PRACTICE EXAM 24: ALGEBRA II REGENTS SIMULATION

PART I — Multiple Choice (24 questions × 2 credits = 48 credits)

1. Express the sum $(-3 + 4i) + (5 - 9i)$ in $a + bi$ form.

A. $2 + 5i$

B. $2 - 5i$

C. $-8 + 13i$

D. $8 - 5i$

2. What is the value of 81^(1/4)?

A. 9

B. 20.25

C. 27

D. 3

3. What are the solutions to the equation $x^2 - 2x + 5 = 0$?

A. $x = -1 \pm 2i$

B. $x = 2 \pm 4i$

C. $x = 1 \pm 2i$

D. $x = 1 \pm 4i$

4. Factored completely, the expression $9x^2 + 12x + 4$ is equivalent to

A. $(3x + 2)^2$

B. $(3x - 2)^2$

C. $(3x + 2)(3x - 2)$

D. $(9x + 4)(x + 1)$

5. What is the value of $\log_2(16)$?

A. 8

B. 4

C. 2

D. 5

6. In a geometric sequence, the third term is 20 and the fifth term is 80. What is the common ratio, given that it is positive?

A. 4

B. 3

C. 2

D. 5

7. When $p(x) = 3x^3 - 2x + 1$ is divided by $(x - 1)$, the remainder is

A. 2

B. 0

C. 4

D. −1

8. If $f(x) = x - 5$ and $g(x) = 2x$, what is the value of $f(g(6))$?

A. 12

B. 14

C. 2

D. 7

9. What is the solution to the equation $\sqrt{(4x - 3)} = 5$?

A. $x = 5$

B. $x = 6$

C. $x = 7$

D. $x = 28$

10. What is the average rate of change of $f(x) = x^2 - 4x + 1$ over the interval [0, 4]?

A. 4

B. 0

C. −4

D. 1

11. Which function models a quantity that begins at 10 and grows by a factor of 3 every 2 hours?

A. f(t) = 10(3)^(t/2)

B. f(t) = 10(3)^(2t)

C. f(t) = 10(1/3)^(t/2)

D. f(t) = 10 + 3t

12. What is the solution to the equation $\log_3(2x - 5) = 3$?

A. x = 9

B. x = 13

C. x = 6

D. x = 16

13. Which statement best describes the nature of the solutions of $x^2 + x + 4 = 0$?

A. Two distinct rational solutions

B. Two complex (non-real) solutions

C. One repeated rational solution

D. Two distinct irrational solutions

14. What is the exact value of $\tan(\pi/4)$?

A. $\sqrt{2}/2$

B. $\sqrt{3}$

C. 1

D. 0

15. For the function $y = -(1/2) \sin(3x) + 4$, what is the amplitude?

A. 1/2

B. 3

C. 4

D. −1/2

16. What is the 15th term of an arithmetic sequence whose first term is −4 and whose common difference is 6?

A. 86

B. 90

C. 74

D. 80

17. If $f(x) = 2x^3$, what is $f^{-1}(x)$?

A. $\sqrt[3]{x} / 2$

B. $(x/2)^3$

C. $\sqrt[3]{(x/2)}$

D. $1 / (2x^3)$

18. In how many ways can a committee of 3 people be chosen from a group of 5 people?

A. 15

B. 10

C. 60

D. 20

19. A data set is normally distributed with a mean of 500 and a standard deviation of 50. What is the z-score of a value of 600?

A. 2

B. 100

C. 1

D. 0.5

20. Given that $\cos \theta = -9/41$ and θ is in Quadrant III, what is the value of $\sin \theta$?

A. 40/41

B. 9/40

C. −9/40

D. −40/41

21. Written in vertex form, the expression $x^2 + 2x - 8$ is equivalent to

A. $(x + 1)^2 - 8$

B. $(x - 1)^2 - 9$

C. $(x + 1)^2 - 9$

D. $(x + 2)^2 - 12$

22. What is the sum of the infinite geometric series $6 + 2 + 2/3 + \ldots$?

A. 8

B. 9

C. 12

D. 18

23. What is the solution to the equation $3^x = 80$, rounded to the nearest hundredth?

A. $x = 26.67$

B. $x = 4.38$

C. $x = 3.00$

D. $x = 3.99$

24. A researcher wants to compare the effects of two fertilizers on plant growth while controlling for outside variables. Which study design is best?

A. Randomly assign plants to two groups, one per fertilizer, with identical conditions

B. Use fertilizer A on the healthiest plants and fertilizer B on the rest

C. Survey gardeners about which fertilizer they prefer

D. Apply both fertilizers to every plant at the same time

PART II — Short Constructed Response (8 questions × 2 credits = 16 credits)

Show all work. A correct answer with no supporting work will receive only 1 credit.

25. Solve the equation $x^2 - 8x + 25 = 0$ algebraically. Express your solutions in $a + bi$ form.

26. Solve algebraically for x: $3 / (x - 4) = 1 / (x + 2)$. State any restrictions on the variable and identify any extraneous solutions that must be rejected.

27. Given $f(x) = x^3 + 4x^2 + x - 6$, determine whether $(x + 2)$ is a factor of $f(x)$. Justify your answer using the Remainder Theorem or the Factor Theorem.

28. Write an equation, in the form $g(x) = a \cdot f(x - h) + k$, for the function obtained by vertically stretching $f(x) = x^3$ by a factor of 2 and then translating the result 3 units to the left.

29. Solve algebraically for x: $32^x = 8$.

30. Express $4 / (x - 2) + 3 / (x + 2)$ as a single rational expression in simplest form. State any restrictions on the variable.

31. A sequence is defined recursively by $a_1 = 1$, $a_2 = 4$, and $a_n = a_{n-1} - a_{n-2}$ for $n \geq 3$. Find the value of a_5.

32. Given that $\sin \theta = -24/25$ and θ terminates in Quadrant III, find the exact value of $\tan \theta$. Show the algebraic work that justifies your answer.

PART III — Extended Constructed Response (3 questions × 4 credits = 12 credits)

Show all work. Partial credit is awarded according to the scoring rubric.

33. A bacteria culture grows according to the function $N(t) = 400(1.25)^t$, where N(t) is the number of bacteria and t is the time in hours.

(a) State the initial number of bacteria and the hourly percent rate of growth.

(b) Algebraically determine the number of hours it will take for the culture to grow to 1000 bacteria. Round your answer to the nearest tenth of an hour.

34. The table below shows the height h(t), in meters, of a kicked ball at time t seconds after the kick.

t (seconds)	0	1	2	3	4
h(t) (meters)	3	9	11	9	3

(a) Using regression, write a quadratic function $h(t) = at^2 + bt + c$ that best models the data. Round each coefficient to the nearest tenth.

(b) Use the regression model from part (a) to determine the maximum height reached by the ball and the time at which the maximum height occurs. Round each answer to the nearest tenth.

35. A circle is defined by the equation $x^2 + y^2 = 21$, and a line is defined by the equation $y = -x + 3$.

(a) Algebraically determine all points of intersection between the circle and the line. Express coordinates in exact form (radicals permitted).

(b) Verify your answer by substituting one of the intersection points back into both original equations. Show the substitution clearly.

PART IV — Long Constructed Response (1 question × 6 credits = 6 credits)

Show all work. This problem requires multiple steps and integrates concepts from several chapters.

36. A reflector is attached to the rim of a bicycle wheel of radius 0.35 meters. As the wheel rolls, the reflector's height above the ground oscillates between a maximum of 0.7 meters and a minimum of 0 meters. The wheel completes one full rotation every 1.5 seconds. At time $t = 0$, the reflector is at the top of the wheel (maximum height).

(a) Write a function h(t) that models the height of the reflector in meters above the ground as a function of time t in seconds.

(b) State the amplitude, period, and midline of the function h(t), and explain what each represents in the context of the rolling wheel.

(c) Algebraically determine all times during the first 3 seconds at which the reflector is at a height of 0.5 meters above the ground. Round each answer to the nearest hundredth of a second. Show all algebraic work.

ANSWER KEY WITH EXPLANATIONS — PRACTICE EXAM 24

1. B — Combine real and imaginary parts: $(-3 + 5) + (4 - 9)i = 2 - 5i$. Adding complex numbers treats each component independently. The result is $2 - 5i$.

2. D — A fourth root undoes a fourth power: $81^{(1/4)} = 3$, since $3^4 = 81$. The exponent 1/4 takes the fourth root. The value is 3.

3. C — Quadratic formula: $x = [2 \pm \sqrt{(4 - 20)}]/2 = [2 \pm \sqrt{(-16)}]/2 = [2 \pm 4i]/2 = 1 \pm 2i$. The negative discriminant produces the imaginary part. Dividing both terms by 2 gives the simplified form.

4. A — This is a perfect square trinomial: $9x^2 + 12x + 4 = (3x + 2)^2$, since $(3x)^2 = 9x^2$, $2^2 = 4$, and $2(3x)(2) = 12x$. The middle term confirms the perfect square. The factorization is $(3x + 2)^2$.

5. B — A logarithm asks for the exponent on the base: $2^4 = 16$, so $\log_2(16) = 4$. Rewriting in exponential form makes the value clear. The base 2 raised to 4 equals 16.

6. C — Dividing the fifth term by the third gives $r^2 = 80/20 = 4$, so $r = 2$ (positive root). The terms span two ratio steps, hence the square. The common ratio is 2.

7. A — By the Remainder Theorem, the remainder is $p(1) = 3 - 2 + 1 = 2$. Substituting the zero of the divisor avoids long division. The remainder is 2.

8. D — Evaluate the inner function first: $g(6) = 2(6) = 12$, then $f(12) = 12 - 5 = 7$. Composition works from the inside out. Subtracting 5 from 12 gives 7.

9. C — Square both sides: $4x - 3 = 25$, so $4x = 28$ and $x = 7$. Squaring removes the radical cleanly. Checking, $\sqrt{(28 - 3)} = \sqrt{25} = 5$ confirms the solution.

10. B — Average rate of change is $[f(4) - f(0)]/(4 - 0)$. With $f(4) = 1$ and $f(0) = 1$, this is $(1 - 1)/4 = 0$. The equal endpoint values give a rate of change of zero.

11. A — A quantity growing by a factor of 3 every 2 hours uses base 3 with exponent t/2 and initial value 10: $f(t) = 10(3)^{(t/2)}$. The exponent t/2 counts the number of 2-hour tripling periods. This models the growth correctly.

12. D — Rewrite in exponential form: $2x - 5 = 3^3 = 27$, so $2x = 32$ and $x = 16$. Converting the logarithmic equation isolates x. Checking shows the argument stays positive.

13. B — The discriminant is $b^2 - 4ac = 1 - 16 = -15$, which is negative. A negative discriminant produces two complex (non-real) conjugate solutions. The sign of the discriminant classifies the roots.

14. C — Tangent is sine over cosine, and at $\pi/4$ both equal $\sqrt{2}/2$, so $\tan(\pi/4) = 1$. The equal legs of the 45-45-90 reference triangle make the ratio 1. The value is 1.

15. A — In $y = a \sin(bx) + d$, the amplitude is $|a|$, and $|-1/2| = 1/2$. The amplitude is the absolute value of the sine coefficient. The negative sign reflects the graph but does not change the amplitude.

16. D — The nth term is $a_n = a_1 + (n - 1)d = -4 + 14(6) = -4 + 84 = 80$. Adding fourteen common differences of 6 raises the value. The fifteenth term is 80.

17. C — To invert, solve $y = 2x^3$ for x: $x^3 = y/2$, so $x = \sqrt[3]{(y/2)}$ and $f^{-1}(x) = \sqrt[3]{(x/2)}$. The inverse undoes multiplication by 2 and cubing in reverse order. The cube root applies to x/2.

18. B — Order does not matter for a committee, so use combinations: $C(5, 3) = 5!/(3!2!) = 10$. Choosing members is a combination, not a permutation. There are 10 possible committees.

19. A — The z-score measures standard deviations from the mean: $z = (600 - 500)/50 = 100/50 = 2$. The value lies two standard deviations above the mean. The z-score is 2.

20. D — In Quadrant III both sine and cosine are negative, and the 9-40-41 triangle gives the sine ratio. Therefore $\sin \theta = -40/41$. The quadrant fixes the negative sign on sine.

21. C — Complete the square: $x^2 + 2x - 8 = (x^2 + 2x + 1) - 1 - 8 = (x + 1)^2 - 9$. Half of 2 squared is 1, added and subtracted to preserve value. The vertex form reveals the vertex at $(-1, -9)$.

22. B — The common ratio is 1/3, and an infinite geometric series with $|r| < 1$ sums to $a_1/(1 - r) = 6/(1 - 1/3) = 6/(2/3) = 9$. The ratio ensures convergence. The series sums to 9.

23. D — Take the log of both sides: $x = \log_3(80) = \ln(80)/\ln(3) \approx 3.99$. The change-of-base formula converts to natural logs. The result rounds to 3.99.

24. A — Randomly assigning plants to two groups under identical conditions isolates the fertilizer as the only varying factor, allowing a valid cause-and-effect comparison. Random assignment balances confounding variables across groups. The other designs introduce bias or fail to separate the treatments.

Part II (Short Constructed Response)

25. $x = 4 \pm 3i$ — Quadratic formula: $x = [8 \pm \sqrt{(64 - 100)}]/2 = [8 \pm \sqrt{(-36)}]/2 = [8 \pm 6i]/2 = 4 \pm 3i$. The negative discriminant produces complex conjugates. Dividing each term by 2 gives the simplified a + bi form.

26. $x = -5$; no extraneous solutions — Cross-multiplying gives $3(x + 2) = (x - 4)$, which expands to $3x + 6 = x - 4$ and simplifies to $2x = -10$, so $x = -5$. The restrictions are $x \neq 4$ and $x \neq -2$, and $x = -5$ violates neither, so it is valid.

27. Yes, $(x + 2)$ is a factor — By the Factor Theorem, $f(-2) = -8 + 16 - 2 - 6 = 0$. A remainder of zero confirms that $(x + 2)$ divides $f(x)$ evenly. A nonzero value would have meant it is not a factor.

28. $g(x) = 2(x + 3)^3$ — A vertical stretch by 2 multiplies the function by 2, giving $2x^3$; shifting 3 units left replaces x with $(x + 3)$. Combining these produces $2(x + 3)^3$. The horizontal shift moves opposite the sign inside.

29. $x = 3/5$ — Write both sides with base 2: $32^x = 2^{(5x)}$ and $8 = 2^3$, so $5x = 3$. Solving gives $x = 3/5$. Equal bases allow the exponents to be set equal.

30. $(7x + 2) / [(x - 2)(x + 2)]$, $x \neq 2$, $x \neq -2$ — Over the common denominator $(x - 2)(x + 2)$: $4(x + 2) + 3(x - 2) = 4x + 8 + 3x - 6 = 7x + 2$. The numerator combines to $7x + 2$. Restrictions come from the original denominators.

31. $a_5 = -4$ — Apply the recursion: $a_3 = 4 - 1 = 3$, $a_4 = 3 - 4 = -1$, $a_5 = -1 - 3 = -4$. Each term is the previous term minus the one before it. Building up term by term reaches −4.

32. $\tan \theta = 24/7$ — With $\sin \theta = -24/25$ in Quadrant III, the 7-24-25 triangle gives $\cos \theta = -7/25$ because cosine is negative there. Then $\tan \theta = \sin/\cos = (-24/25)/(-7/25) = 24/7$. The two negatives make the tangent positive.

Part III (Extended Constructed Response)

33. (a) 400, 25% growth; (b) ≈ 4.1 hours — The coefficient 400 is the initial count, and the base $1.25 = 1 + 0.25$ indicates 25% hourly growth. For part (b), $1000 = 400(1.25)^t$ gives $1.25^t = 2.5$, so $t = \ln(2.5)/\ln(1.25) \approx 4.1$ hours. Logarithms isolate the exponent.

34. (a) $h(t) = -2.0t^2 + 8.0t + 3.0$; (b) 11.0 m at $t = 2.0$ s — Quadratic regression on the data returns $a = -2.0$, $b = 8.0$, $c = 3.0$. The vertex occurs at $t = -b/(2a) = -8/(-4) = 2.0$ seconds, and $h(2) = -8 + 16 + 3 = 11.0$ meters. The negative leading coefficient confirms a maximum.

35. (a) $((3 + \sqrt{33})/2, (3 - \sqrt{33})/2)$ and $((3 - \sqrt{33})/2, (3 + \sqrt{33})/2)$ — Substituting $y = -x + 3$ into the circle gives $x^2 + (-x + 3)^2 = 21$, which simplifies to $2x^2 - 6x - 12 = 0$, then $x^2 - 3x - 6 = 0$, so $x = (3 \pm \sqrt{33})/2$. The y-values follow from $y = -x + 3$. **(b)** Checking $((3 + \sqrt{33})/2, (3 - \sqrt{33})/2)$: $x^2 + y^2 = (42 + 6\sqrt{33})/4 + (42 - 6\sqrt{33})/4 = 84/4 = 21$, and $y = -x + 3 = -(3 + \sqrt{33})/2 + 3 = (3 - \sqrt{33})/2$, so both equations hold.

Part IV (Long Constructed Response)

36. (a) $h(t) = 0.35 \cos(4\pi t/3) + 0.35$ — The amplitude is $(0.7 - 0)/2 = 0.35$ and the midline is $(0.7 + 0)/2 = 0.35$. The period of 1.5 seconds gives $b = 2\pi/1.5 = 4\pi/3$, and starting at the top at $t = 0$ calls for a positive cosine. This yields $h(t) = 0.35 \cos(4\pi t/3) + 0.35$.

(b) Amplitude 0.35, period 1.5 s, midline h = 0.35 — The amplitude of 0.35 meters equals the wheel's radius, the distance from the center height to the top or bottom. The period of 1.5 seconds is the time for one full rotation. The midline $h = 0.35$ meters is the height of the wheel's center, about which the reflector oscillates.

(c) $t \approx 0.27$, 1.23, 1.77, and 2.73 seconds — Setting $0.5 = 0.35 \cos(4\pi t/3) + 0.35$ gives $\cos(4\pi t/3) = 0.4286$. Over the first 3 seconds, $4\pi t/3 = 1.1279$, 5.1553, 7.4111, and 11.4385 radians, so $t = 0.27$, 1.23, 1.77, and 2.73 seconds. Each pair corresponds to the reflector passing through 0.5 meters while descending and rising on each rotation.