PRACTICE EXAM 26: ALGEBRA II REGENTS SIMULATION

PART I — Multiple Choice (24 questions × 2 credits = 48 credits)

1. Express the difference $(6 - i) - (2 + 4i)$ in $a + bi$ form.

A. $4 + 5i$

B. $8 + 3i$

C. $4 + 3i$

D. $4 - 5i$

2. What is the value of 64^(1/3)?

A. 8

B. 4

C. 21.3

D. 16

3. What are the solutions to the equation $x^2 - 6x + 13 = 0$?

A. $x = 3 \pm 2i$

B. $x = -3 \pm 2i$

C. $x = 3 \pm 4i$

D. $x = 6 \pm 2i$

4. Factored completely, the expression $x^3 - 2x^2 + 5x - 10$ is equivalent to

A. $(x - 2)(x^2 - 5)$

B. $(x + 2)(x^2 + 5)$

C. $(x - 2)(x^2 + 5)$

D. $(x - 2)(x + 5)$

5. What is the value of $\log_7(49)$?

A. 7

B. 14

C. 1/2

D. 2

6. What is the 4th term of a geometric sequence whose first term is 2 and whose common ratio is 5?

A. 250

B. 1250

C. 50

D. 500

7. When $p(x) = 2x^3 + x^2 - 3$ is divided by $(x - 1)$, the remainder is

A. 4

B. −3

C. 0

D. 2

8. If $f(x) = x^2 - 2$ and $g(x) = x + 1$, what is the value of $f(g(3))$?

A. 8

B. 14

C. 10

D. 11

9. What is the solution to the equation $3\sqrt{(x + 1)} = 12$?

A. $x = 15$

B. $x = 16$

C. $x = 3$

D. $x = 4$

10. What is the average rate of change of $f(x) = x^3$ over the interval [1, 3]?

A. 26

B. 9

C. 27

D. 13

11. Which function models a quantity that begins at 1000 and decreases by half every 5 years?

A. $f(t) = 1000(2)^{(t/5)}$

B. $f(t) = 1000(1/2)^{(5t)}$

C. $f(t) = 1000(1/2)^{(t/5)}$

D. $f(t) = 1000 - (1/2)t$

12. What is the solution to the equation $\log_2(3x + 1) = 4$?

A. $x = 4$

B. $x = 5$

C. $x = 7$

D. $x = 6$

13. Which statement best describes the nature of the solutions of $9x^2 + 6x + 1 = 0$?

A. Two distinct rational solutions

B. Two distinct irrational solutions

C. Two complex (non-real) solutions

D. One repeated rational solution

14. What is the exact value of $\sin(\pi/6)$?

A. 1/2

B. $\sqrt{3}/2$

C. $\sqrt{2}/2$

D. −1/2

15. For the function $y = 7\cos(x) - 3$, what is the amplitude?

A. −3

B. 3

C. 7

D. 1

16. In an arithmetic sequence, the second term is 9 and the sixth term is 25. What is the first term?

A. 7

B. 5

C. 4

D. 6

17. If $f(x) = 3x + 6$, what is $f^{-1}(x)$?

A. $(x - 6) / 3$

B. $3x - 6$

C. $(x + 6) / 3$

D. $1 / (3x + 6)$

18. Two cards are drawn at random from a standard 52-card deck, with the first card replaced before the second is drawn. What is the probability that both cards are aces?

A. 1/13

B. 1/16

C. 2/52

D. 1/169

19. A data set is normally distributed with a mean of 100 and a standard deviation of 10. Approximately what percent of the data is greater than 110?

A. 34%

B. 16%

C. 84%

D. 68%

20. Given that $\sin \theta = -3/5$ and θ is in Quadrant IV, what is the value of $\tan \theta$?

A. 3/4

B. 4/3

C. −3/4

D. −4/3

21. Written in vertex form, the expression $x^2 + 12x + 40$ is equivalent to

A. $(x + 6)^2 + 4$

B. $(x + 6)^2 - 4$

C. $(x - 6)^2 + 4$

D. $(x + 12)^2 - 104$

22. What is the sum of the infinite geometric series 20 + 5 + 1.25 + …?

A. 25

B. 30

C. 24

D. 26.67

23. What is the solution to the equation $8^x = 500$, rounded to the nearest hundredth?

A. x = 62.50

B. x = 3.50

C. x = 2.99

D. x = 2.08

24. A newspaper conducts a poll in which readers may call a phone number to vote on an issue. Why might the results be biased?

A. The sample is too large to be accurate

B. The sample is self-selected and not representative of all readers

C. Phone polls are always more accurate than online polls

D. The margin of error is automatically zero for call-in polls

PART II — Short Constructed Response (8 questions × 2 credits = 16 credits)

Show all work. A correct answer with no supporting work will receive only 1 credit.

25. Solve the equation $x^2 + 8x + 41 = 0$ algebraically. Express your solutions in a + bi form.

26. Solve algebraically for x: $6 / (x - 1) = 2 / (x + 3)$. State any restrictions on the variable and identify any extraneous solutions that must be rejected.

27. Given $f(x) = x^3 - 2x^2 - 5x + 6$, determine whether $(x + 1)$ is a factor of $f(x)$. Justify your answer using the Remainder Theorem or the Factor Theorem.

28. Write an equation, in the form $g(x) = a \cdot f(x) + k$, for the function obtained by vertically stretching $f(x) = x^2$ by a factor of 4 and then translating the result 7 units down.

29. Solve algebraically for x: 5^(2x) = 125.

30. Express 1 / (x + 2) + 3 / (x − 4) as a single rational expression in simplest form. State any restrictions on the variable.

31. A sequence is defined recursively by $a_1 = 1$ and $a_n = 5a_{n-1} - 3$ for $n \geq 2$. Find the value of a_4.

32. Given that tan θ = −8/15 and θ terminates in Quadrant II, find the exact value of sin θ. Show the algebraic work that justifies your answer.

PART III — Extended Constructed Response (3 questions × 4 credits = 12 credits)

Show all work. Partial credit is awarded according to the scoring rubric.

33. The value of a car is modeled by the function $V(t) = 32000(0.85)^t$, where V(t) is the value in dollars and t is the time in years.

(a) State the initial value of the car and the annual percent rate of depreciation.

(b) Algebraically determine the number of years it will take for the car's value to fall to \$16000. Round your answer to the nearest tenth of a year.

34. The table below shows the height $h(x)$, in meters, of a water stream from a fountain at horizontal distance x meters from the nozzle.

x (meters)	0	1	2	3	4
h(x) (meters)	5	19	29	35	37

(a) Using regression, write a quadratic function $h(x) = ax^2 + bx + c$ that best models the data. Round each coefficient to the nearest tenth.

(b) Use the regression model from part (a) to determine the maximum height of the water stream and the horizontal distance at which the maximum height occurs. Round each answer to the nearest tenth.

35. A circle is defined by the equation $x^2 + y^2 = 44$, and a line is defined by the equation $y = x - 4$.

(a) Algebraically determine all points of intersection between the circle and the line. Express coordinates in exact form (radicals permitted).

(b) Verify your answer by substituting one of the intersection points back into both original equations. Show the substitution clearly.

PART IV — Long Constructed Response (1 question × 6 credits = 6 credits)

Show all work. This problem requires multiple steps and integrates concepts from several chapters.

36. A yo-yo moves up and down at the end of its string. Its height above the floor oscillates between a maximum of 90 cm and a minimum of 10 cm, completing one full up-and-down cycle every 2 seconds. At time $t = 0$, the yo-yo is at its maximum height.

(a) Write a function $h(t)$ that models the height of the yo-yo in centimeters above the floor as a function of time t in seconds.

(b) State the amplitude, period, and midline of the function $h(t)$, and explain what each represents in the context of the moving yo-yo.

(c) Algebraically determine all times during the first 4 seconds at which the yo-yo is at a height of 70 cm above the floor. Round each answer to the nearest hundredth of a second. Show all algebraic work.

ANSWER KEY WITH EXPLANATIONS – PRACTICE EXAM 26

1. D — Combine real and imaginary parts, distributing the subtraction: $(6 - 2) + (-1 - 4)i = 4 - 5i$. Subtracting $4i$ lowers the imaginary part to $-5i$. The result is $4 - 5i$.

2. B — A cube root undoes a cube: $64^{(1/3)} = 4$, since $4^3 = 64$. The exponent 1/3 takes the cube root. The value is 4.

3. A — Quadratic formula: $x = [6 \pm \sqrt{(36 - 52)}]/2 = [6 \pm \sqrt{(-16)}]/2 = [6 \pm 4i]/2 = 3 \pm 2i$. The negative discriminant produces the imaginary part. Dividing both terms by 2 gives the simplified form.

4. C — Group and factor: $x^2(x - 2) + 5(x - 2) = (x - 2)(x^2 + 5)$. The common binomial $(x - 2)$ factors out. The quadratic factor $x^2 + 5$ does not factor over the reals.

5. D — A logarithm asks for the exponent on the base: $7^2 = 49$, so $\log_7(49) = 2$. Rewriting in exponential form makes the value clear. The base 7 raised to 2 equals 49.

6. A — The nth term is $a_n = a_1 \cdot r^{(n-1)}$, so $a_4 = 2(5^3) = 2(125) = 250$. The ratio applied three times scales the term. The fourth term is 250.

7. C — By the Remainder Theorem, the remainder is $p(1) = 2 + 1 - 3 = 0$. A remainder of zero means $(x - 1)$ divides the polynomial evenly. Substituting the zero of the divisor gives 0.

8. B — Evaluate the inner function first: $g(3) = 3 + 1 = 4$, then $f(4) = 4^2 - 2 = 14$. Composition works from the inside out. Squaring 4 and subtracting 2 gives 14.

9. A — Divide by 3, then square: $\sqrt{x + 1} = 4$, so $x + 1 = 16$ and $x = 15$. Isolating the radical before squaring is essential. Checking, $3\sqrt{16} = 3(4) = 12$ confirms the solution.

10. D — Average rate of change is $[f(3) - f(1)]/(3 - 1) = (27 - 1)/2 = 26/2 = 13$. This is the slope of the secant line over the interval. The cubes 27 and 1 drive the result.

11. C — A quantity that halves uses base 1/2, and halving every 5 years puts t/5 in the exponent, with 1000 as the initial value: $f(t) = 1000(1/2)^{(t/5)}$. The exponent t/5 counts the number of 5-year halving periods. This models the decay correctly.

12. B — Rewrite in exponential form: $3x + 1 = 2^4 = 16$, so $3x = 15$ and $x = 5$. Converting the logarithmic equation isolates x. Checking shows the argument stays positive.

13. D — The discriminant is $b^2 - 4ac = 36 - 36 = 0$, indicating one repeated real solution. Since $9x^2 + 6x + 1 = (3x + 1)^2$, the repeated root $-1/3$ is rational. A zero discriminant always yields a single repeated solution.

14. A — The sine of $\pi/6$ (30°) is a standard unit-circle value equal to 1/2. The 30-60-90 reference triangle gives this ratio. The value is 1/2.

15. C — In $y = a \cos(x) + d$, the amplitude is $|a|$, which is 7. The amplitude is the coefficient of the cosine term. The vertical shift of -3 does not affect it.

16. B — The common difference is $d = (25 - 9)/(6 - 2) = 16/4 = 4$. Working back, $a_1 = a_2 - d = 9 - 4 = 5$. Subtracting one common difference recovers the first term.

17. A — To invert, solve $y = 3x + 6$ for x: $x = (y - 6)/3$, so $f^{-1}(x) = (x - 6)/3$. The inverse undoes multiplication by 3 and addition of 6 in reverse order. The result is $(x - 6)/3$.

18. D — With replacement, the draws are independent: $(4/52)(4/52) = (1/13)^2 = 1/169$. Replacing the first card keeps the deck at 52 with 4 aces. The probability both are aces is 1/169.

19. B — A value of 110 is one standard deviation above the mean of 100. Since 50% lies above the mean and 34% lies between the mean and +1 SD, about 16% lies above +1 SD. The empirical rule gives the tail percentage.

20. C — In Quadrant IV cosine is positive while sine is negative, and the 3-4-5 triangle gives the ratios. Then $\tan \theta = \sin/\cos = (-3/5)/(4/5) = -3/4$. The quadrant makes the tangent negative.

21. A — Complete the square: $x^2 + 12x + 40 = (x^2 + 12x + 36) - 36 + 40 = (x + 6)^2 + 4$. Half of 12 squared is 36, added and subtracted to preserve value. The vertex form reveals the vertex at (−6, 4).

22. D — The common ratio is 1/4, and an infinite geometric series with $|r| < 1$ sums to $a_1/(1 - r) = 20/(1 - 1/4) = 20/(3/4) \approx 26.67$. The ratio ensures convergence. The series sums to about 26.67.

23. C — Take the log of both sides: $x = \log_8(500) = \ln(500)/\ln(8) \approx 2.99$. The change-of-base formula converts to natural logs. The result rounds to 2.99.

24. B — A call-in poll captures only readers motivated enough to phone in, producing a self-selected, non-representative sample. Those with strong opinions are overrepresented. This voluntary-response bias distorts the results.

Part II (Short Constructed Response)

25. $x = -4 \pm 5i$ — Quadratic formula: $x = [-8 \pm \sqrt{(64 - 164)}]/2 = [-8 \pm \sqrt{(-100)}]/2 = [-8 \pm 10i]/2 = -4 \pm 5i$. The negative discriminant produces complex conjugates. Dividing each term by 2 gives the simplified a + bi form.

26. $x = -5$; no extraneous solutions — Cross-multiplying gives $6(x + 3) = 2(x - 1)$, which expands to $6x + 18 = 2x - 2$ and simplifies to $4x = -20$, so $x = -5$. The restrictions are $x \neq 1$ and $x \neq -3$, and $x = -5$ violates neither, so it is valid.

27. No, (x + 1) is not a factor — By the Remainder Theorem, $f(-1) = -1 - 2 + 5 + 6 = 8$. A nonzero remainder means (x + 1) does not divide f(x) evenly. Only a remainder of zero would confirm a factor.

28. $g(x) = 4x^2 - 7$ — A vertical stretch by 4 multiplies the function by 4, giving $4x^2$; translating 7 units down subtracts 7. Combining these produces $4x^2 - 7$. The stretch acts on the leading coefficient.

29. $x = 3/2$ — Write the right side as a power of 5: $125 = 5^3$, so $5^{(2x)} = 5^3$ requires $2x = 3$. Solving gives $x = 3/2$. Equal bases allow the exponents to be set equal.

30. $(4x + 2) / [(x + 2)(x - 4)]$, $x \neq -2$, $x \neq 4$ — Over the common denominator $(x + 2)(x - 4)$: $(x - 4) + 3(x + 2) = x - 4 + 3x + 6 = 4x + 2$. The numerator combines to $4x + 2$, which factors as $2(2x + 1)$. Restrictions come from the original denominators.

31. $a_4 = 32$ — Apply the recursion: $a_2 = 5(1) - 3 = 2$, $a_3 = 5(2) - 3 = 7$, $a_4 = 5(7) - 3 = 32$. Each term multiplies the previous term by 5 and subtracts 3. Building up term by term reaches 32.

32. $\sin \theta = 8/17$ — With $\tan \theta = -8/15$, the reference triangle has legs 8 and 15 with hypotenuse 17. In Quadrant II sine is positive, so $\sin \theta = 8/17$. The quadrant fixes the positive sign.

Part III (Extended Constructed Response)

33. (a) \$32,000, 15% depreciation; (b) ≈ 4.3 years — The coefficient 32000 is the initial value, and the base 0.85 = 1 − 0.15 indicates 15% annual depreciation. For part (b), $16000 = 32000(0.85)^t$ gives $0.85^t = 0.5$, so t = ln(0.5)/ln(0.85) ≈ 4.3 years. Logarithms isolate the exponent.

34. (a) $h(x) = -2.0x^2 + 16.0x + 5.0$; (b) 37.0 m at x = 4.0 m — Quadratic regression on the data returns a = −2.0, b = 16.0, c = 5.0. The vertex occurs at x = −b/(2a) = −16/(−4) = 4.0 meters, and h(4) = −32 + 64 + 5 = 37.0 meters. The negative leading coefficient confirms a maximum.

35. (a) (2 + 3√2, −2 + 3√2) and (2 − 3√2, −2 − 3√2) — Substituting y = x − 4 into the circle gives $x^2 + (x - 4)^2 = 44$, which simplifies to $2x^2 - 8x - 28 = 0$, then $x^2 - 4x - 14 = 0$, so x = 2 ± 3√2. The y-values follow from y = x − 4. **(b)** Checking (2 + 3√2, −2 + 3√2): $x^2 + y^2 = (22 + 12\sqrt{2}) + (22 - 12\sqrt{2}) = 44$, and y = x − 4 = (2 + 3√2) − 4 = −2 + 3√2, so both equations hold.

Part IV (Long Constructed Response)

36. (a) h(t) = 40 cos(πt) + 50 — The amplitude is (90 − 10)/2 = 40 and the midline is (90 + 10)/2 = 50. The period of 2 seconds gives b = 2π/2 = π, and starting at the maximum height at t = 0 calls for a positive cosine. This yields h(t) = 40 cos(πt) + 50.

(b) Amplitude 40, period 2 s, midline h = 50 — The amplitude of 40 cm is the distance from the midline to the highest or lowest point of the motion. The period of 2 seconds is the time for one complete up-and-down cycle. The midline h = 50 cm is the average height about which the yo-yo oscillates.

(c) t ≈ 0.33, 1.67, 2.33, and 3.67 seconds — Setting 70 = 40 cos(πt) + 50 gives cos(πt) = 0.5. Over the first 4 seconds, πt = π/3, 5π/3, 7π/3, and 11π/3, so t = 1/3, 5/3, 7/3, and 11/3. These round to 0.33, 1.67, 2.33, and 3.67 seconds.