PRACTICE EXAM 20: ALGEBRA II REGENTS SIMULATION

PART I — Multiple Choice (24 questions × 2 credits = 48 credits)

1. If $f(x) = x^2 - 3x$ and $g(x) = 2x + 1$, what is the value of $f(2) + g(-1)$?

A. 5

B. −5

C. −3

D. 3

2. What is the value of $\log_2(32)$?

A. 4

B. 5

C. 6

D. 16

3. What is the product of $(3 - 2i)(3 + 2i)$?

A. 13

B. 5

C. $9 + 4i$

D. $9 - 4i$

4. Which equation represents exponential decay?

A. $y = 2(1.5)^x$

B. $y = 5x^2$

C. $y = 3^x$

D. $y = 100(0.75)^x$

5. The expression $\sin(\pi/2 - x)$ is equivalent to

A. $\cos(x)$

B. $-\sin(x)$

C. $\tan(x)$

D. $\sec(x)$

6. What are the solutions to the equation $4x^2 - 16 = 0$?

A. $x = 4$ only

B. $x = \pm 4$

C. $x = \pm 2$

D. x = 2 only

7. The graph of g(x) = f(x + 5) is obtained from the graph of f(x) by

A. shifting right 5 units

B. shifting left 5 units

C. shifting up 5 units

D. shifting down 5 units

8. What is the geometric mean of 8 and 18?

A. 26

B. 13

C. 9

D. 12

9. The function f(x) = (2x − 6) / (x − 3) is undefined at x =

A. 0

B. −3

C. 3

D. 6

10. Solve algebraically for x: $\log_2(x) - \log_2(3) = 4$.

A. 48

B. 16

C. 12

D. 64

11. What is the value of the summation Σ (from k = 2 to 5) of (k + 1)?

A. 14

B. 18

C. 20

D. 25

12. Solve for x in the interval $[0, 2\pi)$: $2 \cos(x) + 1 = 0$.

A. $x = \pi/6$ and $x = 11\pi/6$

B. $x = \pi/3$ and $x = 5\pi/3$

C. $x = 5\pi/6$ and $x = 7\pi/6$

D. $x = 2\pi/3$ and $x = 4\pi/3$

13. What is the inverse of the function $f(x) = \sqrt{(x - 1)}$, defined for $x \geq 1$?

A. $f^{-1}(x) = x^2 + 1, x \geq 0$

B. $f^{-1}(x) = (x + 1)^2, x \geq 0$

C. $f^{-1}(x) = x^2 - 1, x \geq 0$

D. $f^{-1}(x) = \sqrt{(x + 1)}, x \geq 0$

14. In a normal distribution with mean 50 and standard deviation 10, approximately what percent of values fall between 40 and 60?

A. 50%

B. 68%

C. 95%

D. 34%

15. Factored completely over the real numbers, the expression $x^4 - 81$ is equivalent to

A. $(x^2 - 9)(x^2 + 9)$

B. $(x - 3)^2(x + 3)^2$

C. $(x - 3)(x + 3)(x^2 + 9)$

D. $(x^2 - 81)$

16. The middle term of the binomial expansion of $(2x - 3)^4$ is

A. $216x^2$

B. $-216x^2$

C. $24x^2$

D. $8x^2$

17. A fair six-sided die is rolled 4 times. What is the probability of rolling exactly two sixes in the four rolls?

A. 1/6

B. 1/36

C. 25/1296

D. 25/216

18. Which expression is equivalent to $\tan^2(\theta) + 1$?

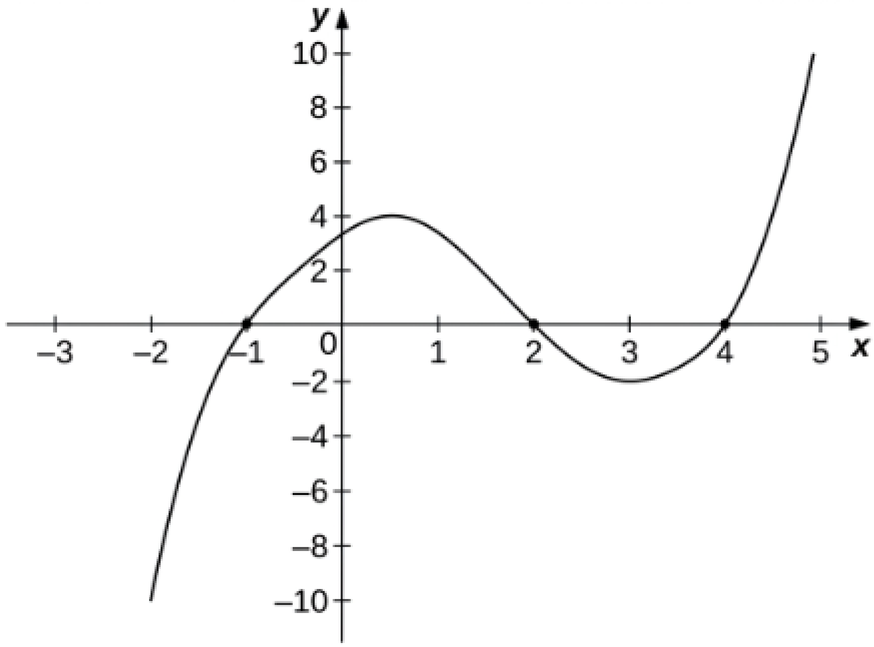
A. $\sin^2(\theta)$

B. $\cos^2(\theta)$

C. $\sec^2(\theta)$

D. $\csc^2(\theta)$

19. The graph of a polynomial function p(x) is shown below. Which statement about p(x) is true?



A. p(x) has degree 4

B. p(x) has three real zeros

C. p(x) has end behavior such that $p(x) \to +\infty$ as $x \to -\infty$

D. p(x) has no real zeros

20. An initial deposit of $1,000 is invested at an annual interest rate of 5 percent, compounded annually. To the nearest dollar, what is the value of the investment after 6 years?

A. $1,300

B. $1,250

C. $1,320

D. $1,340

21. What is the average rate of change of $g(x) = x^3 - x$ on the interval $[-1, 2]$?

A. 2

B. 3

C. 6

D. 4

22. What is the solution to the equation $2^{(x - 1)} = 32$?

A. $x = 4$

B. $x = 5$

C. $x = 6$

D. $x = 16$

23. The function $y = -4 \sin(\pi x) + 7$ has midline

A. $y = -4$

B. $y = 4$

C. $y = \pi$

D. $y = 7$

24. A 95 percent confidence interval for a population proportion is reported as (0.42, 0.58). What is the sample proportion used to construct this interval?

A. 0.42

B. 0.50

C. 0.58

D. 0.16

PART II — Short Constructed Response (8 questions × 2 credits = 16 credits)

Show all work. A correct answer with no supporting work will receive only 1 credit.

25. Solve the equation $x^2 - 6x + 13 = 0$ algebraically. Express your solutions in $a + bi$ form.

26. Solve algebraically for x: $\sqrt{(x + 5)} = x - 1$. Identify and reject any extraneous solutions.

27. Algebraically determine all real zeros of the polynomial $f(x) = x^3 + 2x^2 - 9x - 18$, using the method of factoring by grouping. Show all algebraic steps.

28. Solve the system of equations algebraically:

$y = x^2 - 1$

$y = 2x + 2$

State all solution points as ordered pairs.

29. Express the polynomial $f(x) = 3(x - 1)^2 - 12$ in standard form $ax^2 + bx + c$, and identify the values of a, b, and c.

30. Determine the equation of the inverse of the function $f(x) = 2 / (x + 3)$, and state the domain of $f^{-1}(x)$.

31. In a survey of high school students, 80 percent reported studying mathematics, 60 percent reported studying science, and 50 percent reported studying both subjects. Find the probability that a randomly selected student studies mathematics, given that the student studies science. Express your answer as a fraction in simplest form.

32. Verify the trigonometric identity $\cos^2(\theta) - \sin^2(\theta) = 1 - 2\sin^2(\theta)$. Show all algebraic steps that justify your work.

PART III — Extended Constructed Response (3 questions × 4 credits = 12 credits)

Show all work. Partial credit is awarded according to the scoring rubric.

33. The height $H(t)$, in feet, of a piston in a rotating engine wheel is given by $H(t) = 8\cos(2\pi t) + 10$, where t is the time in seconds since the piston was at its highest point.

(a) State the maximum and minimum heights of the piston, and algebraically determine the first time (in seconds) at which the piston reaches its minimum height.

(b) Algebraically determine all times during the first second of motion at which the piston is at a height of 14 feet. Round each answer to the nearest hundredth of a second.

34. The table below shows the stopping distance D, in feet, of a car at various speeds s, in miles per hour, on a particular road surface.

s (mph)	10	20	30	40	50
D (ft)	11	44	99	176	275

(a) Using quadratic regression, write a function $D(s) = as^2 + bs + c$ that best models the data. Round each coefficient to the nearest hundredth.

(b) Use the regression model from part (a) to predict the stopping distance at a speed of 60 mph and to determine the speed at which the stopping distance equals 400 feet. Round each answer to the nearest tenth.

35. Consider the polynomial function $f(x) = x^3 - 6x^2 + 11x - 6$.

(a) Use synthetic division to verify that $(x - 1)$ is a factor of $f(x)$. Show the complete synthetic division procedure.

(b) Using the quotient from part (a), factor $f(x)$ completely as a product of linear factors and state all real zeros of $f(x)$.

PART IV — Long Constructed Response (1 question × 6 credits = 6 credits)

Show all work. This problem requires multiple steps and integrates concepts from several chapters.

36. A water tank is being filled by a flow that gradually slows as the tank fills. The volume $V(t)$ of water in the tank, in liters, t minutes after filling begins is modeled by $V(t) = 200(1 - e^{-0.08t})$.

(a) State the maximum capacity of the tank as t becomes very large, and explain how you determined this value from the structure of the function $V(t)$.

(b) Algebraically determine the volume of water in the tank exactly 10 minutes after filling begins. Round your answer to the nearest tenth of a liter.

(c) Algebraically determine the time required for the tank to reach 90 percent of its maximum capacity. Round your answer to the nearest tenth of a minute, and show all algebraic work, including the use of natural logarithms to solve the resulting equation.

PRACTICE EXAM 20 – ANSWER KEY WITH EXPLANATIONS

1. C — Evaluate each function independently: $f(2) = 2^2 - 3(2) = 4 - 6 = -2$ and $g(-1) = 2(-1) + 1 = -1$. The sum is $-2 + (-1) = -3$. Function notation requires substituting the input value into the variable expression and following the order of operations.

2. B — The logarithm log_b(N) asks "to what power must b be raised to produce N?" Since $2^5 = 32$, $\log_2(32) = 5$. Recognizing N as a known power of the base is the fastest route to evaluating a logarithm.

3. A — The product $(3 - 2i)(3 + 2i)$ matches the conjugate pattern $(a - b)(a + b) = a^2 - b^2$. Substituting: $3^2 - (2i)^2 = 9 - 4i^2 = 9 - 4(-1) = 13$. Multiplying conjugates always eliminates the imaginary part, yielding a purely real result.

4. D — Exponential decay requires the base of the exponential expression to lie strictly between 0 and 1. Only $y = 100(0.75)^x$ has a base (0.75) in that range; the other exponential options have bases greater than 1 (growth) or are not exponential at all. A base below 1 produces values that decrease toward 0 as x increases.

5. A — The co-function identity states $\sin(\pi/2 - x) = \cos(x)$, reflecting the complementary relationship between sine and cosine. Co-function identities arise because $\pi/2$ radians (90°) is the complement of x within a right triangle, where sine of one acute angle equals cosine of the other.

6. C — Adding 16 to both sides gives $4x^2 = 16$, then dividing by 4 yields $x^2 = 4$. Taking the square root of both sides gives $x = \pm 2$ (both signs because squaring eliminates sign information). The ± must always appear when solving by extracting the square root.

7. B — In the form $g(x) = f(x + h)$, positive h inside the parentheses shifts the graph horizontally by $-h$. Here $h = +5$, so the graph shifts 5 units to the left. The horizontal shift convention is the opposite of the apparent sign inside the function argument.

8. D — The geometric mean of two positive numbers is the square root of their product: $GM = \sqrt{8 \cdot 18} = \sqrt{144} = 12$. The geometric mean is the appropriate "average" for proportional or multiplicative quantities, unlike the arithmetic mean used for additive ones.

9. C — A rational function is undefined where the denominator equals zero. Setting $x - 3 = 0$ gives $x = 3$. Even though the numerator $2x - 6 = 2(x - 3)$ shares this factor, the original function is still undefined at $x = 3$ (the point appears as a hole rather than a vertical asymptote).

10. A — Apply the quotient rule for logarithms: $\log_2(x) - \log_2(3) = \log_2(x/3) = 4$. Converting to exponential form gives $x/3 = 2^4 = 16$, so $x = 48$. Logarithm properties consolidate sums or differences into a single logarithm before exponential conversion.

11. B — Substitute $k = 2, 3, 4, 5$ into $(k + 1)$ and add: $(2 + 1) + (3 + 1) + (4 + 1) + (5 + 1) = 3 + 4 + 5 + 6 = 18$. Sigma notation compactly represents finite sums that must be evaluated term-by-term and totaled.

12. D — Solving $2\cos(x) + 1 = 0$ gives $\cos(x) = -1/2$. Cosine equals $-1/2$ at reference angle $\pi/3$ in Quadrants II and III, producing $x = \pi - \pi/3 = 2\pi/3$ and $x = \pi + \pi/3 = 4\pi/3$ within $[0, 2\pi)$. Combining sign analysis with reference-angle work locates all unit-circle solutions in the specified interval.

13. A — To invert $y = \sqrt{x - 1}$, swap variables and solve: $x = \sqrt{y - 1}$, so squaring gives $x^2 = y - 1$ and $y = x^2 + 1$. The domain of the inverse is $x \geq 0$ because the range of the original (the principal square root) is $y \geq 0$. Domain and range swap when functions are inverted.

14. B — The values 40 and 60 are each exactly 1 standard deviation from the mean (since $(60 - 50)/10 = 1$ and $(40 - 50)/10 = -1$). By the empirical rule, approximately 68 percent of normally distributed data falls within 1 SD of the mean. The 68-95-99.7 rule applies whenever boundaries align with whole-number standard deviations.

15. C — Apply the difference of squares twice: $x^4 - 81 = (x^2)^2 - 9^2 = (x^2 - 9)(x^2 + 9)$, then factor $x^2 - 9$ further as $(x - 3)(x + 3)$. The factor $x^2 + 9$ cannot be factored over the real numbers because its discriminant is negative. Complete factorization requires applying every applicable pattern until no further real factoring is possible.

16. A — In the expansion of $(a + b)^n$, the $(k + 1)^{th}$ term is $C(n, k) \cdot a^{n-k} \cdot b^k$. For $(2x - 3)^4$ with 5 terms total, the middle term corresponds to $k = 2$: $C(4, 2)(2x)^2(-3)^2 = 6 \cdot 4x^2 \cdot 9 = 216x^2$. The middle of an even-indexed expansion sits at the symmetric center of the binomial coefficients.

17. D — Apply the binomial probability formula $P(X = k) = C(n, k) \cdot p^k \cdot (1 - p)^{n-k}$. With $n = 4$, $k = 2$, $p = 1/6$: $P = C(4, 2) \cdot (1/6)^2 \cdot (5/6)^2 = 6 \cdot (1/36) \cdot (25/36) = 150/1296 = 25/216$. Each roll is independent, and the binomial coefficient counts the number of arrangements producing exactly two sixes.

18. C — Dividing the Pythagorean identity $\sin^2\theta + \cos^2\theta = 1$ by $\cos^2\theta$ yields $\tan^2\theta + 1 = \sec^2\theta$. This identity is one of the three standard Pythagorean identities and is the basis for many trigonometric simplifications involving the tangent and secant ratios.

19. B — A polynomial's real zeros correspond to its x-axis crossings. The graph shows the curve crossing the x-axis at three distinct points ($x = -1$, $x = 2$, and $x = 4$), giving exactly three real zeros. The end behavior (rising from $-\infty$ on the left to $+\infty$ on the right) and the cubic shape are consistent with degree 3 and positive leading coefficient.

20. D — Use the compound interest formula $A = P(1 + r/n)^{(nt)}$ with $P = 1000$, $r = 0.05$, $n = 1$, $t = 6$: $A = 1000(1.05)^6 \approx 1000(1.34010) \approx \$1,340.10$, which rounds to \$1,340. Annual compounding applies the rate exactly once per year, so $n = 1$.

21. A — Average rate of change equals $(f(b) - f(a))/(b - a)$. With $g(-1) = -1 + 1 = 0$ and $g(2) = 8 - 2 = 6$: rate $= (6 - 0)/(2 - (-1)) = 6/3 = 2$. This value represents the slope of the secant line connecting the two endpoints of the function on the interval.

22. C — Rewrite both sides as powers of 2: $32 = 2^5$, so $2^{(x - 1)} = 2^5$. Equating exponents gives $x - 1 = 5$, so $x = 6$. The one-to-one property of exponential functions allows direct comparison of exponents once the bases match.

23. D — For $y = A \sin(Bx) + D$, the midline is the horizontal line $y = D$, representing the vertical center of oscillation. With $D = 7$, the midline is $y = 7$. The midline is unaffected by amplitude or period and represents the function's mean output value.

24. B — A confidence interval for a proportion is symmetric about the sample proportion, so $\hat{p}$ equals the midpoint of the interval: $(0.42 + 0.58)/2 = 1.00/2 = 0.50$. The margin of error 0.08 spreads equally above and below this center to form the confidence bounds.

PART II — Short Constructed Response Solutions

25. Apply the quadratic formula with $a = 1$, $b = -6$, $c = 13$: $x = (6 \pm \sqrt{36 - 52})/2 = (6 \pm \sqrt{-16})/2 = (6 \pm 4i)/2 =$ $\mathbf{3 \pm 2i}$. Solutions: $x = 3 + 2i$ and $x = 3 - 2i$.

26. Domain requirement: $x - 1 \geq 0$ (the principal square root is nonnegative), so $x \geq 1$. Squaring both sides: $x + 5 = (x - 1)^2 = x^2 - 2x + 1$ $0 = x^2 - 3x - 4$ $(x - 4)(x + 1) = 0 \rightarrow x = 4$ or $x = -1$. Check $x = 4$: $\sqrt{9} = 3$, and $4 - 1 = 3$ ✓ Check $x = -1$: $\sqrt{4} = 2$, but $-1 - 1 = -2 \rightarrow$ **$x = -1$ rejected as extraneous**. Only valid solution: **$x = 4$**.

27. Group the four terms in pairs: $f(x) = x^3 + 2x^2 - 9x - 18 = (x^3 + 2x^2) + (-9x - 18) = x^2(x + 2) - 9(x + 2) = (x + 2)(x^2 - 9)$ Factor $x^2 - 9$ as a difference of squares: $f(x) = (x + 2)(x - 3)(x + 3)$. **Real zeros: $x = -2$, $x = 3$, $x = -3$**.

28. Set the two expressions for y equal: $x^2 - 1 = 2x + 2$ $x^2 - 2x - 3 = 0$ $(x - 3)(x + 1) = 0 \rightarrow x = 3$ or $x = -1$. For $x = 3$: $y = 2(3) + 2 = 8 \rightarrow (3, 8)$. For $x = -1$: $y = 2(-1) + 2 = 0 \rightarrow (-1, 0)$. Solution points: **(3, 8) and (−1, 0)**.

29. Expand the vertex form: $f(x) = 3(x - 1)^2 - 12 = 3(x^2 - 2x + 1) - 12 = 3x^2 - 6x + 3 - 12 =$ $\mathbf{3x^2 - 6x - 9}$. **$a = 3$, $b = -6$, $c = -9$**.

30. Solve $y = 2/(x + 3)$ for x: $y(x + 3) = 2$ $x + 3 = 2/y$ $x = 2/y - 3$. Swap variables: **$f^{-1}(x) = 2/x - 3$**. The variable x appears in a denominator, so $x \neq 0$. **Domain of $f^{-1}(x)$: $x \neq 0$** (all real numbers except 0).

31. Apply the conditional probability formula $P(M \mid S) = P(M \cap S)/P(S)$: $P(M \mid S) = 0.50 / 0.60 = 50/60 =$ **5/6**.

32. Begin with the left-hand side: $\cos^2\theta - \sin^2\theta$ Apply the Pythagorean identity $\sin^2\theta + \cos^2\theta = 1$, so $\cos^2\theta = 1 - \sin^2\theta$. LHS = $(1 - \sin^2\theta) - \sin^2\theta$ = $\mathbf{1 - 2\sin^2\theta}$ = RHS ✓. The identity is verified.

PART III — Extended Constructed Response Solutions

33. (a) The cosine function ranges between −1 and 1, so $8\cos(2\pi t)$ ranges between −8 and 8, and H(t) ranges between 10 − 8 and 10 + 8. **Maximum height: 18 feet. Minimum height: 2 feet.** The minimum occurs when $\cos(2\pi t) = -1$, which requires $2\pi t = \pi$, so **t = 1/2 second**.

(b) Set H(t) = 14: $14 = 8\cos(2\pi t) + 10$ $4 = 8\cos(2\pi t)$ $\cos(2\pi t) = 1/2$. Within one cycle ($0 \leq 2\pi t < 2\pi$), cosine equals 1/2 at $2\pi t = \pi/3$ and $2\pi t = 5\pi/3$. Solving: $t = 1/6 \approx 0.1667$ and $t = 5/6 \approx 0.8333$. Rounded to the nearest hundredth: **t ≈ 0.17 second and t ≈ 0.83 second**.

34. (a) Compute ratios D/s^2: 11/100 = 0.11, 44/400 = 0.11, 99/900 = 0.11, 176/1600 = 0.11, 275/2500 = 0.11. The ratio is constant at 0.11, indicating a pure quadratic model with no linear or constant terms.

Rounded to the nearest hundredth: $\mathbf{D(s) = 0.11s^2 + 0.00s + 0.00}$.

(b) Predict D(60): $D(60) = 0.11(60)^2 = 0.11(3600) = 396$. Rounded to the nearest tenth: **D(60) ≈ 396.0 feet**.

Solve D(s) = 400: $400 = 0.11s^2$ $s^2 = 400/0.11 \approx 3636.36$ $s \approx \sqrt{3636.36} \approx 60.302$. Rounded to the nearest tenth: **s ≈ 60.3 mph**.

35. (a) Synthetic division of $x^3 - 6x^2 + 11x - 6$ by $(x - 1)$, using c = 1:

1 | 1 −6 11 −6 | 1 −5 6 | 1 −5 6 0

The remainder is **0**, confirming by the Factor Theorem that **(x − 1) is a factor** of f(x).

(b) The quotient from the synthetic division is $x^2 - 5x + 6$. Factor: $x^2 - 5x + 6 = (x - 2)(x - 3)$. Complete factorization: **f(x) = (x − 1)(x − 2)(x − 3)**. **Real zeros: x = 1, x = 2, x = 3**.

PART IV — Long Constructed Response Solution

36. (a) As t becomes very large, the term $e^{-0.08t}$ approaches 0 because the exponent is large and negative. The model therefore approaches V → 200(1 − 0) = **200 liters**. The structure of $V(t) = 200(1 - e^{-0.08t})$ reveals this limit: the factor 200 is multiplied by a quantity that grows from 0 (at t = 0) toward 1 (as $t \to \infty$), so V is bounded above by 200, which represents the maximum capacity of the tank.

(b) Evaluate V(10): $V(10) = 200(1 - e^{-0.08 \cdot 10}) = 200(1 - e^{-0.8})$ $e^{-0.8} \approx 0.4493$ $V(10) \approx 200(1 - 0.4493) = 200(0.5507) \approx 110.134$. Rounded to the nearest tenth: **V(10) ≈ 110.1 liters**.

(c) Ninety percent of the maximum capacity equals 0.90 × 200 = 180 liters. Set V(t) = 180: $180 = 200(1 - e^{-0.08t})$ $0.90 = 1 - e^{-0.08t}$ $e^{-0.08t} = 0.10$.

Take the natural logarithm of both sides: $-0.08t = \ln(0.10) = -\ln(10) \approx -2.3026$ $t = 2.3026/0.08 \approx 28.782$. Rounded to the nearest tenth: **$t \approx 28.8$ minutes**.