PRACTICE EXAM 25: ALGEBRA II REGENTS SIMULATION

PART I — Multiple Choice (24 questions × 2 credits = 48 credits)

1. Express the product $(2 + 3i)(1 - i)$ in $a + bi$ form.

A. $5 + i$

B. $5 - i$

C. $-1 + i$

D. $2 - 3i$

2. Simplify the expression $(x^8)^{(3/4)}$.

A. $x^{(11/4)}$

B. $x^{(32/3)}$

C. x^6

D. x^2

3. What are the solutions to the equation $x^2 + 16 = 0$?

A. $x = \pm 16i$

B. $x = \pm 4i$

C. $x = \pm 4$

D. $x = \pm\sqrt{4}$

4. Factored completely, the expression $8x^3 - 27$ is equivalent to

A. $(2x - 3)(4x^2 - 6x + 9)$

B. $(2x + 3)(4x^2 - 6x + 9)$

C. $(2x - 3)(2x^2 + 9)$

D. $(2x - 3)(4x^2 + 6x + 9)$

5. What is the value of $\log_{10}(0.001)$?

A. −3

B. 3

C. −0.001

D. 1/3

6. What is the sum of the finite geometric series 3 + 9 + 27 + 81 + 243?

A. 243

B. 729

C. 363

D. 486

7. When $p(x) = x^3 - 4x^2 + 2$ is divided by $(x + 1)$, the remainder is

A. 3

B. −1

C. 1

D. −3

8. If $f(x) = 3x + 2$ and $g(x) = x - 4$, what is the value of $g(f(1))$?

A. 5

B. 1

C. 3

D. −2

9. What is the solution to the equation $\sqrt{x} - 2 = 3$?

A. $x = 5$

B. $x = 1$

C. $x = 25$

D. $x = 10$

10. What is the average rate of change of $f(x) = 2^x$ over the interval [1, 4]?

A. 14/3

B. 14

C. 4

D. 7

11. A quantity is modeled by $f(t) = 5000(0.94)^t$, where t is the time in years. What is the annual percent rate of decay?

A. 94%

B. 9.4%

C. 0.6%

D. 6%

12. What is the solution to the equation $\log_4(x) = 3$?

A. x = 12

B. x = 64

C. x = 81

D. x = 16

13. Which statement best describes the nature of the solutions of $x^2 - 2x - 5 = 0$?

A. Two distinct rational solutions

B. One repeated rational solution

C. Two distinct irrational solutions

D. Two complex (non-real) solutions

14. What is the exact value of $\cos(2\pi/3)$?

A. $-1/2$

B. $1/2$

C. $\sqrt{3}/2$

D. $-\sqrt{3}/2$

15. For the function $y = 3 \sin(\pi x/2)$, what is the period?

A. 2

B. 4

C. π

D. $\pi/2$

16. What is the sum of the first 15 terms of an arithmetic sequence whose first term is 2 and whose common difference is 4?

A. 420

B. 480

C. 225

D. 450

17. Which function is the inverse of $f(x) = \log_3(x)$?

A. $f^{-1}(x) = 3^x$

B. $f^{-1}(x) = x^3$

C. $f^{-1}(x) = \log_x(3)$

D. $f^{-1}(x) = (1/3)^x$

18. A bag contains 4 white balls and 6 black balls. If two balls are drawn at random without replacement, what is the probability that both are black?

A. 3/5

B. 9/25

C. 1/3

D. 2/3

19. A data set is normally distributed with a mean of 12 and a standard deviation of 2. Approximately what percent of the data is less than 12?

A. 34%

B. 50%

C. 68%

D. 16%

20. Given that $\tan \theta = 3/4$ and θ is in Quadrant III, what is the value of $\cos \theta$?

A. 4/5

B. 3/5

C. −3/5

D. −4/5

21. Written in vertex form, the expression $x^2 - 6x + 2$ is equivalent to

A. $(x - 3)^2 + 2$

B. $(x + 3)^2 - 7$

C. $(x - 3)^2 - 7$

D. $(x - 6)^2 - 34$

22. What is the sum of the infinite geometric series $12 + 4 + 4/3 + \ldots$?

A. 18

B. 16

C. 24

D. 36

23. What is the solution to the equation $5^x = 90$, rounded to the nearest hundredth?

A. x = 18.00

B. x = 4.50

C. x = 3.00

D. x = 2.80

24. To estimate the proportion of registered voters in a state who support a ballot measure, which sample is most representative?

A. People leaving a single political party's rally

B. A random sample of registered voters from across the state

C. Visitors to one campaign's website

D. Residents of the state capital only

PART II — Short Constructed Response (8 questions × 2 credits = 16 credits)

Show all work. A correct answer with no supporting work will receive only 1 credit.

25. Solve the equation $x^2 + 2x + 26 = 0$ algebraically. Express your solutions in a + bi form.

26. Solve algebraically for x: $5 / (x + 1) = 4 / (x - 1)$. State any restrictions on the variable and identify any extraneous solutions that must be rejected.

27. Given $f(x) = x^3 - 6x^2 + 3x + 10$, determine whether $(x - 5)$ is a factor of $f(x)$. Justify your answer using the Remainder Theorem or the Factor Theorem.

28. Write an equation, in the form $g(x) = a \cdot f(x - h) + k$, for the function obtained by reflecting $f(x) = \sqrt{x}$ across the x-axis and then translating the result 4 units to the right.

29. Solve algebraically for x: 2^(3x) = 64.

30. Express $2 / (x - 3) - 5 / (x^2 - 9)$ as a single rational expression in simplest form. State any restrictions on the variable.

31. A sequence is defined recursively by $a_1 = 3$ and $a_n = 2a_{n-1} + n$ for $n \geq 2$. Find the value of a_4.

32. Given that $\cos \theta = 24/25$ and θ terminates in Quadrant IV, find the exact value of $\tan \theta$. Show the algebraic work that justifies your answer.

PART III — Extended Constructed Response (3 questions × 4 credits = 12 credits)

Show all work. Partial credit is awarded according to the scoring rubric.

33. The value of an investment is modeled by the function $A(t) = 1000(1.08)^t$, where $A(t)$ is the value in dollars and t is the time in years.

(a) State the initial value of the investment and the annual percent rate of growth.

(b) Algebraically determine the number of years it will take for the investment to double in value. Round your answer to the nearest tenth of a year.

34. The table below shows the height $h(t)$, in meters, of a model rocket at time t seconds after launch.

t (seconds)	0	1	2	3	4
h(t) (meters)	2	17	22	17	2

(a) Using regression, write a quadratic function $h(t) = at^2 + bt + c$ that best models the data. Round each coefficient to the nearest tenth.

(b) Use the regression model from part (a) to determine the maximum height reached by the rocket and the time at which the maximum height occurs. Round each answer to the nearest tenth.

35. A circle is defined by the equation $x^2 + y^2 = 26$, and a line is defined by the equation $y = x + 2$.

(a) Algebraically determine all points of intersection between the circle and the line. Express coordinates in exact form (radicals permitted).

(b) Verify your answer by substituting one of the intersection points back into both original equations. Show the substitution clearly.

PART IV — Long Constructed Response (1 question × 6 credits = 6 credits)

Show all work. This problem requires multiple steps and integrates concepts from several chapters.

36. A paddleboat wheel has a radius of 1.2 meters, and its center is 0.5 meters above the surface of the water. The wheel completes one full rotation every 3 seconds. A marked paddle on the rim begins at the highest point of its rotation at time $t = 0$.

(a) Write a function $h(t)$ that models the height of the marked paddle in meters relative to the water surface as a function of time t in seconds.

(b) State the amplitude, period, and midline of the function $h(t)$, and explain what each represents in the context of the paddleboat wheel.

(c) Algebraically determine all times during the first 6 seconds at which the marked paddle is exactly at the water surface (height 0 meters). Round each answer to the nearest hundredth of a second. Show all algebraic work.

ANSWER KEY WITH EXPLANATIONS – PRACTICE EXAM 25

1. A — Distribute and apply $i^2 = -1$: $(2 + 3i)(1 - i) = 2 - 2i + 3i - 3i^2 = 2 + i + 3 = 5 + i$. The $-3i^2$ term becomes +3, raising the real part to 5. The middle terms combine to +i.

2. C — A power raised to a power multiplies exponents: $(x^8)^{(3/4)} = x^{(8\cdot 3/4)} = x^6$. Multiplying 8 by 3/4 gives 6. The result is x^6.

3. B — Isolate the square: $x^2 = -16$, so $x = \pm\sqrt{(-16)} = \pm 4i$. The negative value under the root introduces the imaginary unit. Both roots are pure imaginary.

4. D — This is a difference of cubes, $(2x)^3 - 3^3 = (2x - 3)(4x^2 + 6x + 9)$. The middle term of the quadratic factor is $+ab = +6x$, making it positive. The sign pattern distinguishes it from a sum of cubes.

5. A — Rewrite the argument as a power of 10: $0.001 = 10^{-3}$, so $\log_{10}(0.001) = -3$. The negative exponent reflects a value below 1. The result is −3.

6. C — Using $S = a_1(r^n - 1)/(r - 1) = 3(3^5 - 1)/(3 - 1) = 3(242)/2 = 363$. The common ratio is 3 and there are five terms. Direct addition of the terms also gives 363.

7. D — By the Remainder Theorem, the remainder is $p(-1) = -1 - 4 + 2 = -3$. Substituting the zero of the divisor avoids long division. The remainder is −3.

8. B — Evaluate the inner function first: $f(1) = 3(1) + 2 = 5$, then $g(5) = 5 - 4 = 1$. Composition works from the inside out. The result is 1.

9. C — Add 2, then square: $\sqrt{x} = 5$, so $x = 25$. Isolating the radical before squaring is essential. Checking, $\sqrt{25} - 2 = 5 - 2 = 3$ confirms the solution.

10. A — Average rate of change is $[f(4) - f(1)]/(4 - 1) = (16 - 2)/3 = 14/3$. This is the slope of the secant line over the interval. The exponential values 16 and 2 drive the result.

11. D — In $a(1 - r)^t$, the base 0.94 equals $1 - 0.06$, so the decay rate is 6%. A base below 1 signals decay, and 1 minus the base gives the rate. The decimal 0.06 converts to 6%.

12. B — Rewrite in exponential form: $x = 4^3 = 64$. Converting the logarithmic equation isolates x. The base 4 raised to 3 equals 64.

13. C — The discriminant is $b^2 - 4ac = 4 + 20 = 24$, which is positive but not a perfect square. This produces two distinct irrational real solutions. The nature of the discriminant classifies the roots.

14. A — The angle $2\pi/3$ lies in Quadrant II with reference angle $\pi/3$, where cosine is negative. Since $\cos(\pi/3) = 1/2$, $\cos(2\pi/3) = -1/2$. The quadrant determines the negative sign.

15. B — The period of $\sin(bx)$ is $2\pi/b$; with $b = \pi/2$, the period is $2\pi/(\pi/2) = 4$. The π cancels in the division. The period is 4.

16. D — Using $S = n/2(2a_1 + (n - 1)d) = 15/2(4 + 56) = 15/2(60) = 450$. The formula sums the arithmetic series directly. The sum of the first 15 terms is 450.

17. A — The inverse of a logarithm with base 3 is the exponential function with base 3: $f^{-1}(x) = 3^x$. Exponentials and logarithms are inverse operations. The inverse is 3^x.

18. C — Without replacement, multiply the probabilities: $(6/10)(5/9) = 30/90 = 1/3$. The second draw has one fewer black ball and one fewer total. The probability both are black is 1/3.

19. B — The value 12 is the mean, and a normal distribution is symmetric about its mean. Exactly 50% of the data lies below the mean. The symmetry splits the distribution evenly.

20. D — In Quadrant III cosine is negative, and the 3-4-5 triangle gives the cosine ratio. Therefore $\cos \theta = -4/5$. The quadrant fixes the negative sign on cosine.

21. C — Complete the square: $x^2 - 6x + 2 = (x^2 - 6x + 9) - 9 + 2 = (x - 3)^2 - 7$. Half of −6 squared is 9, added and subtracted to preserve value. The vertex form reveals the vertex at (3, −7).

22. A — The common ratio is 1/3, and an infinite geometric series with $|r| < 1$ sums to $a_1/(1 - r) = 12/(1 - 1/3) = 12/(2/3) = 18$. The ratio ensures convergence. The series sums to 18.

23. D — Take the log of both sides: $x = \log_5(90) = \ln(90)/\ln(5) \approx 2.80$. The change-of-base formula converts to natural logs. The result rounds to 2.80.

24. B — A random sample of registered voters from across the entire state gives every voter an equal chance of selection, producing a representative, unbiased estimate. The other options draw from groups skewed toward one viewpoint or location. Random statewide sampling avoids that bias.

Part II (Short Constructed Response)

25. $x = -1 \pm 5i$ — Quadratic formula: $x = [-2 \pm \sqrt{4 - 104}]/2 = [-2 \pm \sqrt{-100}]/2 = [-2 \pm 10i]/2 = -1 \pm 5i$. The negative discriminant produces complex conjugates. Dividing each term by 2 gives the simplified $a + bi$ form.

26. $x = 9$; no extraneous solutions — Cross-multiplying gives $5(x - 1) = 4(x + 1)$, which expands to $5x - 5 = 4x + 4$ and simplifies to $x = 9$. The restrictions are $x \neq -1$ and $x \neq 1$, and $x = 9$ violates neither, so it is valid.

27. Yes, $(x - 5)$ is a factor — By the Factor Theorem, $f(5) = 125 - 150 + 15 + 10 = 0$. A remainder of zero confirms that $(x - 5)$ divides $f(x)$ evenly. A nonzero value would have meant it is not a factor.

28. $g(x) = -\sqrt{x - 4}$ — Reflecting across the x-axis negates the function to $-\sqrt{x}$; shifting 4 units right replaces x with $(x - 4)$. Combining these produces $-\sqrt{x - 4}$. The horizontal shift moves opposite the sign inside the radical.

29. $x = 2$ — Write the right side as a power of 2: $64 = 2^6$, so $2^{3x} = 2^6$ requires $3x = 6$. Solving gives $x = 2$. Equal bases allow the exponents to be set equal.

30. $(2x + 1) / [(x - 3)(x + 3)]$, $x \neq 3$, $x \neq -3$ — Factor the second denominator as $(x - 3)(x + 3)$, then use it as the common denominator: $2(x + 3)/[(x - 3)(x + 3)] - 5/[(x - 3)(x + 3)] = (2x + 6 - 5)/[(x - 3)(x + 3)] = (2x + 1)/[(x - 3)(x + 3)]$. The numerator combines to $2x + 1$.

31. $a_4 = 42$ — Apply the recursion: $a_2 = 2(3) + 2 = 8$, $a_3 = 2(8) + 3 = 19$, $a_4 = 2(19) + 4 = 42$. Each term doubles the previous term and adds the index. Building up term by term reaches 42.

32. $\tan \theta = -7/24$ — With $\cos \theta = 24/25$ in Quadrant IV, the 7-24-25 triangle gives $\sin \theta = -7/25$ because sine is negative there. Then $\tan \theta = \sin/\cos = (-7/25)/(24/25) = -7/24$. The quadrant makes the tangent negative.

Part III (Extended Constructed Response)

33. (a) \$1000, 8% growth; (b) ≈ 9.0 years — The coefficient 1000 is the initial value, and the base 1.08 = 1 + 0.08 indicates 8% annual growth. For part (b), $2000 = 1000(1.08)^t$ gives $1.08^t = 2$, so $t = \ln(2)/\ln(1.08) \approx 9.0$ years. Logarithms isolate the exponent.

34. (a) $h(t) = -5.0t^2 + 20.0t + 2.0$; (b) 22.0 m at t = 2.0 s — Quadratic regression on the data returns $a = -5.0$, $b = 20.0$, $c = 2.0$. The vertex occurs at $t = -b/(2a) = -20/(-10) = 2.0$ seconds, and $h(2) = -20 + 40 + 2 = 22.0$ meters. The negative leading coefficient confirms a maximum.

35. (a) $(-1 + 2\sqrt{3}, 1 + 2\sqrt{3})$ and $(-1 - 2\sqrt{3}, 1 - 2\sqrt{3})$ — Substituting $y = x + 2$ into the circle gives $x^2 + (x + 2)^2 = 26$, which simplifies to $2x^2 + 4x - 22 = 0$, then $x^2 + 2x - 11 = 0$, so $x = -1 \pm 2\sqrt{3}$. The y-values follow from $y = x + 2$. **(b)** Checking $(-1 + 2\sqrt{3}, 1 + 2\sqrt{3})$: $x^2 + y^2 = (13 - 4\sqrt{3}) + (13 + 4\sqrt{3}) = 26$, and $y = x + 2 = (-1 + 2\sqrt{3}) + 2 = 1 + 2\sqrt{3}$, so both equations hold.

Part IV (Long Constructed Response)

36. (a) $h(t) = 1.2\cos(2\pi t/3) + 0.5$ — The amplitude is the 1.2-meter radius and the midline is the 0.5-meter center height. The period of 3 seconds gives $b = 2\pi/3$, and starting at the highest point at $t = 0$ calls for a positive cosine. This yields $h(t) = 1.2\cos(2\pi t/3) + 0.5$.

(b) Amplitude 1.2, period 3 s, midline h = 0.5 — The amplitude of 1.2 meters equals the wheel's radius, the distance from the center to the rim. The period of 3 seconds is the time for one full rotation. The midline $h = 0.5$ meters is the center height above the water, about which the paddle oscillates.

(c) t ≈ 0.96, 2.04, 3.96, and 5.04 seconds — Setting $0 = 1.2\cos(2\pi t/3) + 0.5$ gives $\cos(2\pi t/3) = -0.4167$. Over the first 6 seconds, $2\pi t/3 = 2.0005, 4.2827, 8.2837$, and 10.5659 radians, so $t = 0.96, 2.04, 3.96$, and 5.04 seconds. Each pair corresponds to the paddle passing through the water surface while descending and rising on each rotation.