PRACTICE EXAM 27: ALGEBRA II REGENTS SIMULATION

Multiple Choice (24 questions × 2 credits = 48 credits)

1. Express the sum $(-2 + 6i) + (7 - 6i)$ in $a + bi$ form.

A. $5 + 12i$

B. 5

C. $-9 + 12i$

D. 9

2. What is the value of 125^(2/3)?

A. 50

B. 250/3

C. 25

D. 10

3. What are the solutions to the equation $x^2 - 4x + 20 = 0$?

A. $x = 2 \pm 4i$

B. $x = -2 \pm 4i$

C. $x = 2 \pm 8i$

D. $x = 4 \pm 4i$

4. Factored completely, the expression $2x^3 - 18x$ is equivalent to

A. $2x(x^2 - 9)$

B. $2(x - 3)(x + 3)$

C. $x(2x - 6)(x + 3)$

D. $2x(x - 3)(x + 3)$

5. What is the value of $\log_3(81)$?

A. 27

B. 4

C. 3

D. 9

6. What is the sum of the finite geometric series $4 + 12 + 36 + 108$?

A. 160

B. 324

C. 108

D. 120

7. When $p(x) = x^3 - 2x^2 + x + 1$ is divided by $(x - 2)$, the remainder is

A. 1

B. −1

C. 5

D. 3

8. If $f(x) = 5x$ and $g(x) = x - 2$, what is the value of $f(g(4))$?

A. 18

B. 20

C. 10

D. 8

9. What is the solution to the equation $\sqrt{(2x + 1)} = 7$?

A. $x = 24$

B. $x = 25$

C. $x = 48$

D. $x = 7$

10. What is the average rate of change of $f(x) = x^2 + 3x$ over the interval [0, 2]?

A. 10

B. 5

C. 3

D. 7

11. A quantity is modeled by $f(t) = 200(1.035)^t$, where t is the time in years. What is the annual percent rate of growth?

A. 35%

B. 1.035%

C. 103.5%

D. 3.5%

12. What is the solution to the equation $\log_5(x - 2) = 2$?

A. x = 12

B. x = 23

C. x = 27

D. x = 10

13. Which statement best describes the nature of the solutions of $x^2 + 4x + 4 = 0$?

A. One repeated rational solution

B. Two distinct rational solutions

C. Two distinct irrational solutions

D. Two complex (non-real) solutions

14. What is the exact value of $\cos(\pi/3)$?

A. $\sqrt{3}/2$

B. 1/2

C. $\sqrt{2}/2$

D. −1/2

15. For the function $y = 2 \sin(4\pi x)$, what is the period?

A. 4π

B. 2

C. 1/2

D. $\pi/2$

16. What is the 10th term of an arithmetic sequence whose first term is 20 and whose common difference is −4?

A. −20

B. −12

C. 56

D. −16

17. If $f(x) = x^2 - 3$ for $x \geq 0$, what is $f^{-1}(x)$?

A. $\sqrt{x} - 3$

B. $\sqrt{(x + 3)}$

C. $(x + 3)^2$

D. $x^2 + 3$

18. In how many ways can 2 students be chosen from a group of 8 students?

A. 28

B. 56

C. 16

D. 64

19. A data set is normally distributed with a mean of 250 and a standard deviation of 30. Approximately what percent of the data falls between 190 and 310?

A. 68%

B. 99.7%

C. 95%

D. 50%

20. Given that $\cos \theta = 8/17$ and θ is in Quadrant I, what is the value of $\sin \theta$?

A. 17/15

B. 8/15

C. −15/17

D. 15/17

21. Written in vertex form, the expression $x^2 - 2x - 6$ is equivalent to

A. $(x - 1)^2 - 7$

B. $(x - 1)^2 - 6$

C. $(x + 1)^2 - 7$

D. $(x - 2)^2 - 10$

22. What is the sum of the infinite geometric series 27 + 9 + 3 + …?

A. 36

B. 45

C. 40.5

D. 54

23. What is the solution to the equation $9^x = 200$, rounded to the nearest hundredth?

A. $x = 22.22$

B. $x = 3.00$

C. $x = 2.20$

D. $x = 2.41$

24. A study tracks people who choose to exercise regularly and finds they have lower blood pressure. Why can we not conclude that exercise causes the lower blood pressure?

A. The sample size is always too small in observational studies

B. It is observational, so confounding variables may explain the difference

C. Blood pressure cannot be measured accurately

D. Correlation and causation are always identical

PART II — Short Constructed Response (8 questions × 2 credits = 16 credits)

Show all work. A correct answer with no supporting work will receive only 1 credit.

25. Solve the equation $x^2 - 10x + 29 = 0$ algebraically. Express your solutions in $a + bi$ form.

26. Solve algebraically for x: $3 / (x + 4) = 5 / (x - 2)$. State any restrictions on the variable and identify any extraneous solutions that must be rejected.

27. Given $f(x) = x^3 - 4x^2 + x + 6$, determine whether $(x - 3)$ is a factor of $f(x)$. Justify your answer using the Remainder Theorem or the Factor Theorem.

28. Write an equation, in the form $g(x) = a \cdot f(x - h) + k$, for the function obtained by translating $f(x) = |x|$ to the right 5 units and up 2 units.

29. Solve algebraically for x: $3^{(x + 1)} = 81$.

30. Express $4 / (x + 1) - 2 / (x - 1)$ as a single rational expression in simplest form. State any restrictions on the variable.

31. A sequence is defined recursively by $a_1 = 2$ and $a_n = (a_{n-1})^2 - 3$ for $n \geq 2$. Find the value of a_4.

32. Given that $\sin \theta = 20/29$ and θ terminates in Quadrant II, find the exact value of $\cos \theta$. Show the algebraic work that justifies your answer.

PART III — Extended Constructed Response (3 questions × 4 credits = 12 credits)

Show all work. Partial credit is awarded according to the scoring rubric.

33. The population of a town is modeled by the function $P(t) = 8000(1.015)^t$, where $P(t)$ is the population and t is the time in years.

(a) State the initial population of the town and the annual percent rate of growth.

(b) Algebraically determine the number of years it will take for the population to grow to 10000. Round your answer to the nearest tenth of a year.

34. The table below shows the height $h(t)$, in meters, of a launched object at time t seconds after release.

t (seconds)	0	1	2	3	4
h(t) (meters)	2	17	26	29	26

(a) Using regression, write a quadratic function $h(t) = at^2 + bt + c$ that best models the data. Round each coefficient to the nearest tenth.

(b) Use the regression model from part (a) to determine the maximum height reached by the object and the time at which the maximum height occurs. Round each answer to the nearest tenth.

35. A circle is defined by the equation $x^2 + y^2 = 38$, and a line is defined by the equation $y = x + 4$.

(a) Algebraically determine all points of intersection between the circle and the line. Express coordinates in exact form (radicals permitted).

(b) Verify your answer by substituting one of the intersection points back into both original equations. Show the substitution clearly.

PART IV — Long Constructed Response (1 question × 6 credits = 6 credits)

Show all work. This problem requires multiple steps and integrates concepts from several chapters.

36. The depth of the water in a shipping channel varies with the tides. The maximum depth is 14 meters and the minimum depth is 6 meters. One complete tidal cycle takes 12 hours. At time $t = 0$ (midnight), the water is at its minimum depth.

(a) Write a function d(t) that models the depth of the water in meters as a function of time t in hours.

(b) State the amplitude, period, and midline of the function d(t), and explain what each represents in the context of the tides.

(c) Algebraically determine all times during the first 12 hours at which the water depth is 13 meters. Round each answer to the nearest hundredth of an hour. Show all algebraic work.

ANSWER KEY WITH EXPLANATIONS — PRACTICE EXAM 27

1. B — Combine real and imaginary parts: $(-2 + 7) + (6 - 6)i = 5 + 0i = 5$. The imaginary parts cancel exactly. The result is the real number 5.

2. C — Apply the cube root, then square: $125^{(2/3)} = (125^{(1/3)})^2 = 5^2 = 25$. The denominator 3 takes the cube root and the numerator 2 squares it. The value is 25.

3. A — Quadratic formula: $x = [4 \pm \sqrt{(16 - 80)}]/2 = [4 \pm \sqrt{(-64)}]/2 = [4 \pm 8i]/2 = 2 \pm 4i$. The negative discriminant produces the imaginary part. Dividing both terms by 2 gives the simplified form.

4. D — Factor out the GCF 2x, then the difference of squares: $2x(x^2 - 9) = 2x(x - 3)(x + 3)$. Complete factoring requires breaking down $x^2 - 9$. The fully factored form has three factors.

5. B — A logarithm asks for the exponent on the base: $3^4 = 81$, so $\log_3(81) = 4$. Rewriting in exponential form makes the value clear. The base 3 raised to 4 equals 81.

6. A — Using $S = a_1(r^n - 1)/(r - 1) = 4(3^4 - 1)/(3 - 1) = 4(80)/2 = 160$. The common ratio is 3 and there are four terms. Direct addition of the terms also gives 160.

7. D — By the Remainder Theorem, the remainder is $p(2) = 8 - 8 + 2 + 1 = 3$. Substituting the zero of the divisor avoids long division. The remainder is 3.

8. C — Evaluate the inner function first: $g(4) = 4 - 2 = 2$, then $f(2) = 5(2) = 10$. Composition works from the inside out. Multiplying 2 by 5 gives 10.

9. A — Square both sides: $2x + 1 = 49$, so $2x = 48$ and $x = 24$. Squaring removes the radical. Checking, $\sqrt{(2 \cdot 24 + 1)} = \sqrt{49} = 7$ confirms the solution.

10. B — Average rate of change is $[f(2) - f(0)]/(2 - 0) = (10 - 0)/2 = 5$. This is the slope of the secant line over the interval. With $f(2) = 10$ and $f(0) = 0$, the result is 5.

11. D — In $a(1 + r)^t$, the base 1.035 equals 1 + 0.035, so the growth rate is 3.5%. A base above 1 signals growth, and the base minus 1 gives the rate. The decimal 0.035 converts to 3.5%.

12. C — Rewrite in exponential form: $x - 2 = 5^2 = 25$, so $x = 27$. Converting the logarithmic equation isolates x. The base 5 raised to 2 equals 25.

13. A — The discriminant is $b^2 - 4ac = 16 - 16 = 0$, indicating one repeated real solution. Since $x^2 + 4x + 4 = (x + 2)^2$, the repeated root −2 is rational. A zero discriminant always yields a single repeated solution.

14. B — The cosine of $\pi/3$ (60°) is a standard unit-circle value equal to 1/2. The 30-60-90 reference triangle gives this ratio. The value is 1/2.

15. C — The period of $\sin(bx)$ is $2\pi/b$; with $b = 4\pi$, the period is $2\pi/(4\pi) = 1/2$. The coefficient of x sets the period. The period is 1/2.

16. D — The nth term is $a_n = a_1 + (n - 1)d$, so $a_{10} = 20 + 9(-4) = 20 - 36 = -16$. The common difference applied nine times lowers the term. The tenth term is −16.

17. B — To invert, solve $y = x^2 - 3$ for x: $x^2 = y + 3$, so $x = \sqrt{(y + 3)}$ and $f^{-1}(x) = \sqrt{(x + 3)}$. The domain restriction $x \geq 0$ makes the positive root the inverse. The result is $\sqrt{(x + 3)}$.

18. A — Order does not matter, so use combinations: $C(8, 2) = (8 \cdot 7)/(2 \cdot 1) = 28$. The combination formula counts unordered selections. There are 28 ways.

19. C — The values 190 and 310 are two standard deviations below and above the mean of 250. By the empirical rule, about 95% of data lies within ±2 SD. The interval captures 95%.

20. D — In Quadrant I all ratios are positive, and the 8-15-17 triangle gives the sine ratio. Therefore $\sin \theta = 15/17$. The Pythagorean triple supplies the missing leg of 15.

21. A — Complete the square: $x^2 - 2x - 6 = (x^2 - 2x + 1) - 1 - 6 = (x - 1)^2 - 7$. Half of −2 squared is 1, added and subtracted to preserve value. The vertex form reveals the vertex at (1, −7).

22. C — The common ratio is 1/3, and an infinite geometric series with $|r| < 1$ sums to $a_1/(1 - r) = 27/(1 - 1/3) = 27/(2/3) = 40.5$. The ratio ensures convergence. The series sums to 40.5.

23. D — Take the log of both sides: $x = \log_9(200) = \ln(200)/\ln(9) \approx 2.41$. The change-of-base formula converts to natural logs. The result rounds to 2.41.

24. B — Because participants chose whether to exercise, the study is observational, and confounding variables such as diet or age may account for the lower blood pressure. Without random assignment, causation cannot be established. Only a randomized experiment can isolate the effect of exercise.

Part II (Short Constructed Response)

25. $x = 5 \pm 2i$ — Quadratic formula: $x = [10 \pm \sqrt{(100 - 116)}]/2 = [10 \pm \sqrt{(-16)}]/2 = [10 \pm 4i]/2 = 5 \pm 2i$. The negative discriminant produces complex conjugates. Dividing each term by 2 gives the simplified a + bi form.

26. $x = -13$; no extraneous solutions — Cross-multiplying gives $3(x - 2) = 5(x + 4)$, which expands to $3x - 6 = 5x + 20$ and simplifies to $-2x = 26$, so $x = -13$. The restrictions are $x \neq -4$ and $x \neq 2$, and $x = -13$ violates neither, so it is valid.

27. Yes, $(x - 3)$ is a factor — By the Factor Theorem, $f(3) = 27 - 36 + 3 + 6 = 0$. A remainder of zero confirms that $(x - 3)$ divides $f(x)$ evenly. A nonzero value would have meant it is not a factor.

28. $g(x) = |x - 5| + 2$ — Translating right 5 units replaces x with $(x - 5)$; translating up 2 units adds 2 outside the absolute value. Combining these gives $|x - 5| + 2$. The horizontal shift moves opposite the sign inside.

29. $x = 3$ — Write the right side as a power of 3: $81 = 3^4$, so $3^{(x + 1)} = 3^4$ requires $x + 1 = 4$. Solving gives $x = 3$. Equal bases allow the exponents to be set equal.

30. $(2x - 6) / [(x + 1)(x - 1)]$, $x \neq -1$, $x \neq 1$ — Over the common denominator $(x + 1)(x - 1)$: $4(x - 1) - 2(x + 1) = 4x - 4 - 2x - 2 = 2x - 6$. The numerator combines to $2x - 6$, which factors as $2(x - 3)$. Restrictions come from the original denominators.

31. $a_4 = 1$ — Apply the recursion: $a_2 = (2)^2 - 3 = 1$, $a_3 = (1)^2 - 3 = -2$, $a_4 = (-2)^2 - 3 = 1$. Each term squares the previous term and subtracts 3. Building up term by term reaches 1.

32. $\cos \theta = -21/29$ — With $\sin \theta = 20/29$, the 20-21-29 Pythagorean triple gives the adjacent leg of 21. In Quadrant II cosine is negative, so $\cos \theta = -21/29$. The quadrant fixes the negative sign.

Part III (Extended Constructed Response)

33. (a) 8,000 people, 1.5% growth; (b) ≈ 15.0 years — The coefficient 8000 is the initial population, and the base $1.015 = 1 + 0.015$ indicates 1.5% annual growth. For part (b), $10000 = 8000(1.015)^t$ gives $1.015^t = 1.25$, so $t = \ln(1.25)/\ln(1.015) \approx 15.0$ years. Logarithms isolate the exponent.

34. (a) $h(t) = -3.0t^2 + 18.0t + 2.0$; (b) 29.0 m at $t = 3.0$ s — Quadratic regression on the data returns $a = -3.0$, $b = 18.0$, $c = 2.0$. The vertex occurs at $t = -b/(2a) = -18/(-6) = 3.0$ seconds, and $h(3) = -27 + 54 + 2 = 29.0$ meters. The negative leading coefficient confirms a maximum.

35. (a) $(-2 + \sqrt{15}, 2 + \sqrt{15})$ and $(-2 - \sqrt{15}, 2 - \sqrt{15})$ — Substituting $y = x + 4$ into the circle gives $x^2 + (x + 4)^2 = 38$, which simplifies to $2x^2 + 8x - 22 = 0$, then $x^2 + 4x - 11 = 0$, so $x = -2 \pm \sqrt{15}$. The y-values follow from $y = x + 4$. **(b)** Checking $(-2 + \sqrt{15}, 2 + \sqrt{15})$: $x^2 + y^2 = (19 - 4\sqrt{15}) + (19 + 4\sqrt{15}) = 38$, and $y = x + 4 = (-2 + \sqrt{15}) + 4 = 2 + \sqrt{15}$, so both equations hold.

Part IV (Long Constructed Response)

36. (a) $d(t) = -4\cos(\pi t/6) + 10$ — The amplitude is $(14 - 6)/2 = 4$ and the midline is $(14 + 6)/2 = 10$. The period of 12 hours gives $b = 2\pi/12 = \pi/6$, and starting at the minimum depth at $t = 0$ calls for a negative cosine. This yields $d(t) = -4\cos(\pi t/6) + 10$.

(b) Amplitude 4, period 12 h, midline d = 10 — The amplitude of 4 meters is the distance from the midline to the maximum or minimum depth. The period of 12 hours is the time for one complete tidal cycle. The midline $d = 10$ meters is the average depth about which the water level oscillates.

(c) $t \approx 4.62$ and 7.38 hours — Setting $13 = -4\cos(\pi t/6) + 10$ gives $\cos(\pi t/6) = -0.75$. Over the first 12 hours, $\pi t/6 = 2.4189$ and 3.8643 radians, so $t = 4.62$ and 7.38 hours. These are the two times the rising and falling tide passes a depth of 13 meters.