

PRACTICE EXAM 21: ALGEBRA II

REGENTS SIMULATION

PART I — Multiple Choice (24 questions \times 2 credits = 48 credits)

1. Express the difference $(7 + 2i) - (4 - 6i)$ in $a + bi$ form.

A. $3 - 4i$

B. $11 - 4i$

C. $3 + 8i$

D. $11 + 8i$

2. What is the value of $32^{(2/5)}$?

A. 4

B. 8

C. $64/5$

D. 16

3. What are the solutions to the equation $x^2 - 8x + 20 = 0$?

A. $x = -4 \pm 2i$

B. $x = 4 \pm 4i$

C. $x = 8 \pm 2i$

D. $x = 4 \pm 2i$

4. Factored completely, the expression $x^2 - 16x + 64$ is equivalent to

A. $(x - 8)(x + 8)$

B. $(x - 8)^2$

C. $(x + 8)^2$

D. $(x - 16)(x - 4)$

5. What is the value of $\log_2(1/8)$?

A. -3

B. 3

C. $-1/3$

D. 4

6. What is the 5th term of a geometric sequence whose first term is 4 and whose common ratio is $1/2$?

A. $1/2$

B. $1/8$

C. $1/4$

D. 2

7. When $p(x) = x^3 - 5x + 2$ is divided by $(x - 2)$, the remainder is

A. 4

B. -2

C. 6

D. 0

8. If $f(x) = x^2 + 1$ and $g(x) = x - 3$, what is the value of $f(g(5))$?

A. 23

B. 5

C. 27

D. 2

9. What is the solution to the equation $2\sqrt{x} = 10$?

A. $x = 5$

B. $x = 10$

C. $x = 25$

D. $x = 100$

10. What is the average rate of change of $f(x) = 3^x$ over the interval $[0, 2]$?

A. 4

B. 8

C. 3

D. 9

11. Which function models a quantity that begins at 1200 and increases by 15% each year?

A. $f(x) = 1200(0.15)^x$

B. $f(x) = 1200(1.15)^x$

C. $f(x) = 1200(15)^x$

D. $f(x) = 1200 + 15x$

12. What is the solution to the equation $\log_2(x + 3) = 5$?

A. $x = 7$

B. $x = 13$

C. $x = 32$

D. $x = 29$

13. Which statement best describes the nature of the solutions of $x^2 + 3x + 1 = 0$?

- A. Two distinct irrational solutions
- B. Two distinct rational solutions
- C. One repeated rational solution
- D. Two complex (non-real) solutions

14. What is the exact value of $\tan(\pi/6)$?

- A. $\sqrt{3}$
- B. $1/2$
- C. $\sqrt{3}/3$
- D. 1

15. For the function $y = -6 \sin(2x) + 1$, what is the amplitude?

- A. -6
- B. 6
- C. 2
- D. 1

16. What is the 10th term of an arithmetic sequence whose first term is 12 and whose common difference is -3?

- A. -18
- B. -12

C. 39

D. -15

17. If $f(x) = 5x - 2$, what is $f^{-1}(x)$?

A. $5x + 2$

B. $(x - 2) / 5$

C. $(x + 2) / 5$

D. $1 / (5x - 2)$

18. A fair six-sided die is rolled twice. What is the probability that both rolls show a 6?

A. $1/36$

B. $1/12$

C. $1/6$

D. $2/6$

19. A data set is normally distributed with a mean of 150 and a standard deviation of 20. Approximately what percent of the data falls between 110 and 190?

A. 68%

B. 34%

C. 99.7%

D. 95%

20. Given that $\cos \theta = -12/13$ and θ is in Quadrant II, what is the value of $\sin \theta$?

A. $-5/13$

B. $5/13$

C. $12/5$

D. $-12/5$

21. Written in vertex form, the expression $x^2 + 8x + 10$ is equivalent to

A. $(x + 4)^2 - 6$

B. $(x + 4)^2 + 10$

C. $(x - 4)^2 - 6$

D. $(x + 8)^2 - 54$

22. What is the sum of the infinite geometric series $50 + 10 + 2 + \dots$?

A. 60

B. 75

C. 62.5

D. 100

23. What is the solution to the equation $7^x = 200$, rounded to the nearest hundredth?

- A. $x = 28.57$
- B. $x = 2.72$
- C. $x = 3.00$
- D. $x = 1.95$

24. A data set contains one value far greater than all the others. Which measure of center is least affected by this outlier?

- A. The mean
- B. The range
- C. The standard deviation
- D. The median

PART II — Short Constructed Response (8 questions \times 2 credits = 16 credits)

Show all work. A correct answer with no supporting work will receive only 1 credit.

25. Solve the equation $x^2 - 2x + 17 = 0$ algebraically. Express your solutions in a + bi form.

26. Solve algebraically for x : $4 / (x - 3) = 3 / (x - 2)$. State any restrictions on the variable and identify any extraneous solutions that must be rejected.

27. Given $f(x) = x^3 + 5x^2 + 6x$, determine whether $(x + 2)$ is a factor of $f(x)$. Justify your answer using the Remainder Theorem or the Factor Theorem.

28. Write an equation, in the form $g(x) = a \cdot f(x - h) + k$, for the function obtained by translating $f(x) = x^3$ to the right 2 units and down 5 units.

29. Solve algebraically for x : $8^{(x - 1)} = 16$.

30. Express $\frac{3}{x - 1} - \frac{2}{x + 4}$ as a single rational expression in simplest form. State any restrictions on the variable.

31. A sequence is defined recursively by $a_1 = 100$ and $a_n = a_{n-1} / 2 - 10$ for $n \geq 2$. Find the value of a_4 .

32. Given that $\sin \theta = -8/17$ and θ terminates in Quadrant IV, find the exact value of $\cos \theta$. Show the algebraic work that justifies your answer.

PART III — Extended Constructed Response (3 questions \times 4 credits = 12 credits)

Show all work. Partial credit is awarded according to the scoring rubric.

33. The population of a city is modeled by the function $P(t) = 5000(1.02)^t$, where $P(t)$ is the population and t is the time in years.

(a) State the initial population of the city and the annual percent rate of growth.

(b) Algebraically determine the number of years it will take for the population to grow to 6000. Round your answer to the nearest tenth of a year.

34. The table below shows the height $h(x)$, in meters, of a water arc from a sprinkler at horizontal distance x meters from the nozzle.

x (meters) 0 1 2 3 4
--- --- --- --- --- ---
$h(x)$ (meters) 1 11 17 19 17

(a) Using regression, write a quadratic function $h(x) = ax^2 + bx + c$ that best models the data. Round each coefficient to the nearest tenth.

(b) Use the regression model from part (a) to determine the maximum height of the water arc and the horizontal distance at which the maximum height occurs. Round each answer to the nearest tenth.

35. A circle is defined by the equation $x^2 + y^2 = 28$, and a line is defined by the equation $y = -x + 2$.

(a) Algebraically determine all points of intersection between the circle and the line. Express coordinates in exact form (radicals permitted).

(b) Verify your answer by substituting one of the intersection points back into both original equations. Show the substitution clearly.

PART IV — Long Constructed Response (1 question \times 6 credits = 6 credits)

Show all work. This problem requires multiple steps and integrates concepts from several chapters.

36. The depth of the water at a dock varies with the tides. The maximum depth is 9 meters and the minimum depth is 3 meters. One complete tidal cycle takes 12 hours. At time $t = 0$ (midnight), the water is at its maximum depth.

(a) Write a function $d(t)$ that models the depth of the water in meters as a function of time t in hours.

(b) State the amplitude, period, and midline of the function $d(t)$, and explain what each represents in the context of the tides.

(c) Algebraically determine all times during the first 12 hours at which the water depth is 7 meters. Round each answer to the nearest hundredth of an hour. Show all algebraic work.

ANSWER KEY WITH EXPLANATIONS – PRACTICE EXAM 21

1. C — Combine real and imaginary parts, distributing the subtraction: $(7 - 4) + (2 - (-6))i = 3 + 8i$. Subtracting $-6i$ adds $6i$ to the imaginary part. The result is $3 + 8i$.

2. A — A rational exponent applies the root then the power: $32^{(2/5)} = (32^{(1/5)})^2 = 2^2 = 4$. The fifth root of 32 is 2, squared to give 4. Evaluating the root first keeps the numbers small.

3. D — Quadratic formula: $x = [8 \pm \sqrt{(64 - 80)}]/2 = [8 \pm \sqrt{(-16)}]/2 = [8 \pm 4i]/2 = 4 \pm 2i$. The negative discriminant produces the imaginary part. Dividing both terms by 2 gives the simplified form.

4. B — This is a perfect square trinomial: $x^2 - 16x + 64 = (x - 8)^2$, since $8^2 = 64$ and $2(8) = 16$. The repeated factor reflects the perfect square. The factorization is $(x - 8)^2$.

5. A — Rewrite the argument as a power of 2: $1/8 = 2^{-3}$, so $\log_2(1/8) = -3$. The negative exponent reflects the reciprocal. The value is -3 .

6. C — The n th term is $a_n = a_1 \cdot r^{(n-1)}$, so $a_5 = 4(1/2)^4 = 4(1/16) = 1/4$. The ratio applied four times shrinks the term. The fifth term is $1/4$.

7. D — By the Remainder Theorem, the remainder is $p(2) = 8 - 10 + 2 = 0$. A remainder of zero means $(x - 2)$ divides the polynomial evenly. Substituting the zero of the divisor gives 0.

8. B — Evaluate the inner function first: $g(5) = 5 - 3 = 2$, then $f(2) = 2^2 + 1 = 5$. Composition works from the inside out. Squaring 2 and adding 1 gives 5.

- 9. C** — Divide by 2, then square: $\sqrt{x} = 5$, so $x = 25$. Isolating the radical before squaring is essential. Checking, $2\sqrt{25} = 2(5) = 10$ confirms the solution.
- 10. A** — Average rate of change is $[f(2) - f(0)]/(2 - 0) = (9 - 1)/2 = 8/2 = 4$. This is the slope of the secant line over the interval. The exponential values 9 and 1 drive the result.
- 11. B** — A quantity increasing by 15% uses base $1 + 0.15 = 1.15$ with the initial value 1200 as the coefficient, giving $f(x) = 1200(1.15)^x$. The base above 1 signals growth. Multiplying by 1.15 each year produces the 15% increase.
- 12. D** — Rewrite in exponential form: $x + 3 = 2^5 = 32$, so $x = 29$. Converting the logarithmic equation isolates x . Checking shows the argument stays positive.
- 13. A** — The discriminant is $b^2 - 4ac = 9 - 4 = 5$, which is positive but not a perfect square. This produces two distinct irrational real solutions. The nature of the discriminant classifies the roots.
- 14. C** — Tangent is sine over cosine, and at $\pi/6$ this is $(1/2)/(\sqrt{3}/2) = 1/\sqrt{3} = \sqrt{3}/3$. Rationalizing the denominator gives $\sqrt{3}/3$. The tangent of 30° is $\sqrt{3}/3$.
- 15. B** — In $y = a \sin(bx) + d$, the amplitude is $|a|$, and $|-6| = 6$. The amplitude is the absolute value of the sine coefficient. The negative sign reflects the graph but does not change the amplitude.
- 16. D** — The n th term is $a_n = a_1 + (n - 1)d = 12 + 9(-3) = 12 - 27 = -15$. Adding nine common differences of -3 lowers the value. The tenth term is -15 .
- 17. C** — To invert, solve $y = 5x - 2$ for x : $x = (y + 2)/5$, so $f^{-1}(x) = (x + 2)/5$. The inverse undoes multiplication by 5 and subtraction of 2 in reverse order. The result is $(x + 2)/5$.
- 18. A** — The two rolls are independent, so multiply their probabilities: $1/6 \cdot 1/6 = 1/36$. Each roll has a $1/6$ chance of showing a 6. The combined probability is $1/36$.
- 19. D** — The values 110 and 190 are two standard deviations below and above the mean of 150. By the empirical rule, about 95% of data falls within ± 2 standard deviations. This interval captures the central 95%.
- 20. B** — In Quadrant II sine is positive, and the 5-12-13 triangle gives the sine ratio. Therefore $\sin \theta = 5/13$. The quadrant fixes the positive sign on sine.
- 21. A** — Complete the square: $x^2 + 8x + 10 = (x^2 + 8x + 16) - 16 + 10 = (x + 4)^2 - 6$. Half of 8 squared is 16, added and subtracted to preserve value. The vertex form reveals the vertex at $(-4, -6)$.
- 22. C** — The common ratio is $1/5$, and an infinite geometric series with $|r| < 1$ sums to $a_1/(1 - r) = 50/(1 - 1/5) = 50/(4/5) = 62.5$. The ratio ensures convergence. The series sums to 62.5.
- 23. B** — Take the log of both sides: $x = \log_7(200) = \ln(200)/\ln(7) \approx 2.72$. The change-of-base formula converts to natural logs. The result rounds to 2.72.

24. D — The median is based on position rather than value, so a single extreme outlier barely shifts it. The mean, range, and standard deviation all incorporate the outlier's magnitude and are pulled toward it. The median is the most resistant measure of center.

Part II (Short Constructed Response)

25. $x = 1 \pm 4i$ — Quadratic formula: $x = [2 \pm \sqrt{4 - 68}]/2 = [2 \pm \sqrt{-64}]/2 = [2 \pm 8i]/2 = 1 \pm 4i$. The negative discriminant produces complex conjugates. Dividing each term by 2 gives the simplified $a + bi$ form.

26. $x = -1$; no extraneous solutions — Cross-multiplying gives $4(x - 2) = 3(x - 3)$, which expands to $4x - 8 = 3x - 9$ and simplifies to $x = -1$. The restrictions are $x \neq 3$ and $x \neq 2$, and $x = -1$ violates neither, so it is valid.

27. Yes, $(x + 2)$ is a factor — By the Factor Theorem, $f(-2) = -8 + 20 - 12 = 0$. A remainder of zero confirms that $(x + 2)$ divides $f(x)$ evenly. A nonzero value would have meant it is not a factor.

28. $g(x) = (x - 2)^3 - 5$ — Shifting 2 units right replaces x with $(x - 2)$; shifting 5 units down subtracts 5. Combining these produces $(x - 2)^3 - 5$. The horizontal shift moves opposite the sign inside the parentheses.

29. $x = 7/3$ — Write both sides with base 2: $8^{(x - 1)} = 2^{(3x - 3)}$ and $16 = 2^4$, so $3x - 3 = 4$. Solving gives $3x = 7$, so $x = 7/3$. Equal bases allow the exponents to be set equal.

30. $(x + 14) / [(x - 1)(x + 4)]$, $x \neq 1$, $x \neq -4$ — Over the common denominator $(x - 1)(x + 4)$: $3(x + 4) - 2(x - 1) = 3x + 12 - 2x + 2 = x + 14$. The numerator combines to $x + 14$. Restrictions come from the original denominators.

31. $a_4 = -5$ — Apply the recursion: $a_2 = 100/2 - 10 = 40$, $a_3 = 40/2 - 10 = 10$, $a_4 = 10/2 - 10 = -5$. Each term halves the previous term and subtracts 10. Building up term by term reaches -5 .

32. $\cos \theta = 15/17$ — Using $\sin^2\theta + \cos^2\theta = 1$: $\cos^2\theta = 1 - 64/289 = 225/289$, so $\cos \theta = \pm 15/17$. In Quadrant IV cosine is positive, giving $15/17$. The quadrant fixes the positive sign.

Part III (Extended Constructed Response)

33. (a) 5000, 2% growth; (b) ≈ 9.2 years — The coefficient 5000 is the initial population, and the base $1.02 = 1 + 0.02$ indicates 2% annual growth. For part (b), $6000 = 5000(1.02)^t$ gives $1.02^t = 1.2$, so $t = \ln(1.2)/\ln(1.02) \approx 9.2$ years. Logarithms isolate the exponent.

34. (a) $h(x) = -2.0x^2 + 12.0x + 1.0$; (b) 19.0 m at $x = 3.0$ m — Quadratic regression on the data returns $a = -2.0$, $b = 12.0$, $c = 1.0$. The vertex occurs at $x = -b/(2a) = -12/(-4) = 3.0$ meters, and $h(3) = -18 + 36 + 1 = 19.0$ meters. The negative leading coefficient confirms a maximum.

35. (a) $(1 + \sqrt{13}, 1 - \sqrt{13})$ and $(1 - \sqrt{13}, 1 + \sqrt{13})$ — Substituting $y = -x + 2$ into the circle gives $x^2 + (-x + 2)^2 = 28$, which simplifies to $2x^2 - 4x - 24 = 0$, then $x^2 - 2x - 12 = 0$, so $x = 1 \pm \sqrt{13}$. The y -values

follow from $y = -x + 2$. **(b)** Checking $(1 + \sqrt{13}, 1 - \sqrt{13})$: $x^2 + y^2 = (14 + 2\sqrt{13}) + (14 - 2\sqrt{13}) = 28$, and $y = -x + 2 = -(1 + \sqrt{13}) + 2 = 1 - \sqrt{13}$, so both equations hold.

Part IV (Long Constructed Response)

36. (a) $d(t) = 3 \cos(\pi t/6) + 6$ — The amplitude is $(9 - 3)/2 = 3$ and the midline is $(9 + 3)/2 = 6$. The period of 12 hours gives $b = 2\pi/12 = \pi/6$, and starting at the maximum depth at $t = 0$ calls for a positive cosine. This yields $d(t) = 3 \cos(\pi t/6) + 6$.

(b) Amplitude 3, period 12 hr, midline $d = 6$ — The amplitude of 3 meters is the distance from the midline to the maximum or minimum depth. The period of 12 hours is the time for one complete tidal cycle. The midline $d = 6$ meters is the average depth about which the water level oscillates.

(c) $t \approx 2.35$ hours and $t \approx 9.65$ hours — Setting $7 = 3 \cos(\pi t/6) + 6$ gives $\cos(\pi t/6) = 1/3$. Over the first 12 hours, $\pi t/6 = 1.2310$ and 5.0522 radians, so $t = 2.35$ and 9.65 hours. These are the two times the falling and rising tide passes through a depth of 7 meters.