PRACTICE EXAM 22: ALGEBRA II REGENTS SIMULATION

PART I — Multiple Choice (24 questions × 2 credits = 48 credits)

1. What is the value of $(1 + i)^2$ expressed in a + bi form?

A. 2i

B. 2

C. 1 + 2i

D. 0

2. Simplify the expression x^(3/4) / x^(1/4).

A. x^(3/16)

B. x^(1/2)

C. x

D. x^(1/4)

3. What are the solutions to the equation $x^2 + 2x + 10 = 0$?

A. x = 1 ± 3i

B. $x = -1 \pm 6i$

C. $x = -2 \pm 3i$

D. $x = -1 \pm 3i$

4. Factored completely, the expression $x^3 + 3x^2 + 2x + 6$ is equivalent to

A. $(x + 3)(x^2 - 2)$

B. $(x - 3)(x^2 + 2)$

C. $(x + 3)(x^2 + 2)$

D. $(x + 2)(x^2 + 3)$

5. What is the value of $\log_9(3)$?

A. 1/2

B. 2

C. 3

D. 1/3

6. What is the sum of the finite geometric series $1 + 4 + 16 + 64$?

A. 64

B. 256

C. 128

D. 85

7. When $p(x) = 2x^3 + x - 5$ is divided by $(x + 1)$, the remainder is

A. -4

B. -8

C. -6

D. -2

8. If $f(x) = 4x$ and $g(x) = x + 5$, what is the value of $g(f(2))$?

A. 28

B. 40

C. 13

D. 17

9. What is the solution to the equation $\sqrt{(x - 3)} = 4$?

A. $x = 19$

B. $x = 13$

C. $x = 7$

D. $x = 16$

10. What is the average rate of change of $f(x) = x^2 + x$ over the interval [2, 5]?

A. 24

B. 30

C. 6

D. 8

11. Which function models a quantity that begins at 80 and halves every 3 years?

A. f(t) = 80(1/2)^(3t)

B. f(t) = 80(2)^(t/3)

C. f(t) = 80(1/2)^(t/3)

D. f(t) = 80 − (1/2)t

12. What is the solution to the equation log(x) = 2?

A. x = 20

B. x = 100

C. x = 10

D. x = 1000

13. Which statement best describes the nature of the solutions of $4x^2 - 4x + 1 = 0$?

A. Two distinct irrational solutions

B. Two distinct rational solutions

C. Two complex (non-real) solutions

D. One repeated rational solution

14. What is the exact value of $\cos(\pi/6)$?

A. $\sqrt{3}/2$

B. $1/2$

C. $\sqrt{2}/2$

D. $-\sqrt{3}/2$

15. For the function $y = \sin(x/3)$, what is the period?

A. 3

B. $\pi/3$

C. 6π

D. 2π

16. What is the sum of the first 20 terms of an arithmetic sequence whose first term is 1 and whose common difference is 2?

A. 380

B. 400

C. 200

D. 420

17. If $f(x) = x^2$ for $x \geq 0$, what is $f^{-1}(x)$?

A. $\sqrt{x}$

B. x^2

C. $-\sqrt{x}$

D. $1/x^2$

18. In how many different ways can 4 people be arranged in a line?

A. 4

B. 16

C. 12

D. 24

19. A data set is normally distributed with a mean of 60 and a standard deviation of 12. Approximately what percent of the data is greater than 60?

A. 68%

B. 34%

C. 50%

D. 100%

20. Given that $\tan \theta = 24/7$ and θ is in Quadrant I, what is the value of $\sin \theta$?

A. 7/25

B. 24/25

C. 25/24

D. 7/24

21. Written in vertex form, the expression $x^2 - 4x + 9$ is equivalent to

A. $(x - 2)^2 + 9$

B. $(x - 2)^2 - 5$

C. $(x - 2)^2 + 5$

D. $(x - 4)^2 - 7$

22. What is the sum of the infinite geometric series $100 + 25 + 6.25 + \ldots$?

A. 133.33

B. 125

C. 150

D. 400

23. What is the solution to the equation $2^x = 30$, rounded to the nearest hundredth?

A. x = 15.00

B. x = 5.00

C. x = 3.40

D. x = 4.91

24. Which sampling method is most likely to produce a biased sample when estimating the average daily screen time of all high school students?

A. Randomly selecting students from a complete roster

B. Surveying only students in the computer programming club

C. Using a random number generator to choose students

D. Selecting every 10th student from an alphabetical list

PART II — Short Constructed Response (8 questions × 2 credits = 16 credits)

Show all work. A correct answer with no supporting work will receive only 1 credit.

25. Solve the equation $x^2 - 4x + 8 = 0$ algebraically. Express your solutions in a + bi form.

26. Solve algebraically for x: $2 / (x + 1) + 3 = 5 / (x + 1)$. State any restrictions on the variable and identify any extraneous solutions that must be rejected.

27. Given $f(x) = x^3 - 3x^2 - 6x + 8$, determine whether $(x - 4)$ is a factor of f(x). Justify your answer using the Remainder Theorem or the Factor Theorem.

28. Write an equation, in the form $g(x) = a \cdot f(x - h) + k$, for the function obtained by reflecting $f(x) = |x|$ across the x-axis and then translating the result 2 units to the left.

29. Solve algebraically for x: $9^x = 27^(x - 1)$.

30. Express $5 / (x^2 - 25) + 1 / (x - 5)$ as a single rational expression in simplest form. State any restrictions on the variable.

31. A sequence is defined recursively by $a_1 = 1$, $a_2 = 1$, and $a_n = a_{n-1} + a_{n-2}$ for $n \geq 3$. Find the value of a_6.

32. Given that $\cos \theta = 3/5$ and θ terminates in Quadrant IV, find the exact value of $\tan \theta$. Show the algebraic work that justifies your answer.

PART III — Extended Constructed Response (3 questions × 4 credits = 12 credits)

Show all work. Partial credit is awarded according to the scoring rubric.

33. The balance of a savings account is modeled by the function $A(t) = 800(1.035)^t$, where A(t) is the balance in dollars and t is the time in years.

(a) State the initial balance of the account and the annual percent rate of growth.

(b) Algebraically determine the number of years it will take for the balance to reach \$1200. Round your answer to the nearest tenth of a year.

34. The table below shows a company's profit P(x), in thousands of dollars, as a function of the selling price x, in dollars, of its product.

x (dollars)	0	1	2	3	4
P(x) (thousands)	−5	15	27	31	27

(a) Using regression, write a quadratic function $P(x) = ax^2 + bx + c$ that best models the data. Round each coefficient to the nearest tenth.

(b) Use the regression model from part (a) to determine the maximum profit and the selling price at which the maximum profit occurs. Round each answer to the nearest tenth.

35. A circle is defined by the equation $x^2 + y^2 = 34$, and a line is defined by the equation $y = x + 4$.

(a) Algebraically determine all points of intersection between the circle and the line. Express coordinates in exact form (radicals permitted).

(b) Verify your answer by substituting one of the intersection points back into both original equations. Show the substitution clearly.

PART IV — Long Constructed Response (1 question × 6 credits = 6 credits)

Show all work. This problem requires multiple steps and integrates concepts from several chapters.

36. The average monthly high temperature in a town varies sinusoidally over the year. The warmest month has an average high of 88°F, and the coldest month has an average high of 32°F. One full cycle takes 12 months. At time t = 0 (the warmest point of the year), the temperature is at its maximum.

(a) Write a function T(t) that models the average monthly high temperature in degrees Fahrenheit as a function of time t in months.

(b) State the amplitude, period, and midline of the function T(t), and explain what each represents in the context of the temperature.

(c) Algebraically determine all times during the first 12 months at which the average high temperature is 70°F. Round each answer to the nearest hundredth of a month. Show all algebraic work.

ANSWER KEY WITH EXPLANATIONS – PRACTICE EXAM 22

1. A — Square the binomial and apply $i^2 = -1$: $(1 + i)^2 = 1 + 2i + i^2 = 1 + 2i - 1 = 2i$. The real parts 1 and −1 cancel, leaving only the imaginary term. The result is 2i.

2. B — Dividing like bases subtracts exponents: x^(3/4) / x^(1/4) = x^(3/4 − 1/4) = x^(2/4) = x^(1/2). The fractional exponents subtract to 1/2. The result is x^(1/2).

3. D — Quadratic formula: x = [−2 ± √(4 − 40)]/2 = [−2 ± √(−36)]/2 = [−2 ± 6i]/2 = −1 ± 3i. The negative discriminant produces the imaginary part. Dividing both terms by 2 gives the simplified form.

4. C — Group and factor: $x^2(x + 3) + 2(x + 3) = (x + 3)(x^2 + 2)$. The common binomial (x + 3) factors out. The quadratic factor $x^2 + 2$ does not factor over the reals.

5. A — Since 9^(1/2) = 3, the exponent that takes 9 to 3 is 1/2, so $\log_9(3) = 1/2$. The square root relationship gives the value. The answer is 1/2.

6. D — Using $S = a_1(r^n - 1)/(r - 1) = 1(4^4 - 1)/(4 - 1) = 255/3 = 85$. The common ratio is 4 and there are four terms. Direct addition of the terms also gives 85.

7. B — By the Remainder Theorem, the remainder is $p(-1) = 2(-1) + (-1) - 5 = -2 - 1 - 5 = -8$. Substituting the zero of the divisor avoids long division. The remainder is −8.

8. C — Evaluate the inner function first: f(2) = 4(2) = 8, then g(8) = 8 + 5 = 13. Composition works from the inside out. Adding 5 to 8 gives 13.

9. A — Square both sides: $x - 3 = 16$, so $x = 19$. Squaring removes the radical cleanly. Checking, $\sqrt{(19 - 3)} = \sqrt{16} = 4$ confirms the solution.

10. D — Average rate of change is $[f(5) - f(2)]/(5 - 2)$. With $f(5) = 30$ and $f(2) = 6$, this is $(30 - 6)/3 = 24/3 = 8$. This is the slope of the secant line over the interval.

11. C — A quantity that halves uses base 1/2, and halving every 3 years puts $t/3$ in the exponent, with 80 as the initial value: $f(t) = 80(1/2)^{(t/3)}$. The exponent $t/3$ counts the number of 3-year halving periods. This models the decay correctly.

12. B — A common logarithm equals 2 when the argument is 10^2: $x = 100$. Rewriting in exponential form isolates x. The base-10 log of 100 is 2.

13. D — The discriminant is $b^2 - 4ac = 16 - 16 = 0$, indicating one repeated real solution. Since $4x^2 - 4x + 1 = (2x - 1)^2$, the repeated root 1/2 is rational. A zero discriminant always yields a single repeated solution.

14. A — The cosine of $\pi/6$ (30°) is a standard unit-circle value equal to $\sqrt{3}/2$. The 30-60-90 reference triangle gives this ratio. The value is $\sqrt{3}/2$.

15. C — The period of $\sin(bx)$ is $2\pi/b$; with $b = 1/3$, the period is $2\pi/(1/3) = 6\pi$. The fractional coefficient stretches the graph horizontally. The period is 6π.

16. B — Using $S = n/2(2a_1 + (n - 1)d) = 20/2(2 + 38) = 10(40) = 400$. This sums the first 20 odd integers. The sum of the first 20 terms is 400.

17. A — The inverse of $f(x) = x^2$ restricted to $x \geq 0$ is the principal square root: $f^{-1}(x) = \sqrt{x}$. Restricting the domain makes the squaring function one-to-one. The inverse is $\sqrt{x}$.

18. D — Arranging 4 distinct people in a line is a permutation: $4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$. Order matters, so each position reduces the choices by one. There are 24 arrangements.

19. C — The value 60 is the mean, and a normal distribution is symmetric about its mean. Exactly 50% of the data lies above the mean. The symmetry splits the distribution evenly.

20. B — With $\tan \theta = 24/7$ in Quadrant I, the 7-24-25 triangle applies, and all ratios are positive there. Therefore $\sin \theta = 24/25$. The opposite side 24 over the hypotenuse 25 gives the sine.

21. C — Complete the square: $x^2 - 4x + 9 = (x^2 - 4x + 4) - 4 + 9 = (x - 2)^2 + 5$. Half of −4 squared is 4, added and subtracted to preserve value. The vertex form reveals the vertex at (2, 5).

22. A — The common ratio is 1/4, and an infinite geometric series with $|r| < 1$ sums to $a_1/(1 - r) = 100/(1 - 1/4) = 100/(3/4) \approx 133.33$. The ratio ensures convergence. The series sums to about 133.33.

23. D — Take the log of both sides: $x = \log_2(30) = \ln(30)/\ln(2) \approx 4.91$. The change-of-base formula converts to natural logs. The result rounds to 4.91.

24. B — Surveying only students in the computer programming club draws from a group likely to have atypical screen time, producing a non-representative, biased sample. The other methods use random or systematic selection from the full population. Convenience sampling from a special-interest group introduces bias.

Part II (Short Constructed Response)

25. x = 2 ± 2i — Quadratic formula: x = [4 ± √(16 − 32)]/2 = [4 ± √(−16)]/2 = [4 ± 4i]/2 = 2 ± 2i. The negative discriminant produces complex conjugates. Dividing each term by 2 gives the simplified a + bi form.

26. x = 0; no extraneous solutions — Multiplying through by (x + 1) gives 2 + 3(x + 1) = 5, which simplifies to 3x + 5 = 5, so x = 0. The restriction is $x \neq -1$, and x = 0 does not violate it, so it is valid.

27. Yes, (x − 4) is a factor — By the Factor Theorem, f(4) = 64 − 48 − 24 + 8 = 0. A remainder of zero confirms that (x − 4) divides f(x) evenly. A nonzero value would have meant it is not a factor.

28. g(x) = −|x + 2| — Reflecting across the x-axis negates the function to −|x|; shifting 2 units left replaces x with (x + 2). Combining these produces −|x + 2|. The horizontal shift moves opposite the sign inside.

29. x = 3 — Write both sides with base 3: 9^x = 3^(2x) and 27^(x − 1) = 3^(3x − 3), so 2x = 3x − 3. Solving gives x = 3. Equal bases allow the exponents to be set equal.

30. (x + 10) / [(x − 5)(x + 5)], $x \neq 5$, $x \neq -5$ — Factor the first denominator as (x − 5)(x + 5), then use it as the common denominator: 5/[(x − 5)(x + 5)] + (x + 5)/[(x − 5)(x + 5)] = (5 + x + 5)/[(x − 5)(x + 5)] = (x + 10)/[(x − 5)(x + 5)]. The numerator combines to x + 10.

31. $a_6 = 8$ — Apply the recursion: $a_3 = 1 + 1 = 2$, $a_4 = 2 + 1 = 3$, $a_5 = 3 + 2 = 5$, $a_6 = 5 + 3 = 8$. Each term is the sum of the two preceding terms. Building up term by term reaches 8.

32. tan θ = −4/3 — With cos θ = 3/5 in Quadrant IV, the 3-4-5 triangle gives sin θ = −4/5 because sine is negative there. Then tan θ = sin/cos = (−4/5)/(3/5) = −4/3. The quadrant makes the tangent negative.

Part III (Extended Constructed Response)

33. (a) $800, 3.5% growth; (b) ≈ 11.8 years — The coefficient 800 is the initial balance, and the base 1.035 = 1 + 0.035 indicates 3.5% annual growth. For part (b), $1200 = 800(1.035)^t$ gives $1.035^t = 1.5$, so t = ln(1.5)/ln(1.035) ≈ 11.8 years. Logarithms isolate the exponent.

34. (a) $P(x) = -4.0x^2 + 24.0x - 5.0$; (b) $31 thousand at x = $3 — Quadratic regression on the data returns a = −4.0, b = 24.0, c = −5.0. The vertex occurs at x = −b/(2a) = −24/(−8) = 3 dollars, and P(3) = −36 + 72 − 5 = 31 thousand dollars. The negative leading coefficient confirms a maximum.

35. (a) (−2 + √13, 2 + √13) and (−2 − √13, 2 − √13) — Substituting y = x + 4 into the circle gives $x^2 + (x + 4)^2 = 34$, which simplifies to $2x^2 + 8x - 18 = 0$, then $x^2 + 4x - 9 = 0$, so x = −2 ± √13. The y-values

follow from $y = x + 4$. **(b)** Checking $(-2 + \sqrt{13}, 2 + \sqrt{13})$: $x^2 + y^2 = (17 - 4\sqrt{13}) + (17 + 4\sqrt{13}) = 34$, and $y = x + 4 = (-2 + \sqrt{13}) + 4 = 2 + \sqrt{13}$, so both equations hold.

Part IV (Long Constructed Response)

36. (a) $T(t) = 28\cos(\pi t/6) + 60$ — The amplitude is $(88 - 32)/2 = 28$ and the midline is $(88 + 32)/2 = 60$. The period of 12 months gives $b = 2\pi/12 = \pi/6$, and starting at the maximum at $t = 0$ calls for a positive cosine. This yields $T(t) = 28\cos(\pi t/6) + 60$.

(b) Amplitude 28, period 12 months, midline $T = 60$ — The amplitude of 28°F is the distance from the midline to the warmest or coldest average high. The period of 12 months is the time for one complete yearly cycle. The midline $T = 60$°F is the average temperature about which the value oscillates.

(c) $t \approx 2.30$ months and $t \approx 9.70$ months — Setting $70 = 28\cos(\pi t/6) + 60$ gives $\cos(\pi t/6) = 0.3571$. Over the first 12 months, $\pi t/6 = 1.2059$ and 5.0773 radians, so $t = 2.30$ and 9.70 months. These are the two times the falling and rising temperature passes through 70°F.