

PRACTICE EXAM 29: ALGEBRA II

REGENTS SIMULATION

PART I — Multiple Choice (24 questions \times 2 credits = 48 credits)

1. Express $(1 + i)^2$ in $a + bi$ form.

- A. $2i$
- B. $1 + 2i$
- C. 2
- D. $2 + 2i$

2. What is the value of $32^{(2/5)}$?

- A. 12.8
- B. 64
- C. 8
- D. 4

3. What are the solutions to the equation $x^2 + 2x + 10 = 0$?

- A. $x = 1 \pm 3i$

B. $x = -1 \pm 6i$

C. $x = -1 \pm 3i$

D. $x = -2 \pm 3i$

4. Factored completely, the expression $x^3 + 4x^2 - 9x - 36$ is equivalent to

A. $(x + 4)(x^2 - 9)$

B. $(x + 4)(x - 3)(x + 3)$

C. $(x - 4)(x - 3)(x + 3)$

D. $(x + 4)(x - 9)(x + 1)$

5. What is the value of $\log_5(125)$?

A. 3

B. 25

C. $1/3$

D. 5

6. What is the sum of the finite geometric series $1 + 1/2 + 1/4 + 1/8 + 1/16$?

A. 2

B. $15/16$

C. $31/16$

D. $5/16$

7. When $p(x) = 2x^3 - x^2 + 4$ is divided by $(x - 1)$, the remainder is

A. 4

B. 5

C. 6

D. 3

8. If $f(x) = x - 3$ and $g(x) = x^2 + 1$, what is the value of $g(f(5))$?

A. 23

B. 11

C. 26

D. 5

9. What is the solution to the equation $\sqrt[3]{(x - 1)} = 2$?

A. $x = 7$

B. $x = 5$

C. $x = 9$

D. $x = 3$

10. What is the average rate of change of $f(x) = \sqrt{x}$ over the interval $[4, 16]$?

A. $1/6$

B. $1/4$

C. 6

D. 2

11. \$2000 is invested at an annual interest rate of 4% compounded annually. What is the value of the investment after 3 years?

A. \$2240.00

B. \$2249.73

C. \$2080.00

D. \$2400.00

12. What is the solution to the equation $2\log_3(x) = 4$?

A. $x = 4$

B. $x = 6$

C. $x = 81$

D. $x = 9$

13. Which statement best describes the nature of the solutions of $3x^2 - 5x + 1 = 0$?

- A. Two distinct irrational solutions
- B. Two distinct rational solutions
- C. Two complex (non-real) solutions
- D. One repeated rational solution

14. What is the exact value of $\sin(5\pi/6)$?

- A. $-1/2$
- B. $\sqrt{3}/2$
- C. $1/2$
- D. $-\sqrt{3}/2$

15. For the function $y = -2 \cos(2x/3)$, what is the period?

- A. $2\pi/3$
- B. 3
- C. 2π
- D. 3π

16. In an arithmetic sequence, the third term is 11 and the seventh term is 27. What is the common difference?

- A. 3
- B. 4

C. 5

D. 8

17. If $f(x) = (x + 2)/3$, what is $f^{-1}(x)$?

A. $3x - 2$

B. $(x - 2)/3$

C. $3x + 2$

D. $(3 - x)/2$

18. In how many ways can 4 books be arranged in order on a shelf from a collection of 6 distinct books?

A. 24

B. 15

C. 360

D. 720

19. A data set is normally distributed with a mean of 500 and a standard deviation of 100. Approximately what percent of the data falls between 400 and 600?

A. 34%

B. 68%

C. 95%

D. 50%

20. Given that $\sin \theta = 7/25$ and θ is in Quadrant II, what is the value of $\cos \theta$?

A. $24/25$

B. $7/24$

C. $-7/25$

D. $-24/25$

21. Written in vertex form, the expression $x^2 + 10x + 18$ is equivalent to

A. $(x + 5)^2 + 18$

B. $(x - 5)^2 - 7$

C. $(x + 5)^2 - 7$

D. $(x + 10)^2 - 82$

22. What is the sum of the infinite geometric series $9 + 6 + 4 + \dots$?

A. 27

B. 18

C. 13.5

D. 24

23. What is the solution to the equation $4^x = 75$, rounded to the nearest hundredth?

- A. $x = 18.75$
- B. $x = 3.11$
- C. $x = 2.16$
- D. $x = 4.32$

24. A researcher wants to study the study habits of all students at a high school but surveys only students in the honors program. What is the flaw in this sampling method?

- A. The sample size is too large
- B. There is no flaw; this is a random sample
- C. The survey questions are biased
- D. The sample is not representative of the entire student population

PART II — Short Constructed Response (8 questions \times 2 credits = 16 credits)

Show all work. A correct answer with no supporting work will receive only 1 credit.

25. Solve the equation $x^2 + 4x + 13 = 0$ algebraically. Express your solutions in a + bi form.

26. Solve algebraically for x : $2 / (x - 3) = 8 / (x + 3)$. State any restrictions on the variable and identify any extraneous solutions that must be rejected.

27. Given $f(x) = x^3 + 2x^2 - 5x + 2$, determine whether $(x - 1)$ is a factor of $f(x)$. Justify your answer using the Remainder Theorem or the Factor Theorem.

28. Write an equation, in the form $g(x) = a \cdot f(x - h) + k$, for the function obtained by translating $f(x) = 2^x$ to the right 1 unit and down 3 units.

29. Solve algebraically for x : $9^x = 27$.

30. Express $x / (x + 3) - 2 / (x - 1)$ as a single rational expression in simplest form. State any restrictions on the variable.

31. A sequence is defined recursively by $a_1 = 5$ and $a_n = a_{n-1} + 2n$ for $n \geq 2$. Find the value of a_4 .

32. Given that $\tan \theta = 7/24$ and θ terminates in Quadrant III, find the exact value of $\sin \theta$. Show the algebraic work that justifies your answer.

PART III — Extended Constructed Response (3 questions \times 4 credits = 12 credits)

Show all work. Partial credit is awarded according to the scoring rubric.

33. The amount of a medication in a patient's bloodstream is modeled by the function $M(t) = 80(0.75)^t$, where $M(t)$ is the amount in milligrams and t is the time in hours.

(a) State the initial amount of medication and the hourly percent rate at which it decreases.

(b) Algebraically determine the number of hours it will take for the amount to fall to 20 milligrams. Round your answer to the nearest tenth of an hour.

34. The table below shows the height $h(x)$, in meters, of a kite at horizontal distance x meters from where the flyer stands.

x (meters) 0 1 2 3 4
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$h(x)$ (meters) 3 10 15 18 19

(a) Using regression, write a quadratic function $h(x) = ax^2 + bx + c$ that best models the data. Round each coefficient to the nearest tenth.

(b) Use the regression model from part (a) to determine the maximum height of the kite and the horizontal distance at which the maximum height occurs. Round each answer to the nearest tenth.

35. A circle is defined by the equation $x^2 + y^2 = 50$, and a line is defined by the equation $y = x - 2$.

(a) Algebraically determine all points of intersection between the circle and the line. Express coordinates in exact form (radicals permitted).

(b) Verify your answer by substituting one of the intersection points back into both original equations. Show the substitution clearly.

PART IV — Long Constructed Response (1 question \times 6 credits = 6 credits)

Show all work. This problem requires multiple steps and integrates concepts from several chapters.

36. A Ferris wheel has a radius of 20 meters, and its center is 22 meters above the ground. The wheel completes one full rotation every 40 seconds. A rider boards at the lowest point of the wheel at time $t = 0$.

(a) Write a function $h(t)$ that models the rider's height in meters above the ground as a function of time t in seconds.

(b) State the amplitude, period, and midline of the function $h(t)$, and explain what each represents in the context of the Ferris wheel.

(c) Algebraically determine all times during the first 40 seconds at which the rider is at a height of 32 meters above the ground. Round each answer to the nearest hundredth of a second. Show all algebraic work.

ANSWER KEY WITH EXPLANATIONS – PRACTICE EXAM 29

1. A — Expand the square and apply $i^2 = -1$: $(1 + i)^2 = 1 + 2i + i^2 = 1 + 2i - 1 = 2i$. The real terms 1 and -1 cancel. The result is the pure imaginary number $2i$.

2. D — Apply the fifth root, then square: $32^{(2/5)} = (32^{(1/5)})^2 = 2^2 = 4$. The denominator 5 takes the fifth root and the numerator 2 squares it. The value is 4.

3. C — Quadratic formula: $x = [-2 \pm \sqrt{(4 - 40)}]/2 = [-2 \pm \sqrt{(-36)}]/2 = [-2 \pm 6i]/2 = -1 \pm 3i$. The negative discriminant produces the imaginary part. Dividing both terms by 2 gives the simplified form.

4. B — Group and factor: $x^2(x + 4) - 9(x + 4) = (x + 4)(x^2 - 9)$, then the difference of squares gives $(x + 4)(x - 3)(x + 3)$. Complete factoring requires breaking down $x^2 - 9$. The fully factored form has three factors.

5. A — A logarithm asks for the exponent on the base: $5^3 = 125$, so $\log_5(125) = 3$. Rewriting in exponential form makes the value clear. The base 5 raised to 3 equals 125.

6. C — Using $S = a_1(1 - r^n)/(1 - r) = (1 - (1/2)^5)/(1 - 1/2) = (31/32)/(1/2) = 31/16$. The five terms sum to slightly less than 2. The value is $31/16$.

7. B — By the Remainder Theorem, the remainder is $p(1) = 2 - 1 + 4 = 5$. Substituting the zero of the divisor avoids long division. The remainder is 5.

- 8. D** — Evaluate the inner function first: $f(5) = 5 - 3 = 2$, then $g(2) = 2^2 + 1 = 5$. Composition works from the inside out. Squaring 2 and adding 1 gives 5.
- 9. C** — Cube both sides: $x - 1 = 2^3 = 8$, so $x = 9$. Cubing removes the cube root, and no extraneous roots arise with odd roots. Checking, $\sqrt[3]{8} = 2$ confirms the solution.
- 10. A** — Average rate of change is $[f(16) - f(4)]/(16 - 4) = (4 - 2)/12 = 2/12 = 1/6$. This is the slope of the secant line over the interval. The square-root values 4 and 2 drive the result.
- 11. B** — Apply $A = P(1 + r)^t = 2000(1.04)^3 = 2000(1.124864) \approx 2249.73$. Compounding multiplies by 1.04 each year for three years. The value grows to about \$2249.73.
- 12. D** — Divide by 2, then rewrite in exponential form: $\log_3(x) = 2$, so $x = 3^2 = 9$. Isolating the logarithm before converting is essential. The solution is 9.
- 13. A** — The discriminant is $b^2 - 4ac = 25 - 12 = 13$, which is positive but not a perfect square. This produces two distinct irrational real solutions. The nature of the discriminant classifies the roots.
- 14. C** — The angle $5\pi/6$ lies in Quadrant II with reference angle $\pi/6$, where sine is positive. Since $\sin(\pi/6) = 1/2$, $\sin(5\pi/6) = 1/2$. The quadrant keeps the sine positive.
- 15. D** — The period of $\cos(bx)$ is $2\pi/b$; with $b = 2/3$, the period is $2\pi/(2/3) = 3\pi$. Dividing by the fractional coefficient enlarges the period. The period is 3π .
- 16. B** — The common difference is $d = (27 - 11)/(7 - 3) = 16/4 = 4$. Dividing the change in terms by the change in position gives d . The common difference is 4.
- 17. A** — To invert, solve $y = (x + 2)/3$ for x : $3y = x + 2$, so $x = 3y - 2$ and $f^{-1}(x) = 3x - 2$. The inverse undoes division by 3 and addition of 2 in reverse order. The result is $3x - 2$.
- 18. C** — Order matters, so use permutations: $P(6, 4) = 6 \cdot 5 \cdot 4 \cdot 3 = 360$. The permutation formula counts ordered arrangements. There are 360 ways.
- 19. B** — The values 400 and 600 are one standard deviation below and above the mean of 500. By the empirical rule, about 68% of data lies within ± 1 SD. The interval captures 68%.
- 20. D** — In Quadrant II cosine is negative while sine is positive, and the 7-24-25 triangle gives the cosine ratio. Therefore $\cos \theta = -24/25$. The quadrant makes the cosine negative.
- 21. C** — Complete the square: $x^2 + 10x + 18 = (x^2 + 10x + 25) - 25 + 18 = (x + 5)^2 - 7$. Half of 10 squared is 25, added and subtracted to preserve value. The vertex form reveals the vertex at $(-5, -7)$.
- 22. A** — The common ratio is $2/3$, and an infinite geometric series with $|r| < 1$ sums to $a_1/(1 - r) = 9/(1 - 2/3) = 9/(1/3) = 27$. The ratio ensures convergence. The series sums to 27.

23. B — Take the log of both sides: $x = \log_4(75) = \ln(75)/\ln(4) \approx 3.11$. The change-of-base formula converts to natural logs. The result rounds to 3.11.

24. D — Honors students are not representative of the entire student body, so their study habits cannot be generalized to all students. A valid sample must give every student a chance of selection. This selection bias undermines the conclusions.

Part II (Short Constructed Response)

25. $x = -2 \pm 3i$ — Quadratic formula: $x = [-4 \pm \sqrt{(16 - 52)}]/2 = [-4 \pm \sqrt{-36}]/2 = [-4 \pm 6i]/2 = -2 \pm 3i$. The negative discriminant produces complex conjugates. Dividing each term by 2 gives the simplified $a + bi$ form.

26. $x = 5$; no extraneous solutions — Cross-multiplying gives $2(x + 3) = 8(x - 3)$, which expands to $2x + 6 = 8x - 24$ and simplifies to $-6x = -30$, so $x = 5$. The restrictions are $x \neq 3$ and $x \neq -3$, and $x = 5$ violates neither, so it is valid.

27. Yes, $(x - 1)$ is a factor — By the Factor Theorem, $f(1) = 1 + 2 - 5 + 2 = 0$. A remainder of zero confirms that $(x - 1)$ divides $f(x)$ evenly. A nonzero value would have meant it is not a factor.

28. $g(x) = 2^{x-1} - 3$ — Translating right 1 unit replaces x with $(x - 1)$ in the exponent; translating down 3 units subtracts 3. Combining these gives $2^{x-1} - 3$. The horizontal shift moves opposite the sign inside.

29. $x = 3/2$ — Write both sides as powers of 3: $9^x = 3^{2x}$ and $27 = 3^3$, so $3^{2x} = 3^3$ requires $2x = 3$. Solving gives $x = 3/2$. Equal bases allow the exponents to be set equal.

30. $(x^2 - 3x - 6) / [(x + 3)(x - 1)]$, $x \neq -3$, $x \neq 1$ — Over the common denominator $(x + 3)(x - 1)$: $x(x - 1) - 2(x + 3) = x^2 - x - 2x - 6 = x^2 - 3x - 6$. The numerator combines to $x^2 - 3x - 6$, which does not factor further. Restrictions come from the original denominators.

31. $a_4 = 23$ — Apply the recursion: $a_2 = 5 + 2(2) = 9$, $a_3 = 9 + 2(3) = 15$, $a_4 = 15 + 2(4) = 23$. Each term adds twice the index to the previous term. Building up term by term reaches 23.

32. $\sin \theta = -7/25$ — With $\tan \theta = 7/24$, the 7-24-25 Pythagorean triple gives the hypotenuse 25 and opposite leg 7. In Quadrant III sine is negative, so $\sin \theta = -7/25$. The quadrant fixes the negative sign.

Part III (Extended Constructed Response)

33. (a) 80 mg, 25% decrease per hour; (b) ≈ 4.8 hours — The coefficient 80 is the initial amount, and the base $0.75 = 1 - 0.25$ indicates a 25% hourly decrease. For part (b), $20 = 80(0.75)^t$ gives $0.75^t = 0.25$, so $t = \ln(0.25)/\ln(0.75) \approx 4.8$ hours. Logarithms isolate the exponent.

34. (a) $h(x) = -1.0x^2 + 8.0x + 3.0$; (b) 19.0 m at $x = 4.0$ m — Quadratic regression on the data returns $a = -1.0$, $b = 8.0$, $c = 3.0$. The vertex occurs at $x = -b/(2a) = -8/(-2) = 4.0$ meters, and $h(4) = -16 + 32 + 3 = 19.0$ meters. The negative leading coefficient confirms a maximum.

35. (a) $(1 + 2\sqrt{6}, -1 + 2\sqrt{6})$ and $(1 - 2\sqrt{6}, -1 - 2\sqrt{6})$ — Substituting $y = x - 2$ into the circle gives $x^2 + (x - 2)^2 = 50$, which simplifies to $2x^2 - 4x - 46 = 0$, then $x^2 - 2x - 23 = 0$, so $x = 1 \pm 2\sqrt{6}$. The y -values follow from $y = x - 2$. **(b)** Checking $(1 + 2\sqrt{6}, -1 + 2\sqrt{6})$: $x^2 + y^2 = (25 + 4\sqrt{6}) + (25 - 4\sqrt{6}) = 50$, and $y = x - 2 = (1 + 2\sqrt{6}) - 2 = -1 + 2\sqrt{6}$, so both equations hold.

Part IV (Long Constructed Response)

36. (a) $h(t) = -20 \cos(\pi t/20) + 22$ — The amplitude is the 20-meter radius and the midline is the 22-meter center height. The period of 40 seconds gives $b = 2\pi/40 = \pi/20$, and boarding at the lowest point at $t = 0$ calls for a negative cosine. This yields $h(t) = -20 \cos(\pi t/20) + 22$.

(b) Amplitude 20, period 40 s, midline $h = 22$ — The amplitude of 20 meters equals the wheel's radius, the distance from the center to the rim. The period of 40 seconds is the time for one full rotation. The midline $h = 22$ meters is the center height above the ground, about which the rider's height oscillates.

(c) $t \approx 13.33$ and 26.67 seconds — Setting $32 = -20 \cos(\pi t/20) + 22$ gives $\cos(\pi t/20) = -0.5$. Over the first 40 seconds, $\pi t/20 = 2\pi/3$ and $4\pi/3$, so $t = 40/3$ and $80/3$. These round to 13.33 and 26.67 seconds, the times the rising and falling rider passes a height of 32 meters.