PRACTICE EXAM 30: ALGEBRA II REGENTS SIMULATION

PART I — Multiple Choice (24 questions × 2 credits = 48 credits)

1. Express the product $(3 + 2i)(4 - i)$ in $a + bi$ form.

A. $14 - 5i$

B. $14 + 5i$

C. $10 + 5i$

D. $10 - 5i$

2. Simplify the expression x^(3/4) · x^(2/3).

A. x^(1/12)

B. x^(6/12)

C. x^(5/7)

D. x^(17/12)

3. Which statement best describes the nature of the solutions of $x^2 - 6x + 7 = 0$?

A. Two distinct irrational solutions

B. Two distinct rational solutions

C. One repeated rational solution

D. Two complex (non-real) solutions

4. Factored completely, the expression $x^3 - 2x^2 - 9x + 18$ is equivalent to

A. $(x - 2)(x - 3)(x + 3)$

B. $(x - 2)(x^2 - 9)$

C. $(x + 2)(x - 3)(x + 3)$

D. $(x - 2)(x + 3)^2$

5. The solutions to the equation $x^2 - 6x + 25 = 0$ are

A. $x = -3 \pm 4i$

B. $x = 3 \pm 16i$

C. $x = 3 \pm 4i$

D. $x = 6 \pm 8i$

6. The expression $(x^2 - 4) / (x^2 + 5x + 6)$, in fully simplified form with restrictions, is

A. $(x + 2) / (x + 3)$, $x \neq -3$, $x \neq -2$

B. $(x - 2) / (x + 3)$, $x \neq -3$, $x \neq -2$

C. $(x - 2) / (x - 3)$, $x \neq 3$, $x \neq -2$

D. $(x + 2) / (x - 3), x \neq 3, x \neq -2$

7. What is the solution set of the equation $\sqrt{(2x + 7)} = x - 4$?

A. {1, 9}

B. {1}

C. { }

D. {9}

8. When $p(x) = x^3 + 2x^2 - 5x + 1$ is divided by $(x + 3)$, the remainder is

A. −5

B. 1

C. 7

D. 13

9. How many points of intersection do the line $y = 2x + 1$ and the parabola $y = x^2 - 2$ have?

A. 0

B. 1

C. 2

D. 3

10. The expression $x^3 - 64$ is equivalent to

A. $(x - 4)(x^2 + 4x + 16)$

B. $(x - 4)(x^2 - 4x + 16)$

C. $(x + 4)(x^2 - 4x + 16)$

D. $(x - 4)(x^2 + 16)$

11. If $f(x) = -x^2 + 4x - 3$, what is the value of $f(-1)$?

A. 0

B. −8

C. −2

D. 6

12. The first three terms of a geometric sequence are 5, 15, 45. What is the 6th term?

A. 135

B. 405

C. 729

D. 1215

13. The graph of $f(x) = x^2$ is transformed to produce $g(x) = (x - 2)^2 + 5$. Which best describes the transformation?

A. Shift right 2 and up 5

B. Shift left 2 and up 5

C. Shift right 2 and down 5

D. Shift left 2 and down 5

14. The average rate of change of $f(x) = x^2 - 3x$ on the interval [2, 5] is

A. 12

B. 6

C. 2

D. 4

15. A population grows according to $P(t) = 800(1.07)^t$, where t is the time in years. The annual growth rate is

A. 0.7%

B. 7%

C. 70%

D. 107%

16. What is the solution to the equation $\log_3(x + 2) = 4$?

A. $x = 7$

B. $x = 14$

C. $x = 79$

D. $x = 81$

17. The value of $\sin(5\pi/6)$ is

A. $1/2$

B. $-1/2$

C. $\sqrt{3}/2$

D. $-\sqrt{3}/2$

18. For the function $y = 4\cos(3x) - 2$, what is the period?

A. 3

B. 4

C. $2\pi/3$

D. 2π

19. If $f(x) = 2x - 7$, then $f^{-1}(x)$ is equal to

A. $1 / (2x - 7)$

B. $(x + 7) / 2$

C. $(x - 7) / 2$

D. $2x + 7$

20. Given that cos θ = −8/17 and θ is in Quadrant III, the value of sin θ is

A. 15/17

B. 8/15

C. −8/15

D. −15/17

21. Battery lifespans for a manufacturer are normally distributed with a mean of 70 hours and a standard deviation of 10 hours. Approximately what percent of batteries last between 60 and 80 hours?

A. 68%

B. 95%

C. 34%

D. 50%

22. Which study design is most appropriate for establishing whether a new fertilizer causes increased plant growth?

A. An observational study of gardens already using the fertilizer

B. A survey asking gardeners about their fertilizer choices

C. A randomized controlled experiment with random treatment assignment

D. A census of all plants grown in the region this season

23. A scatter plot of an investment's value shows that the amount appears to multiply by a constant factor each year. Which regression model is most appropriate for this data?

A. Linear regression

B. Quadratic regression

C. Power regression

D. Exponential regression

24. A single card is drawn at random from a standard 52-card deck. What is the probability that the card drawn is a heart?

A. 1/13

B. 1/4

C. 1/2

D. 3/13

PART II — Short Constructed Response (8 questions × 2 credits = 16 credits)

Show all work. A correct answer with no supporting work will receive only 1 credit.

25. Solve the equation $x^2 + 4x + 29 = 0$ algebraically. Express your solutions in $a + bi$ form.

26. Solve algebraically for x: $12 / (x^2 - 4) + 1 = 3 / (x - 2)$. State any extraneous solutions and indicate which (if any) are rejected.

27. Given $f(x) = 2x^3 - 3x^2 - 8x + 12$, determine whether $(x - 2)$ is a factor of $f(x)$. Justify your answer using the Remainder Theorem or the Factor Theorem.

28. Write an equation, in the form $g(x) = a \cdot f(x - h) + k$, for the function obtained by reflecting $f(x) = \sqrt{x}$ across the x-axis and then translating the resulting graph 3 units to the left and 1 unit down.

29. Solve algebraically for x: $5^{(2x - 3)} = 125$.

30. Express $3 / (x - 2) + 4 / (x + 1)$ as a single rational expression in simplest form. State any restrictions on the variable.

31. A sequence is defined recursively by $a_1 = 3$ and $a_n = 3a_{n-1} - 2$ for $n \geq 2$. Find the value of a_4.

32. Given that $\sin \theta = 5/13$ and θ terminates in Quadrant II, find the exact value of $\cos \theta$. Show the algebraic work that justifies your answer.

PART III — Extended Constructed Response (3 questions × 4 credits = 12 credits)

Show all work. Partial credit is awarded according to the scoring rubric.

33. A radioactive substance decays exponentially according to the function $A(t) = 300(0.88)^t$, where $A(t)$ is the amount remaining in grams and t is the time elapsed in years.

(a) State the initial amount of the substance and the annual percent rate of decay.

(b) Algebraically determine the number of years it will take for the amount of the substance to decay to 75 grams. Round your answer to the nearest tenth of a year.

34. The table below shows the height $h(t)$, in meters, of a projectile launched from a platform at time t seconds after launch.

t (seconds)	0	1	2	3	4
h(t) (meters)	10	35	50	55	50

(a) Using regression, write a quadratic function $h(t) = at^2 + bt + c$ that best models the data. Round each coefficient to the nearest tenth.

(b) Use the regression model from part (a) to determine the maximum height reached by the projectile and the time at which the maximum height occurs. Round each answer to the nearest tenth.

35. A circle is defined by the equation $x^2 + y^2 = 16$, and a line is defined by the equation $y = x + 2$.

(a) Algebraically determine all points of intersection between the circle and the line. Express coordinates in exact form (radicals permitted).

(b) Verify your answer by substituting one of the intersection points back into both original equations. Show the substitution clearly.

PART IV — Long Constructed Response (1 question × 6 credits = 6 credits)

Show all work. This problem requires multiple steps and integrates concepts from several chapters.

36. The depth of water in a harbor varies with the tides and can be modeled by a sinusoidal function. At a certain harbor, the depth at high tide is 12 meters and the depth at low tide is 4 meters. One complete tidal cycle takes 12 hours. At time $t = 0$ (midnight), the water is at low tide.

(a) Write a function $d(t)$ that models the depth of the water in meters as a function of time t in hours.

(b) State the amplitude, period, and midline of the function $d(t)$, and explain what each represents in the context of the tides.

(c) Algebraically determine all times during the first 12 hours at which the water depth is 9 meters. Round each answer to the nearest hundredth of an hour. Show all algebraic work.

ANSWER KEY WITH EXPLANATIONS – PRACTICE EXAM 30

1. B — Distribute and apply $i^2 = -1$: $(3+2i)(4-i) = 12 - 3i + 8i - 2i^2 = 12 + 5i + 2 = 14 + 5i$. The middle terms combine to $+5i$ and the $-2i^2$ becomes $+2$, shifting the real part to 14. Correctly handling i^2 is the deciding step.

2. D — Multiplying powers with the same base adds the exponents: $3/4 + 2/3 = 9/12 + 8/12 = 17/12$, giving $x^{(17/12)}$. Finding a common denominator before adding the fractional exponents is essential. The improper fraction confirms the exponent exceeds 1.

3. A — The discriminant $b^2 - 4ac = (-6)^2 - 4(1)(7) = 36 - 28 = 8$ is positive but not a perfect square. A positive non-perfect-square discriminant yields two distinct irrational real solutions. The sign and "squareness" of the discriminant fully classify the roots.

4. A — Group and factor: $x^2(x - 2) - 9(x - 2) = (x - 2)(x^2 - 9)$, then factor the difference of squares to $(x - 2)(x - 3)(x + 3)$. Complete factoring requires breaking down $x^2 - 9$ fully. Stopping at $(x - 2)(x^2 - 9)$ leaves the expression not fully factored.

5. C — Apply the quadratic formula: $x = [6 \pm \sqrt{(36 - 100)}]/2 = [6 \pm \sqrt{(-64)}]/2 = [6 \pm 8i]/2 = 3 \pm 4i$. The negative radicand $\sqrt{(-64)} = 8i$ produces the imaginary part. Dividing both terms by 2 gives the simplified form.

6. B — Factor numerator and denominator: $(x - 2)(x + 2) / [(x + 2)(x + 3)]$, then cancel the common $(x + 2)$ to get $(x - 2)/(x + 3)$. Restrictions come from the original denominator, so $x \neq -2$ and $x \neq -3$. The canceled factor must still be excluded from the domain.

7. D — Squaring gives $2x + 7 = x^2 - 8x + 16$, so $x^2 - 10x + 9 = 0$ and $(x - 1)(x - 9) = 0$. Testing both, $x = 1$ fails because $\sqrt{9} = 3 \neq -3$, while $x = 9$ satisfies $\sqrt{25} = 5 = 5$. Only $x = 9$ survives, so the solution set is $\{9\}$.

8. C — By the Remainder Theorem, dividing by $(x + 3)$ gives the remainder $p(-3)$: $(-27) + 2(9) - 5(-3) + 1 = -27 + 18 + 15 + 1 = 7$. Substituting the zero of the divisor avoids long division. The result is the remainder, not a factor.

9. C — Setting $x^2 - 2 = 2x + 1$ gives $x^2 - 2x - 3 = 0$, which factors to $(x - 3)(x + 1) = 0$. Two real solutions mean the line crosses the parabola at two points. The number of real roots equals the number of intersection points.

10. A — This is a difference of cubes, $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$, with $a = x$ and $b = 4$: $(x - 4)(x^2 + 4x + 16)$. The quadratic factor uses $+ab$ and is not further factorable over the reals. The sign pattern distinguishes it from a sum of cubes.

11. B — Substitute $x = -1$: $f(-1) = -(-1)^2 + 4(-1) - 3 = -1 - 4 - 3 = -8$. The squared term stays negative because the negation applies after squaring. Careful sign tracking gives -8.

12. D — The common ratio is $15/5 = 3$, and the nth term is $a_n = a_1 \cdot r^{(n-1)}$. The 6th term is $5 \cdot 3^5 = 5 \cdot 243 = 1215$. Using $n - 1 = 5$ as the exponent is the key step.

13. A — In $(x - h)^2 + k$, the value $h = 2$ inside shifts the graph right 2, and $k = +5$ shifts it up 5. The horizontal shift is opposite the sign inside the parentheses. So the transformation is right 2 and up 5.

14. D — Average rate of change is $[f(5) - f(2)]/(5 - 2)$. With $f(5) = 25 - 15 = 10$ and $f(2) = 4 - 6 = -2$, this is $[10 - (-2)]/3 = 12/3 = 4$. It represents the slope of the secant line over the interval.

15. B — In the model $a(1 + r)^t$, the base 1.07 equals $1 + r$, so $r = 0.07 = 7\%$. The growth rate is the decimal portion above 1 converted to a percent. The base being greater than 1 confirms growth rather than decay.

16. C — Rewrite in exponential form: $x + 2 = 3^4 = 81$, so $x = 79$. Converting the logarithmic equation to exponential form isolates x. Checking shows $x - 1 = 79$ keeps the argument positive and valid.

17. A — The angle $5\pi/6$ lies in Quadrant II with reference angle $\pi/6$, and sine is positive there. Since $\sin(\pi/6) = 1/2$, $\sin(5\pi/6) = 1/2$. The reference angle and quadrant sign together determine the value.

18. C — For $y = a \cos(bx) + d$, the period is $2\pi/b$. With $b = 3$, the period is $2\pi/3$. The vertical shift and amplitude do not affect the period.

19. B — To find the inverse, swap x and y in $y = 2x - 7$ and solve: $x = 2y - 7$ gives $y = (x + 7)/2$. The inverse undoes the operations in reverse order. The result is a linear function, not a reciprocal.

20. D — Using $\sin^2\theta + \cos^2\theta = 1$: $\sin^2\theta = 1 - 64/289 = 225/289$, so $\sin \theta = \pm 15/17$. In Quadrant III sine is negative, giving $-15/17$. The quadrant determines the sign of the result.

21. A — The values 60 and 80 are exactly one standard deviation below and above the mean of 70. By the empirical rule, about 68% of data falls within one standard deviation of the mean. This interval captures the central 68% of a normal distribution.

22. C — Only a randomized controlled experiment with random assignment can establish causation, because randomization balances confounding variables across groups. Observational studies and surveys can show association but not cause. Random treatment assignment is the defining feature for causal inference.

23. D — A constant multiplicative factor per time unit is the signature of exponential growth, modeled by $y = a \cdot b^x$. Linear models add a constant amount, not multiply by one. The repeated multiplication pattern matches exponential regression.

24. B — A standard deck has 13 hearts out of 52 cards, so the probability is 13/52 = 1/4. Dividing favorable outcomes by total equally likely outcomes gives the probability. Each suit represents one-fourth of the deck.

Part II (Short Constructed Response)

25. $x = -2 \pm 5i$ — Quadratic formula: $x = [-4 \pm \sqrt{(16 - 116)}]/2 = [-4 \pm \sqrt{(-100)}]/2 = [-4 \pm 10i]/2 = -2 \pm 5i$. The negative discriminant produces a pair of complex conjugates. Dividing each term by 2 yields the simplified a + bi form.

26. $x = 1$; $x = 2$ rejected — Multiplying through by $(x - 2)(x + 2)$: $12 + (x^2 - 4) = 3(x + 2)$, which simplifies to $x^2 - 3x + 2 = 0$ and factors to $(x - 1)(x - 2) = 0$. The candidate $x = 2$ makes the original denominator zero, so it is extraneous and rejected, leaving $x = 1$.

27. Yes, $(x - 2)$ is a factor — By the Factor Theorem, $f(2) = 2(8) - 3(4) - 8(2) + 12 = 16 - 12 - 16 + 12 = 0$. A remainder of 0 confirms $(x - 2)$ divides $f(x)$ evenly. A nonzero result would have meant it is not a factor.

28. $g(x) = -\sqrt{(x + 3)} - 1$ — Reflecting across the x-axis negates the function to $-\sqrt{x}$; translating 3 units left replaces x with $(x + 3)$; translating 1 unit down subtracts 1. Combining these gives $-\sqrt{(x + 3)} - 1$. Horizontal shifts move opposite the sign inside the radical.

29. $x = 3$ — Express both sides with base 5: $125 = 5^3$, so $5^{(2x - 3)} = 5^3$ requires $2x - 3 = 3$. Solving gives $2x = 6$ and $x = 3$. Equal bases allow setting the exponents equal.

30. $(7x - 5)/[(x - 2)(x + 1)]$, $x \neq 2$, $x \neq -1$ — Over the common denominator $(x - 2)(x + 1)$: $3(x + 1) + 4(x - 2) = 3x + 3 + 4x - 8 = 7x - 5$. The numerator combines to $7x - 5$ with no further cancellation. Restrictions come from the original denominators.

31. $a_4 = 55$ — Apply the recursion: $a_2 = 3(3) - 2 = 7$, $a_3 = 3(7) - 2 = 19$, $a_4 = 3(19) - 2 = 55$. Each term depends on the one before it. Building up term by term reaches $a_4 = 55$.

32. $\cos\theta = -12/13$ — Using $\sin^2\theta + \cos^2\theta = 1$: $\cos^2\theta = 1 - 25/169 = 144/169$, so $\cos\theta = \pm 12/13$. In Quadrant II cosine is negative, giving −12/13. The quadrant fixes the sign of the answer.

Part III (Extended Constructed Response)

33. (a) 300 g, 12% decay; (b) ≈ 10.8 years — The coefficient 300 is the initial amount, and the base 0.88 = 1 − 0.12 indicates a 12% annual decay rate. For part (b), $75 = 300(0.88)^t$ gives $0.25 = 0.88^t$, so $t = \ln(0.25)/\ln(0.88) \approx 10.8$ years. Taking logarithms of both sides isolates the exponent.

34. (a) $h(t) = -5.0t^2 + 30.0t + 10.0$; (b) 55.0 m at t = 3.0 s — Quadratic regression on the data returns a = −5.0, b = 30.0, c = 10.0. The vertex occurs at $t = -b/(2a) = -30/(-10) = 3.0$ seconds, and $h(3) = -45 + 90 + 10 = 55.0$ meters. The negative leading coefficient confirms the vertex is a maximum.

35. (a) $(-1 + \sqrt{7}, 1 + \sqrt{7})$ and $(-1 - \sqrt{7}, 1 - \sqrt{7})$ — Substituting y = x + 2 into the circle: $x^2 + (x + 2)^2 = 16$ gives $2x^2 + 4x - 12 = 0$, then $x^2 + 2x - 6 = 0$, so $x = -1 \pm \sqrt{7}$. The corresponding y-values are $1 \pm \sqrt{7}$. **(b)** Checking $(-1 + \sqrt{7}, 1 + \sqrt{7})$: $(-1 + \sqrt{7})^2 + (1 + \sqrt{7})^2 = (8 - 2\sqrt{7}) + (8 + 2\sqrt{7}) = 16$, and $y = x + 2 = (-1 + \sqrt{7}) + 2 = 1 + \sqrt{7}$, so both equations hold.

Part IV (Long Constructed Response)

36. (a) $d(t) = -4\cos(\pi t/6) + 8$ — The amplitude is (12 − 4)/2 = 4 and the midline is (12 + 4)/2 = 8. The period of 12 hours gives $b = 2\pi/12 = \pi/6$, and the negative cosine starts at the minimum (low tide) at t = 0. This produces $d(t) = -4\cos(\pi t/6) + 8$.

(b) Amplitude 4, period 12 hr, midline d = 8 — The amplitude of 4 is half the difference between high and low tide, the vertical distance from the midline to each extreme. The period of 12 hours is the time for one complete tidal cycle. The midline d = 8 is the average water depth around which the tide oscillates.

(c) t ≈ 3.48 hours and t ≈ 8.52 hours — Set $9 = -4\cos(\pi t/6) + 8$, giving $\cos(\pi t/6) = -1/4$. Within one period, $\pi t/6 = \arccos(-1/4) \approx 1.8235$ or $2\pi - 1.8235 \approx 4.4597$. Solving for t gives t ≈ 3.48 and t ≈ 8.52 hours, the two times the depth reaches 9 meters.