PRACTICE EXAM 28: ALGEBRA II REGENTS SIMULATION

PART I — Multiple Choice (24 questions × 2 credits = 48 credits)

1. What is the value of i^{47}?

A. i

B. 1

C. −i

D. −1

2. What is the value of 16^(3/4)?

A. 12

B. 8

C. 64

D. 4

3. What are the solutions to the equation $2x^2 + 8 = 0$?

A. $x = \pm 4i$

B. $x = \pm 2$

C. $x = \pm\sqrt{2}\, i$

D. $x = \pm 2i$

4. Factored completely, the expression $27x^3 + 1$ is equivalent to

A. $(3x + 1)(9x^2 - 3x + 1)$

B. $(3x + 1)(9x^2 + 3x + 1)$

C. $(3x - 1)(9x^2 + 3x + 1)$

D. $(3x + 1)(3x^2 - 1)$

5. What is the value of $\log_2(1/8)$?

A. 3

B. $-1/3$

C. -3

D. 4

6. What is the 6th term of a geometric sequence whose first term is 3 and whose common ratio is 2?

A. 192

B. 48

C. 64

D. 96

7. When $p(x) = x^4 - 3x^2 + 2$ is divided by $(x + 1)$, the remainder is

A. 0

B. 6

C. -2

D. 2

8. If $f(x) = 2x + 1$ and $g(x) = x^2$, what is the value of $g(f(2))$?

A. 9

B. 25

C. 17

D. 11

9. What is the solution to the equation $\sqrt{(x + 6)} = x$?

A. $x = 3$ and $x = -2$

B. $x = -2$

C. $x = 9$

D. $x = 3$

10. What is the average rate of change of $f(x) = 3^x$ over the interval [0, 2]?

A. 9

B. 8

C. 4

D. 3

11. A 500 mg sample of a substance has a half-life of 8 hours. Which function models the amount remaining after t hours?

A. f(t) = 500(1/2)^(t/8)

B. f(t) = 500(1/2)^(8t)

C. f(t) = 500(2)^(t/8)

D. f(t) = 500 − 8t

12. What is the solution to the equation log(x) + log(4) = 2?

A. x = 20

B. x = 25

C. x = 96

D. x = 50

13. Which statement best describes the nature of the solutions of $2x^2 + 3x + 5 = 0$?

A. Two distinct rational solutions

B. Two distinct irrational solutions

C. Two complex (non-real) solutions

D. One repeated rational solution

14. What is the exact value of $\tan(\pi/4)$?

A. $\sqrt{2}/2$

B. $\sqrt{3}$

C. 1/2

D. 1

15. For the function $y = 5\sin(x) + 8$, what is the equation of the midline?

A. $y = 5$

B. $y = 8$

C. $y = 13$

D. $y = 0$

16. What is the sum of the first 20 terms of an arithmetic sequence whose first term is 1 and whose common difference is 3?

A. 590

B. 580

C. 600

D. 1180

17. If $f(x) = 2^x$, what is $f^{-1}(x)$?

A. $(1/2)^x$

B. x^2

C. $\log_2(x)$

D. $\sqrt{x}$

18. For two events, $P(A) = 0.6$, $P(B) = 0.5$, and $P(A \cap B) = 0.3$. Which statement is true?

A. A and B are mutually exclusive

B. P(A or B) = 1.1

C. A and B are dependent

D. A and B are independent because $P(A) \cdot P(B) = P(A \cap B)$

19. A data set is normally distributed with a mean of 70 and a standard deviation of 5. What is the z-score for a value of 80?

A. 2

B. 10

C. 1.6

D. 0.5

20. Given that $\tan \theta = -5/12$ and θ is in Quadrant II, what is the value of $\sin \theta$?

A. 12/13

B. 5/13

C. −5/13

D. −12/13

21. Written in vertex form, the expression $2x^2 - 8x + 3$ is equivalent to

A. $2(x - 2)^2 + 3$

B. $(x - 4)^2 - 5$

C. $2(x - 2)^2 - 5$

D. $2(x - 4)^2 - 13$

22. What is the sum of the infinite geometric series $16 - 8 + 4 - 2 + \ldots$?

A. 32/3

B. 8

C. 16

D. 24

23. What is the solution to the equation $6^x = 1000$, rounded to the nearest hundredth?

A. $x = 166.67$

B. $x = 4.00$

C. $x = 2.79$

D. $x = 3.86$

24. A data set consists of the values 4, 7, 9, 10, and 12. If the value 12 is changed to 120, which measure of the data is least affected by this change?

A. The mean

B. The median

C. The range

D. The standard deviation

PART II — Short Constructed Response (8 questions × 2 credits = 16 credits)

Show all work. A correct answer with no supporting work will receive only 1 credit.

25. Solve the equation $x^2 - 6x + 25 = 0$ algebraically. Express your solutions in a + bi form.

26. Solve algebraically for x: $7 / (x + 2) = 3 / (x - 2)$. State any restrictions on the variable and identify any extraneous solutions that must be rejected.

27. Given $f(x) = x^3 + 3x^2 - 4x - 12$, determine whether $(x + 2)$ is a factor of $f(x)$. Justify your answer using the Remainder Theorem or the Factor Theorem.

28. Write an equation, in the form $g(x) = a \cdot f(x - h) + k$, for the function obtained by reflecting $f(x) = x^3$ across the x-axis and then translating the result 3 units to the left.

29. Solve algebraically for x: $4^{(2x - 1)} = 64$.

30. Express $3 / (x^2 - 4) + 1 / (x + 2)$ as a single rational expression in simplest form. State any restrictions on the variable.

31. A sequence is defined recursively by $a_1 = -1$ and $a_n = 3a_{n-1} + 4$ for $n \geq 2$. Find the value of a_4.

32. Given that $\cos \theta = -9/41$ and θ terminates in Quadrant III, find the exact value of $\sin \theta$. Show the algebraic work that justifies your answer.

PART III — Extended Constructed Response (3 questions × 4 credits = 12 credits)

Show all work. Partial credit is awarded according to the scoring rubric.

33. A bacterial culture begins with 150 cells and doubles every 3 hours. The number of cells is modeled by the function $N(t) = 150(2)^{(t/3)}$, where t is the time in hours.

(a) State the initial number of cells and describe what the exponent t/3 represents in the model.

(b) Algebraically determine the number of hours it will take for the culture to grow to 1000 cells. Round your answer to the nearest tenth of an hour.

34. The table below shows the height h(x), in meters, of a thrown ball at horizontal distance x meters from the thrower.

x (meters)	0	1	2	3	4
h(x) (meters)	1	21	33	37	33

(a) Using regression, write a quadratic function $h(x) = ax^2 + bx + c$ that best models the data. Round each coefficient to the nearest tenth.

(b) Use the regression model from part (a) to determine the maximum height of the ball and the horizontal distance at which the maximum height occurs. Round each answer to the nearest tenth.

35. A circle is defined by the equation $x^2 + y^2 = 24$, and a line is defined by the equation $y = x + 4$.

(a) Algebraically determine all points of intersection between the circle and the line. Express coordinates in exact form (radicals permitted).

(b) Verify your answer by substituting one of the intersection points back into both original equations. Show the substitution clearly.

PART IV — Long Constructed Response (1 question × 6 credits = 6 credits)

Show all work. This problem requires multiple steps and integrates concepts from several chapters.

36. A mass on a spring oscillates vertically. Its height above a reference point ranges from a maximum of 25 cm to a minimum of 5 cm, completing one full oscillation every 4 seconds. At time t = 0, the mass is at its maximum height.

(a) Write a function h(t) that models the height of the mass in centimeters above the reference point as a function of time t in seconds.

(b) State the amplitude, period, and midline of the function h(t), and explain what each represents in the context of the oscillating mass.

(c) Algebraically determine all times during the first 8 seconds at which the mass is at a height of 20 cm. Round each answer to the nearest hundredth of a second. Show all algebraic work.

ANSWER KEY WITH EXPLANATIONS — PRACTICE EXAM 28

1. C — Powers of i cycle every four: divide 47 by 4 to get remainder 3, so $i^{47} = i^3 = -i$. The remainder determines the equivalent power. The value is −i.

2. B — Apply the fourth root, then cube: 16^(3/4) = (16^(1/4))³ = 2³ = 8. The denominator 4 takes the fourth root and the numerator 3 cubes it. The value is 8.

3. D — Isolate the square: $2x^2 = -8$, so $x^2 = -4$ and $x = \pm 2i$. The negative value under the root introduces the imaginary unit. Both roots are pure imaginary.

4. A — This is a sum of cubes, $(3x)^3 + 1^3 = (3x + 1)(9x^2 - 3x + 1)$. The middle term of the quadratic factor is $-ab = -3x$, making it negative. The sign pattern distinguishes it from a difference of cubes.

5. C — Rewrite the argument as a power of 2: $1/8 = 2^{-3}$, so $\log_2(1/8) = -3$. The negative exponent reflects a value below 1. The result is −3.

6. D — The nth term is $a_n = a_1 \cdot r^{(n-1)}$, so $a_6 = 3(2^5) = 3(32) = 96$. The ratio applied five times scales the term. The sixth term is 96.

7. A — By the Remainder Theorem, the remainder is $p(-1) = 1 - 3 + 2 = 0$. A remainder of zero means (x + 1) divides the polynomial evenly. Substituting −1 gives 0.

8. B — Evaluate the inner function first: $f(2) = 2(2) + 1 = 5$, then $g(5) = 5^2 = 25$. Composition works from the inside out. Squaring 5 gives 25.

9. D — Squaring gives $x + 6 = x^2$, so $x^2 - x - 6 = 0$ and $(x - 3)(x + 2) = 0$, yielding $x = 3$ or $x = -2$. Checking, $x = -2$ fails because $\sqrt{4} = 2 \neq -2$, leaving $x = 3$ as the only valid solution. Squaring can introduce extraneous roots that must be tested.

10. C — Average rate of change is $[f(2) - f(0)]/(2 - 0) = (9 - 1)/2 = 4$. This is the slope of the secant line over the interval. The exponential values 9 and 1 drive the result.

11. A — A half-life uses base 1/2, and a half-life of 8 hours puts $t/8$ in the exponent, with 500 as the initial amount: $f(t) = 500(1/2)^{(t/8)}$. The exponent $t/8$ counts the number of 8-hour halving periods. This models the decay correctly.

12. B — Combine the logs: $\log(4x) = 2$, so $4x = 10^2 = 100$ and $x = 25$. The product rule for logarithms merges the terms. Solving the exponential form gives 25.

13. C — The discriminant is $b^2 - 4ac = 9 - 40 = -31$, which is negative. A negative discriminant produces two complex conjugate solutions. No real solutions exist.

14. D — The tangent of $\pi/4$ (45°) is a standard unit-circle value equal to 1. In a 45-45-90 triangle the legs are equal, so the ratio is 1. The value is 1.

15. B — In $y = a \sin(x) + d$, the midline is $y = d$, which is $y = 8$. The vertical shift sets the midline. The amplitude of 5 does not affect it.

16. A — Using $S = n/2(2a_1 + (n - 1)d) = 20/2(2 + 19 \cdot 3) = 10(2 + 57) = 10(59) = 590$. The formula sums the arithmetic series directly. The sum of the first 20 terms is 590.

17. C — The inverse of an exponential with base 2 is the logarithm with base 2: $f^{-1}(x) = \log_2(x)$. Exponentials and logarithms are inverse operations. The inverse is $\log_2(x)$.

18. D — Two events are independent when $P(A) \cdot P(B) = P(A \cap B)$; here $0.6 \times 0.5 = 0.3$, which equals the given intersection. The equality confirms independence. The occurrence of one event does not change the probability of the other.

19. A — The z-score is $(\text{value} - \text{mean})/\text{SD} = (80 - 70)/5 = 2$. It measures how many standard deviations the value lies from the mean. A value of 80 is 2 standard deviations above the mean.

20. B — In Quadrant II sine is positive while cosine is negative, and the 5-12-13 triangle gives the sine ratio. Therefore $\sin \theta = 5/13$. The quadrant makes the sine positive.

21. C — Factor 2 from the variable terms, then complete the square: $2(x^2 - 4x) + 3 = 2(x^2 - 4x + 4) - 8 + 3 = 2(x - 2)^2 - 5$. The subtracted 8 accounts for the factor of 2 times the added 4. The vertex form reveals the vertex at $(2, -5)$.

22. A — The common ratio is $-1/2$, and an infinite geometric series with $|r| < 1$ sums to $a_1/(1 - r) = 16/(1 - (-1/2)) = 16/(3/2) = 32/3$. The alternating signs come from the negative ratio. The series sums to 32/3.

23. D — Take the log of both sides: $x = \log_6(1000) = \ln(1000)/\ln(6) \approx 3.86$. The change-of-base formula converts to natural logs. The result rounds to 3.86.

24. B — The median depends on the position of the middle value, not the magnitude of an outlier, so changing 12 to 120 leaves it unchanged. The mean, range, and standard deviation all shift substantially with the extreme value. This resistance to outliers makes the median robust.

Part II (Short Constructed Response)

25. x = 3 ± 4i — Quadratic formula: $x = [6 \pm \sqrt{(36 - 100)}]/2 = [6 \pm \sqrt{(-64)}]/2 = [6 \pm 8i]/2 = 3 \pm 4i$. The negative discriminant produces complex conjugates. Dividing each term by 2 gives the simplified a + bi form.

26. x = 5; no extraneous solutions — Cross-multiplying gives $7(x - 2) = 3(x + 2)$, which expands to $7x - 14 = 3x + 6$ and simplifies to $4x = 20$, so $x = 5$. The restrictions are $x \neq -2$ and $x \neq 2$, and $x = 5$ violates neither, so it is valid.

27. Yes, (x + 2) is a factor — By the Factor Theorem, $f(-2) = -8 + 12 + 8 - 12 = 0$. A remainder of zero confirms that $(x + 2)$ divides $f(x)$ evenly. A nonzero value would have meant it is not a factor.

28. $g(x) = -(x + 3)^3$ — Reflecting across the x-axis negates the function to $-x^3$; translating 3 units left replaces x with $(x + 3)$. Combining these produces $-(x + 3)^3$. The horizontal shift moves opposite the sign inside.

29. x = 2 — Write the right side as a power of 4: $64 = 4^3$, so $4^{(2x - 1)} = 4^3$ requires $2x - 1 = 3$. Solving gives $2x = 4$, so $x = 2$. Equal bases allow the exponents to be set equal.

30. $(x + 1) / [(x - 2)(x + 2)]$, $x \neq 2$, $x \neq -2$ — Factor the first denominator as $(x - 2)(x + 2)$, then use it as the common denominator: $3/[(x - 2)(x + 2)] + (x - 2)/[(x - 2)(x + 2)] = (3 + x - 2)/[(x - 2)(x + 2)] = (x + 1)/[(x - 2)(x + 2)]$. The numerator combines to $x + 1$.

31. $a_4 = 25$ — Apply the recursion: $a_2 = 3(-1) + 4 = 1$, $a_3 = 3(1) + 4 = 7$, $a_4 = 3(7) + 4 = 25$. Each term triples the previous term and adds 4. Building up term by term reaches 25.

32. $\sin\theta = -40/41$ — With $\cos\theta = -9/41$, the 9-40-41 Pythagorean triple gives the opposite leg of 40. In Quadrant III sine is negative, so $\sin\theta = -40/41$. The quadrant fixes the negative sign.

Part III (Extended Constructed Response)

33. (a) 150 cells; t/3 counts the number of 3-hour doubling periods; (b) ≈ 8.2 hours — The coefficient 150 is the initial cell count, and dividing t by 3 converts elapsed hours into the number of doublings. For

part (b), 1000 = 150(2)^(t/3) gives 2^(t/3) = 6.667, so t/3 = $\log_2(6.667) \approx 2.737$ and t ≈ 8.2 hours. Logarithms isolate the exponent.

34. (a) h(x) = −4.0x² + 24.0x + 1.0; (b) 37.0 m at x = 3.0 m — Quadratic regression on the data returns a = −4.0, b = 24.0, c = 1.0. The vertex occurs at x = −b/(2a) = −24/(−8) = 3.0 meters, and h(3) = −36 + 72 + 1 = 37.0 meters. The negative leading coefficient confirms a maximum.

35. (a) (−2 + 2√2, 2 + 2√2) and (−2 − 2√2, 2 − 2√2) — Substituting y = x + 4 into the circle gives x² + (x + 4)² = 24, which simplifies to 2x² + 8x − 8 = 0, then x² + 4x − 4 = 0, so x = −2 ± 2√2. The y-values follow from y = x + 4. **(b)** Checking (−2 + 2√2, 2 + 2√2): x² + y² = (12 − 8√2) + (12 + 8√2) = 24, and y = x + 4 = (−2 + 2√2) + 4 = 2 + 2√2, so both equations hold.

Part IV (Long Constructed Response)

36. (a) h(t) = 10 cos(πt/2) + 15 — The amplitude is (25 − 5)/2 = 10 and the midline is (25 + 5)/2 = 15. The period of 4 seconds gives b = 2π/4 = π/2, and starting at the maximum height at t = 0 calls for a positive cosine. This yields h(t) = 10 cos(πt/2) + 15.

(b) Amplitude 10, period 4 s, midline h = 15 — The amplitude of 10 cm is the distance from the midline to the highest or lowest point of the motion. The period of 4 seconds is the time for one complete oscillation. The midline h = 15 cm is the average height about which the mass oscillates.

(c) t ≈ 0.67, 3.33, 4.67, and 7.33 seconds — Setting 20 = 10 cos(πt/2) + 15 gives cos(πt/2) = 0.5. Over the first 8 seconds, πt/2 = π/3, 5π/3, 7π/3, and 11π/3, so t = 2/3, 10/3, 14/3, and 22/3. These round to 0.67, 3.33, 4.67, and 7.33 seconds.