0$, $b = 64.0$, $c = 5.0$. The vertex occurs at $t = -b/(2a) = -64/(-32) = 2.0$ seconds, and $h(2) = -64 + 128 + 5 = 69.0$ feet. The negative leading coefficient confirms a maximum.

35. (a) $((1 + \sqrt{17})/2, (-1 + \sqrt{17})/2)$ and $((1 - \sqrt{17})/2, (-1 - \sqrt{17})/2)$ — Substituting $y = x - 1$ into the circle gives $x^2 + (x - 1)^2 = 9$, which simplifies to $2x^2 - 2x - 8 = 0$, then $x^2 - x - 4 = 0$, so $x = (1 \pm \sqrt{17})/2$. The y-values follow from $y = x - 1$. **(b)** Checking $((1 + \sqrt{17})/2, (-1 + \sqrt{17})/2)$: $x^2 + y^2 = (18 + 2\sqrt{17})/4 + (18 - 2\sqrt{17})/4 = 36/4 = 9$, and $y = x - 1 = (1 + \sqrt{17})/2 - 1 = (-1 + \sqrt{17})/2$, so both equations hold.

Part IV (Long Constructed Response)

36. (a) $h(t) = -30 \cos(\pi t/2) + 50$ — The amplitude is the 30-meter blade length and the midline is the 50-meter hub height. The period of 4 seconds gives $b = 2\pi/4 = \pi/2$, and the negative cosine starts the point at its lowest position at $t = 0$. This yields $h(t) = -30 \cos(\pi t/2) + 50$.

(b) Amplitude 30, period 4 s, midline h = 50 — The amplitude of 30 meters equals the blade length, the distance from the hub to the blade tip. The period of 4 seconds is the time for one full rotation. The midline $h = 50$ is the hub height, the center about which the tip oscillates.

(c) t ≈ 1.33, 2.67, 5.33, and 6.67 seconds — Setting $65 = -30 \cos(\pi t/2) + 50$ gives $\cos(\pi t/2) = -0.5$. Over the first 8 seconds, $\pi t/2 = 2\pi/3, 4\pi/3, 8\pi/3$, and $10\pi/3$, so $t = 4/3, 8/3, 16/3$, and $20/3$. These round to 1.33, 2.67, 5.33, and 6.67 seconds.

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