PRACTICE EXAM 23: ALGEBRA II REGENTS SIMULATION

PART I — Multiple Choice (24 questions × 2 credits = 48 credits)

1. Express the product $(4 + i)(2 - 3i)$ in $a + bi$ form.

A. $8 - 10i$

B. $5 - 10i$

C. $11 + 10i$

D. $11 - 10i$

2. What is the value of $16^{(3/4)}$?

A. 12

B. 8

C. 64

D. 4

3. What are the solutions to the equation $x^2 + 25 = 0$?

A. $x = \pm 5i$

B. $x = \pm 25i$

C. $x = \pm 5$

D. $x = \pm\sqrt{5}\,i$

4. Factored completely, the expression $x^3 + 125$ is equivalent to

A. $(x + 5)(x^2 + 5x + 25)$

B. $(x - 5)(x^2 + 5x + 25)$

C. $(x + 5)(x^2 - 5x + 25)$

D. $(x + 5)(x^2 + 25)$

5. What is the value of $\log_5(1/25)$?

A. 2

B. $-1/2$

C. 5

D. -2

6. What is the 4th term of a geometric sequence whose first term is 3 and whose common ratio is -2?

A. -24

B. 24

C. -48

D. 48

7. When $p(x) = x^3 + x^2 - 4$ is divided by $(x - 2)$, the remainder is

A. 4

B. 0

C. 8

D. -4

8. If $f(x) = 2x - 3$ and $g(x) = x^2$, what is the value of $f(g(2))$?

A. 1

B. 5

C. 8

D. 13

9. What is the solution set of the equation $\sqrt{(x + 2)} = x$?

A. $\{-1, 2\}$

B. $\{-1\}$

C. $\{ \}$

D. $\{2\}$

10. What is the average rate of change of $f(x) = -2x^2 + 3$ over the interval $[-1, 2]$?

A. −2

B. −6

C. 2

D. −4

11. A population is modeled by $P(t) = 300(1.1)^t$, where t is the time in years. What is the population after 2 years?

A. 330

B. 360

C. 363

D. 660

12. What is the solution to the equation $\log_2(x) = \log_2(3) + \log_2(4)$?

A. $x = 7$

B. $x = 12$

C. $x = 24$

D. $x = 5$

13. Which statement best describes the nature of the solutions of $2x^2 + 5x + 2 = 0$?

A. Two distinct rational solutions

B. Two distinct irrational solutions

C. One repeated rational solution

D. Two complex (non-real) solutions

14. What is the exact value of $\sin(5\pi/6)$?

A. $\sqrt{3}/2$

B. $-1/2$

C. $-\sqrt{3}/2$

D. $1/2$

15. For the function $y = 4\cos(2x) - 7$, what is the equation of the midline?

A. $y = 4$

B. $y = 7$

C. $y = -7$

D. $y = -4$

16. In an arithmetic sequence, the third term is 8 and the eighth term is 23. What is the first term?

A. 5

B. 2

C. 4

D. 1

17. If $f(x) = x/4 + 1$, what is $f^{-1}(x)$?

A. $4x + 1$

B. $(x - 1) / 4$

C. $x/4 - 1$

D. $4(x - 1)$

18. A spinner is divided into 10 equal sectors numbered 1 through 10. What is the probability of landing on a number that is not a multiple of 3?

A. 7/10

B. 3/10

C. 1/2

D. 2/5

19. A data set is normally distributed with a mean of 40 and a standard deviation of 5. Approximately what percent of the data falls between 35 and 45?

A. 95%

B. 68%

C. 34%

D. 50%

20. Given that $\sin \theta = 9/41$ and θ is in Quadrant I, what is the value of $\tan \theta$?

A. 40/41

B. 41/40

C. 9/40

D. 40/9

21. Written in vertex form, the expression $x^2 - 10x + 30$ is equivalent to

A. $(x - 5)^2 + 5$

B. $(x - 5)^2 - 5$

C. $(x + 5)^2 + 5$

D. $(x - 10)^2 - 70$

22. What is the sum of the infinite geometric series 80 + 20 + 5 + …?

A. 100

B. 120

C. 96

D. 106.67

23. What is the solution to the equation $4^x = 60$, rounded to the nearest hundredth?

A. x = 15.00

B. x = 3.91

C. x = 2.95

D. x = 1.39

24. A study finds a strong positive correlation between monthly ice cream sales and the number of drowning incidents. Which is the best conclusion?

A. Buying ice cream causes drowning

B. The correlation does not prove causation; a third factor may explain both

C. Drowning causes people to buy ice cream

D. The correlation must be due to a calculation error

PART II — Short Constructed Response (8 questions × 2 credits = 16 credits)

Show all work. A correct answer with no supporting work will receive only 1 credit.

25. Solve the equation $x^2 + 6x + 25 = 0$ algebraically. Express your solutions in a + bi form.

26. Solve algebraically for x: $7 / (x + 3) = 2 / (x - 2)$. State any restrictions on the variable and identify any extraneous solutions that must be rejected.

27. Given $f(x) = 2x^3 - 3x^2 + 4x - 5$, determine whether $(x - 1)$ is a factor of $f(x)$. Justify your answer using the Remainder Theorem or the Factor Theorem.

28. Write an equation, in the form $g(x) = a \cdot f(x) + k$, for the function obtained by vertically stretching $f(x) = \sqrt{x}$ by a factor of 4 and then translating the result 2 units down.

29. Solve algebraically for x: $16^x = 8$.

30. Express $6 / (x^2 + 2x) - 2 / x$ as a single rational expression in simplest form. State any restrictions on the variable.

31. A sequence is defined recursively by $a_1 = 2$ and $a_n = 4a_{n-1} + 1$ for $n \geq 2$. Find the value of a_4.

32. Given that $\tan \theta = -12/5$ and θ terminates in Quadrant II, find the exact value of $\cos \theta$. Show the algebraic work that justifies your answer.

PART III — Extended Constructed Response (3 questions × 4 credits = 12 credits)

Show all work. Partial credit is awarded according to the scoring rubric.

33. A radioactive sample decays according to the function $R(t) = 120(0.5)^{(t/8)}$, where $R(t)$ is the amount remaining in grams and t is the time in days.

(a) State the initial amount of the sample and the number of days that represents the half-life.

(b) Algebraically determine the number of days it will take for the sample to decay to 40 grams. Round your answer to the nearest tenth of a day.

34. The table below shows a company's average cost $C(x)$, in dollars, of producing a unit when x hundred units are made.

x (hundreds)	0	1	2	3	4
C(x) (dollars)	50	25	10	5	10

(a) Using regression, write a quadratic function $C(x) = ax^2 + bx + c$ that best models the data. Round each coefficient to the nearest tenth.

(b) Use the regression model from part (a) to determine the minimum average cost and the number of units at which the minimum cost occurs. Round each answer to the nearest tenth.

35. A circle is defined by the equation $x^2 + y^2 = 40$, and a line is defined by the equation $y = x - 2$.

(a) Algebraically determine all points of intersection between the circle and the line. Express coordinates in exact form (radicals permitted).

(b) Verify your answer by substituting one of the intersection points back into both original equations. Show the substitution clearly.

PART IV — Long Constructed Response (1 question × 6 credits = 6 credits)

Show all work. This problem requires multiple steps and integrates concepts from several chapters.

36. A bungee jumper bounces up and down after the initial fall. The jumper's height above the ground oscillates between a maximum of 45 meters and a minimum of 15 meters, completing one full bounce cycle every 4 seconds. At time $t = 0$, the jumper is at the maximum height.

(a) Write a function $h(t)$ that models the jumper's height in meters above the ground as a function of time t in seconds.

(b) State the amplitude, period, and midline of the function $h(t)$, and explain what each represents in the context of the bouncing jumper.

(c) Algebraically determine all times during the first 8 seconds at which the jumper is at a height of 38 meters above the ground. Round each answer to the nearest hundredth of a second. Show all algebraic work.

ANSWER KEY WITH EXPLANATIONS – PRACTICE EXAM 23

1. D — Distribute and apply $i^2 = -1$: $(4 + i)(2 - 3i) = 8 - 12i + 2i - 3i^2 = 8 - 10i + 3 = 11 - 10i$. The $-3i^2$ term becomes +3, shifting the real part to 11. The middle terms combine to $-10i$.

2. B — A rational exponent applies the root then the power: $16^{(3/4)} = (16^{(1/4)})^3 = 2^3 = 8$. The fourth root of 16 is 2, cubed to give 8. Evaluating the root first keeps the numbers small.

3. A — Isolate the square: $x^2 = -25$, so $x = \pm\sqrt{(-25)} = \pm 5i$. The negative value under the root introduces the imaginary unit. Both roots are pure imaginary.

4. C — This is a sum of cubes, $x^3 + 5^3 = (x + 5)(x^2 - 5x + 25)$. The middle term of the quadratic factor is $-5x$, making it negative. The sign pattern distinguishes it from a difference of cubes.

5. D — Rewrite the argument as a power of 5: $1/25 = 5^{-2}$, so $\log_5(1/25) = -2$. The negative exponent reflects the reciprocal. The value is -2.

6. A — The nth term is $a_n = a_1 \cdot r^{(n-1)}$, so $a_4 = 3(-2)^3 = 3(-8) = -24$. The negative ratio cubed gives a negative result. The fourth term is -24.

7. C — By the Remainder Theorem, the remainder is $p(2) = 8 + 4 - 4 = 8$. Substituting the zero of the divisor avoids long division. The remainder is 8.

8. B — Evaluate the inner function first: $g(2) = 2^2 = 4$, then $f(4) = 2(4) - 3 = 5$. Composition works from the inside out. The result is 5.

9. D — Squaring gives $x + 2 = x^2$, so $x^2 - x - 2 = 0$ and $(x - 2)(x + 1) = 0$. The candidate $x = -1$ fails because $\sqrt{1} = 1 \neq -1$, leaving $x = 2$ as the only valid solution. The check removes the extraneous root.

10. A — Average rate of change is $[f(2) - f(-1)]/(2 - (-1))$. With $f(2) = -5$ and $f(-1) = 1$, this is $(-5 - 1)/3 = -6/3 = -2$. The negative result reflects a decreasing secant slope.

11. C — Substitute $t = 2$: $P(2) = 300(1.1)^2 = 300(1.21) = 363$. Squaring the growth factor accounts for two years. The population is 363.

12. B — Combine the logs using the product rule: $\log_2(3) + \log_2(4) = \log_2(12)$, so $x = 12$. Equal logarithms with the same base require equal arguments. The solution is 12.

13. A — The discriminant is $b^2 - 4ac = 25 - 16 = 9$, which is a positive perfect square. This produces two distinct rational real solutions. A perfect-square discriminant signals rational roots.

14. D — The angle $5\pi/6$ lies in Quadrant II with reference angle $\pi/6$, where sine is positive. Since $\sin(\pi/6) = 1/2$, $\sin(5\pi/6) = 1/2$. The quadrant keeps the value positive.

15. C — In $y = a \cos(bx) + d$, the midline is the vertical shift d, which is −7. The midline is the horizontal line about which the function oscillates. The amplitude of 4 does not affect its location.

16. B — The common difference is $d = (23 - 8)/(8 - 3) = 15/5 = 3$. Working back, $a_1 = a_3 - 2d = 8 - 6 = 2$. Subtracting two common differences recovers the first term.

17. D — To invert, solve $y = x/4 + 1$ for x: $y - 1 = x/4$, so $x = 4(y - 1)$ and $f^{-1}(x) = 4(x - 1)$. The inverse undoes division by 4 and addition of 1 in reverse order. The result is $4(x - 1)$.

18. A — The multiples of 3 from 1 to 10 are 3, 6, and 9, so 3 outcomes are multiples and 7 are not. The probability of not a multiple is 7/10. Subtracting the favorable multiples from the total gives 7.

19. B — The values 35 and 45 are one standard deviation below and above the mean of 40. By the empirical rule, about 68% of data falls within one standard deviation. This interval captures the central 68%.

20. C — With $\sin \theta = 9/41$ in Quadrant I, the 9-40-41 triangle gives $\cos \theta = 40/41$, so $\tan \theta = 9/40$. All ratios are positive in Quadrant I. The opposite over adjacent gives 9/40.

21. A — Complete the square: $x^2 - 10x + 30 = (x^2 - 10x + 25) - 25 + 30 = (x - 5)^2 + 5$. Half of −10 squared is 25, added and subtracted to preserve value. The vertex form reveals the vertex at (5, 5).

22. D — The common ratio is 1/4, and an infinite geometric series with $|r| < 1$ sums to $a_1/(1 - r) = 80/(1 - 1/4) = 80/(3/4) \approx 106.67$. The ratio ensures convergence. The series sums to about 106.67.

23. C — Take the log of both sides: $x = \log_4(60) = \ln(60)/\ln(4) \approx 2.95$. The change-of-base formula converts to natural logs. The result rounds to 2.95.

24. B — Correlation alone does not establish causation, and a confounding variable, warmer weather, increases both ice cream sales and swimming-related drownings. Neither variable causes the other directly. A lurking third factor explains the association.

Part II (Short Constructed Response)

25. $x = -3 \pm 4i$ — Quadratic formula: $x = [-6 \pm \sqrt{(36 - 100)}]/2 = [-6 \pm \sqrt{(-64)}]/2 = [-6 \pm 8i]/2 = -3 \pm 4i$. The negative discriminant produces complex conjugates. Dividing each term by 2 gives the simplified $a + bi$ form.

26. $x = 4$; no extraneous solutions — Cross-multiplying gives $7(x - 2) = 2(x + 3)$, which expands to $7x - 14 = 2x + 6$ and simplifies to $5x = 20$, so $x = 4$. The restrictions are $x \neq -3$ and $x \neq 2$, and $x = 4$ violates neither, so it is valid.

27. No, $(x - 1)$ is not a factor — By the Remainder Theorem, $f(1) = 2 - 3 + 4 - 5 = -2$. A nonzero remainder means $(x - 1)$ does not divide $f(x)$ evenly. Only a remainder of zero would confirm a factor.

28. $g(x) = 4\sqrt{x} - 2$ — A vertical stretch by 4 multiplies the function by 4, giving $4\sqrt{x}$; translating 2 units down subtracts 2. Combining these produces $4\sqrt{x} - 2$. The stretch acts on the leading coefficient.

29. $x = 3/4$ — Write both sides with base 2: $16^x = 2^{(4x)}$ and $8 = 2^3$, so $4x = 3$. Solving gives $x = 3/4$. Equal bases allow the exponents to be set equal.

30. $-2(x - 1) / [x(x + 2)]$, $x \neq 0$, $x \neq -2$ — Factor the first denominator as $x(x + 2)$, then use it as the common denominator: $6/[x(x + 2)] - 2(x + 2)/[x(x + 2)] = [6 - 2(x + 2)]/[x(x + 2)] = (2 - 2x)/[x(x + 2)]$, which factors as $-2(x - 1)/[x(x + 2)]$. The numerator simplifies to $2 - 2x$.

31. $a_4 = 149$ — Apply the recursion: $a_2 = 4(2) + 1 = 9$, $a_3 = 4(9) + 1 = 37$, $a_4 = 4(37) + 1 = 149$. Each term quadruples the previous term and adds 1. Building up term by term reaches 149.

32. $\cos\theta = -5/13$ — With $\tan\theta = -12/5$, the reference triangle has legs 12 and 5 with hypotenuse 13. In Quadrant II cosine is negative, so $\cos\theta = -5/13$. The quadrant fixes the negative sign.

Part III (Extended Constructed Response)

33. (a) 120 g, half-life 8 days; (b) ≈ 12.7 days — The coefficient 120 is the initial amount, and the exponent $t/8$ with base 0.5 means the sample halves every 8 days. For part (b), $40 = 120(0.5)^{(t/8)}$ gives $0.5^{(t/8)} = 1/3$, so $t/8 = \log_{0.5}(1/3) \approx 1.585$ and $t \approx 12.7$ days. Logarithms isolate the exponent.

34. (a) $C(x) = 5.0x^2 - 30.0x + 50.0$; (b) \$5 at $x = 3$ hundred units — Quadratic regression on the data returns $a = 5.0$, $b = -30.0$, $c = 50.0$. The vertex occurs at $x = -b/(2a) = 30/10 = 3$, and $C(3) = 45 - 90 + 50 = 5$ dollars. The positive leading coefficient confirms a minimum.

35. (a) $(1 + \sqrt{19}, -1 + \sqrt{19})$ and $(1 - \sqrt{19}, -1 - \sqrt{19})$ — Substituting $y = x - 2$ into the circle gives $x^2 + (x - 2)^2 = 40$, which simplifies to $2x^2 - 4x - 36 = 0$, then $x^2 - 2x - 18 = 0$, so $x = 1 \pm \sqrt{19}$. The y-values follow from $y = x - 2$. **(b)** Checking $(1 + \sqrt{19}, -1 + \sqrt{19})$: $x^2 + y^2 = (20 + 2\sqrt{19}) + (20 - 2\sqrt{19}) = 40$, and $y = x - 2 = (1 + \sqrt{19}) - 2 = -1 + \sqrt{19}$, so both equations hold.

Part IV (Long Constructed Response)

36. (a) $h(t) = 15 \cos(\pi t/2) + 30$ — The amplitude is $(45 - 15)/2 = 15$ and the midline is $(45 + 15)/2 = 30$. The period of 4 seconds gives $b = 2\pi/4 = \pi/2$, and starting at the maximum height at $t = 0$ calls for a positive cosine. This yields $h(t) = 15 \cos(\pi t/2) + 30$.

(b) Amplitude 15, period 4 s, midline $h = 30$ — The amplitude of 15 meters is the distance from the midline to the highest or lowest point of the bounce. The period of 4 seconds is the time for one complete bounce cycle. The midline $h = 30$ meters is the average height about which the jumper oscillates.

(c) $t \approx 0.64, 3.36, 4.64$, and 7.36 seconds — Setting $38 = 15 \cos(\pi t/2) + 30$ gives $\cos(\pi t/2) = 8/15$. Over the first 8 seconds, $\pi t/2 = 1.0087, 5.2745, 7.2919$, and 11.5577 radians, so $t = 0.64, 3.36, 4.64$, and 7.36 seconds. Each pair corresponds to the jumper passing through 38 meters while descending and rising on each bounce.