PRACTICE EXAM 19: ALGEBRA II REGENTS SIMULATION

PART I — Multiple Choice (24 questions × 2 credits = 48 credits)

1. What is the value of i^{17}?

A. 1

B. i

C. -1

D. $-i$

2. Which expression is equivalent to $(x + 2)^3$?

A. $x^3 + 6x^2 + 12x + 8$

B. $x^3 + 8$

C. $x^3 + 3x^2 + 3x + 8$

D. $x^3 + 2x^2 + 4x + 8$

3. What is the vertex form of $f(x) = x^2 - 6x + 11$?

A. $f(x) = (x - 3)^2 - 2$

B. $f(x) = (x + 3)^2 + 2$

C. $f(x) = (x + 3)^2 - 2$

D. $f(x) = (x - 3)^2 + 2$

4. What is the remainder when $x^3 + 2x^2 - 5x + 4$ is divided by $(x + 3)$?

A. −22

B. −8

C. 4

D. 10

5. Which function has a period of 8?

A. $f(x) = \sin(8x)$

B. $f(x) = \sin(2x)$

C. $f(x) = \sin(\pi x / 4)$

D. $f(x) = \sin(x / 8)$

6. Solve algebraically for x: 9^(x + 1) = 27^(2x).

A. x = 1

B. x = 1/2

C. x = 2

D. x = −1/2

7. What is the inverse of $f(x) = 5^x$?

A. $f^{-1}(x) = \log_5(x)$

B. $f^{-1}(x) = 1 / 5^x$

C. $f^{-1}(x) = x^5$

D. $f^{-1}(x)$ = log_x(5)

8. What is the 12th term of the sequence defined by $a_n = 3n - 7$?

A. 7

B. 36

C. 29

D. 25

9. Which function has end behavior such that $f(x) \to +\infty$ as $x \to -\infty$ and $f(x) \to +\infty$ as $x \to +\infty$?

A. $f(x) = -2x^4 + 3x^2$

B. $f(x) = x^3 + 5x$

C. $f(x) = -3x^2 + 7$

D. $f(x) = 4x^6 - 2x^2 + 1$

10. What is the exact value of $\sin(11\pi / 6)$?

A. 1/2

B. −1/2

C. $\sqrt{3} / 2$

D. $-\sqrt{3} / 2$

11. Heights of adult women are normally distributed with a mean of 64 inches and a standard deviation of 3 inches. Approximately what percent of women are taller than 70 inches?

A. 2.5%

B. 5%

C. 16%

D. 32%

12. Which expression is equivalent to $\ln(8) - 2\ln(2)$?

A. ln(2)

B. ln(4)

C. ln(6)

D. 0

13. If $f(x) = x - 4$ and $g(x) = x^2 + 1$, what is the value of $(g \circ f)(5)$?

A. 22

B. 0

C. 2

D. 5

14. A baseball is thrown vertically and its height in feet is modeled by $h(t) = -16t^2 + 64t + 5$, where t is in seconds. What is the maximum height of the ball, in feet?

A. 5

B. 64

C. 32

D. 69

15. What is the solution set of the equation $\log(x^2) = 4$?

A. $x = \pm 10$

B. $x = \pm 100$

C. $x = 10$ only

D. $x = 100$ only

16. A fair coin is tossed 5 times. What is the probability of obtaining heads on all 5 tosses?

A. 1/5

B. 5/32

C. 1/32

D. 1/10

17. How many complex roots, counted with multiplicity, does the polynomial $f(x) = (x - 2)^3(x^2 + 4)(x + 1)$ have?

A. 6

B. 3

C. 5

D. 4

18. Solve algebraically for x: $x + 6 / x = 7$.

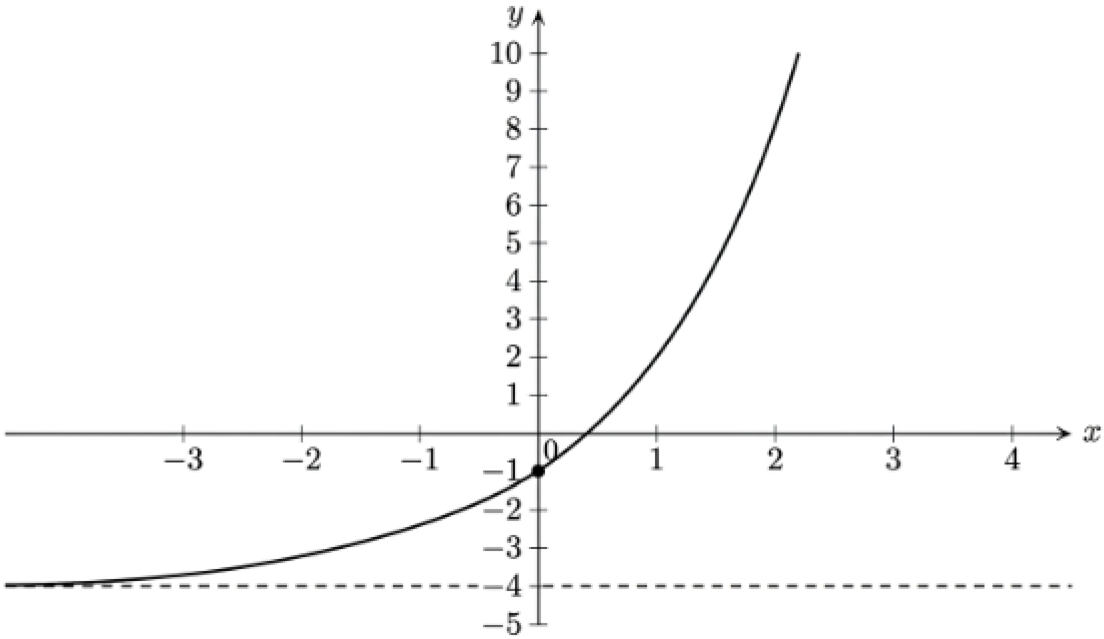
A. $x = 1$ only

B. $x = 6$ only

C. $x = -1$ and $x = -6$

D. $x = 1$ and $x = 6$

19. The exponential function $f(x)$ is graphed below. What is the y-intercept of f?



A. 3

B. −1

C. 0

D. −4

20. Which formula represents the future value A of a principal P invested for t years at an annual interest rate r, compounded n times per year?

A. A = P + rt

B. A = P · e^(rt)

C. A = P(1 + r / n)^(nt)

D. $A = P(1 + r)^t$

21. The expression $1 - 2\sin^2(\theta)$ is equivalent to

A. sin(2θ)

B. $\cos^2(\theta) + \sin^2(\theta)$

C. 1

D. cos(2θ)

22. What is the sum of the first 8 terms of the geometric sequence 1, 2, 4, 8, ...?

A. 255

B. 128

C. 256

D. 511

23. Which of the following best describes the purpose of conducting a simulation in statistics?

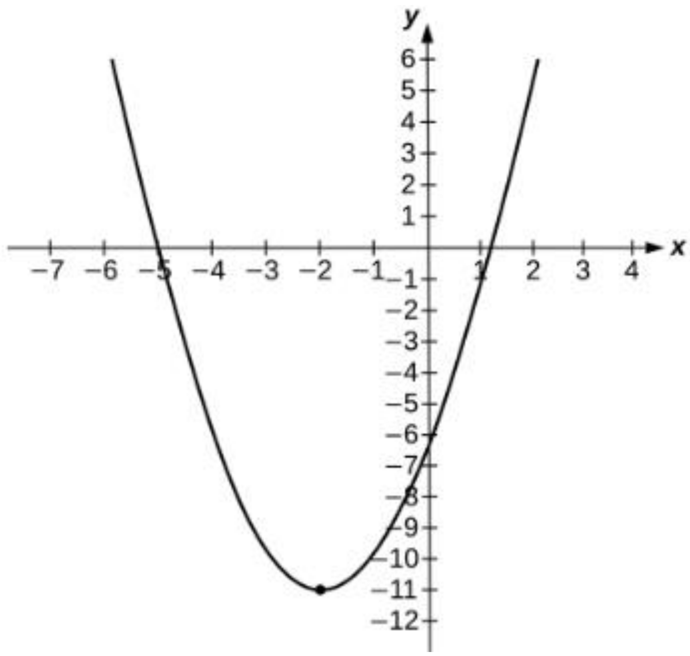
A. To collect raw observational data without statistical bias

B. To approximate probability or model behavior when direct calculation is impractical

C. To replace experimental design entirely for any research study

D. To eliminate sampling error from all observational studies

24. The graph of the parabola $y = ax^2 + bx + c$ is shown below. What is the equation of its axis of symmetry?



A. x = 2

B. x = −8

C. x = −2

D. x = 4

PART II — Short Constructed Response (8 questions × 2 credits = 16 credits)

Show all work. A correct answer with no supporting work will receive only 1 credit.

25. Solve the equation $2x^2 - 6x + 5 = 0$ algebraically. Express your solutions in a + bi form.

26. Solve algebraically for x: $\log_3(x + 5) = 2$.

27. Determine all real zeros of the polynomial $p(x) = x^3 - 4x^2 - 11x + 30$ algebraically. Show all algebraic work, including any factoring or polynomial division.

28. Express the function $f(x) = 2x^2 - 12x + 5$ in vertex form by completing the square. State the coordinates of the vertex.

29. Find the sum of the first 25 terms of the arithmetic sequence whose first term is 7 and common difference is 4. Show the formula used and substitute the given values.

30. A box contains 6 white balls and 4 black balls. Two balls are drawn at random without replacement. Find the probability that one ball is white and the other is black. Express your answer as a fraction in simplest form.

31. Solve the trigonometric equation $2 \sin(\theta) = 1$ algebraically for all values of θ in the interval $0 \leq \theta < 2\pi$. Express each solution in radians as an exact value.

32. Given the complex numbers $z_1 = 2 - 3i$ and $z_2 = 1 + 4i$, compute the product $z_1 \cdot z_2$ and express the result in $a + bi$ form.

PART III — Extended Constructed Response (3 questions × 4 credits = 12 credits)

Show all work. Partial credit is awarded according to the scoring rubric.

33. A cup of coffee cools according to Newton's Law of Cooling. The temperature of the coffee $T(t)$, in degrees Fahrenheit, t minutes after it is poured is modeled by the function $T(t) = 70 + 110\ e^{(-0.05t)}$.

(a) Determine the initial temperature of the coffee (at $t = 0$) and state the room temperature that the coffee approaches as t becomes very large. Justify your answer using the structure of the function.

(b) Algebraically determine the time, in minutes, it takes for the coffee to cool to 100°F. Round your answer to the nearest tenth of a minute.

34. Scores on a standardized state achievement test are normally distributed with a mean of 500 and a standard deviation of 100.

(a) Using the empirical rule, estimate the percentage of test takers who scored between 400 and 600. Justify your reasoning.

(b) A student earned a score of 720 on the test. Algebraically calculate this student's z-score and interpret what the z-score indicates about the student's performance relative to the population.

35. Consider the rational expression $(x^2 - 4) / (x^2 - x - 6)$.

(a) Simplify the expression completely by factoring both the numerator and denominator. State any restrictions on the variable x.

(b) Determine the value of the simplified expression when $x = 5$, and verify your result by also substituting $x = 5$ into the original (unsimplified) expression. Show all algebraic work.

PART IV — Long Constructed Response (1 question × 6 credits = 6 credits)

Show all work. This problem requires multiple steps and integrates concepts from several chapters.

36. Alex deposits $2,000 into Account A, which earns 4 percent annual interest compounded annually. On the same day, Bailey deposits $1,500 into Account B, which earns 6 percent annual interest compounded annually. Neither account has additional deposits or withdrawals.

(a) Write a function A(t) modeling Alex's balance and a function B(t) modeling Bailey's balance, where t represents the number of years after the initial deposit.

(b) Algebraically determine each account balance after 10 years. Round each answer to the nearest cent.

(c) Algebraically determine the value of t, to the nearest tenth of a year, at which Bailey's account balance first exceeds Alex's account balance. Show all algebraic work, including the use of logarithms to solve the resulting equation.

PRACTICE EXAM 19 – ANSWER KEY WITH EXPLANATIONS

1. B — Powers of i cycle every four exponents: $i^1 = i$, $i^2 = -1$, $i^3 = -i$, $i^4 = 1$. Dividing 17 by 4 leaves a remainder of 1, so $i^{17} = i^1 = i$. The cyclical pattern reduces any large integer power of i to one of four standard values.

2. A — Apply the binomial cube formula $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$ with a = x and b = 2: $x^3 + 3x^2(2) + 3x(4) + 8 = x^3 + 6x^2 + 12x + 8$. The cube of a binomial is never just $a^3 + b^3$ — the cross terms are essential to the expansion.

3. D — Take half the linear coefficient −6 (which is −3), square it (9), and rewrite by adding and subtracting 9: $x^2 - 6x + 9 + 11 - 9 = (x - 3)^2 + 2$. The vertex form $f(x) = (x - h)^2 + k$ reveals the vertex at (3, 2) directly.

4. D — By the Remainder Theorem, the remainder when p(x) is divided by (x − c) equals p(c). Here c = −3, so p(−3) = (−27) + 2(9) − 5(−3) + 4 = −27 + 18 + 15 + 4 = 10. The theorem replaces polynomial long division with a single substitution.

5. C — For a sine function in the form y = sin(Bx), the period equals 2π/|B|. With B = π/4, the period is 2π ÷ (π/4) = 8. The smaller B is in magnitude, the longer the period of the resulting sine curve.

6. B — Rewrite both sides with base 3: $9 = 3^2$ and $27 = 3^3$, giving 3^(2x + 2) = 3^(6x). Equating exponents produces 2x + 2 = 6x, so 4x = 2 and x = 1/2. The one-to-one property of exponential functions allows direct comparison of exponents once the bases match.

7. A — The inverse of any exponential function b^x is the logarithm with the same base, log_b(x). Swapping x and y in $y = 5^x$ gives $x = 5^y$, which converts to $y = \log_5(x)$. Exponential and logarithmic functions are inverses by definition.

8. C — The formula $a_n = 3n - 7$ is explicit, so substitute n = 12 directly: $a_{12} = 3(12) - 7 = 36 - 7 = 29$. Explicit formulas allow any term to be evaluated immediately without iterating through earlier terms.

9. D — End behavior is governed by the leading term. Both ends rising to +∞ requires an even degree (so both ends behave alike) and a positive leading coefficient (so they rise rather than fall). The term $4x^6$ satisfies both conditions, while the other options have negative coefficients or odd degree.

10. B — The angle 11π/6 lies in Quadrant IV, where sine is negative. Its reference angle is 2π − 11π/6 = π/6, and sin(π/6) = 1/2, so sin(11π/6) = −1/2. Combining quadrant signs with reference-angle values recovers any unit-circle sine value.

11. A — The value 70 is exactly 2 standard deviations above the mean (since (70 − 64)/3 = 2). By the empirical rule, approximately 95 percent of data lies within 2 SD of the mean, leaving 5 percent in the two tails combined. Half of that, about 2.5 percent, lies in the upper tail above 70 inches.

12. A — Apply the power rule: $2 \ln(2) = \ln(2^2) = \ln(4)$. The expression becomes ln(8) − ln(4) = ln(8/4) = ln(2) by the quotient rule. Logarithm properties convert differences into single logarithms of quotients.

13. C — Function composition evaluates inside-out: $f(5) = 5 - 4 = 1$, then $g(1) = 1^2 + 1 = 2$. The order matters because $(g \circ f)(x)$ means g is applied to the output of f, not the other way around.

14. D — The vertex of $h(t) = -16t^2 + 64t + 5$ occurs at $t = -b/(2a) = -64/(-32) = 2$ seconds. Substituting: $h(2) = -16(4) + 64(2) + 5 = -64 + 128 + 5 = 69$ feet. The vertex of a downward-opening parabola represents its maximum value.

15. B — Converting $\log(x^2) = 4$ to exponential form gives $x^2 = 10^4 = 10{,}000$. Solving yields $x = \pm 100$, since both 100^2 and $(-100)^2$ equal 10,000. The squaring inside the logarithm allows both signs to produce a valid positive argument.

16. C — Coin tosses are independent events, so the probability of 5 heads in a row equals $(1/2)^5 = 1/32$. Independent events multiply probabilities, and each toss has probability 1/2 of landing heads.

17. A — By the Fundamental Theorem of Algebra, a polynomial has exactly as many complex roots (counting multiplicity) as its degree. Multiplying the factors gives degree $3 + 2 + 1 = 6$, so f has 6 complex roots in total. The factor $(x - 2)^3$ alone contributes 3 roots through its multiplicity.

18. D — Multiply both sides of $x + 6/x = 7$ by x to clear the denominator: $x^2 + 6 = 7x$. Rearranging gives $x^2 - 7x + 6 = 0$, which factors as $(x - 1)(x - 6) = 0$, yielding $x = 1$ or $x = 6$. Both solutions are nonzero, so neither is extraneous.

19. B — The y-intercept is the point at which the graph crosses the y-axis ($x = 0$). The graph shows the curve passing through $(0, -1)$ marked with a solid dot, so $f(0) = -1$. The y-intercept is read directly from the y-coordinate of that intersection.

20. C — The general compound interest formula is $A = P(1 + r/n)^{(nt)}$, where the interest rate r is divided by the number of compoundings per year n, and the exponent nt counts the total number of compounding periods. Continuous compounding uses $P \cdot e^{(rt)}$ (option B), while simple interest uses $P + rt$ (option A).

21. D — The double-angle identity for cosine states $\cos(2\theta) = 1 - 2\sin^2(\theta)$. The expression is therefore equivalent to $\cos(2\theta)$. This identity is one of three equivalent forms ($\cos^2\theta - \sin^2\theta$ and $2\cos^2\theta - 1$ being the others) commonly used to simplify expressions involving squared trigonometric ratios.

22. A — Use the geometric sum formula $S_n = a(r^n - 1)/(r - 1)$ with $a = 1$, $r = 2$, $n = 8$: $S_8 = (2^8 - 1)/(2 - 1) = (256 - 1)/1 = 255$. The formula collapses an entire geometric series into a single arithmetic calculation.

23. B — A simulation models a random process using repeated trials (often computer-generated) to approximate probabilities or distributions that would be analytically difficult or impossible to compute. Simulations do not replace experimental design or eliminate sampling error; they are tools for estimating likelihoods when direct mathematical analysis is impractical.

24. C — The axis of symmetry of a parabola is the vertical line passing through its vertex. The graph shows the vertex marked at the point $(-2, -11)$, and the symmetric pair of points $(1, -2)$ and $(-5, -2)$

confirms the line of symmetry at $x = -2$. The axis of symmetry always takes the form x = (x-coordinate of vertex).

PART II — Short Constructed Response Solutions

25. Apply the quadratic formula with $a = 2$, $b = -6$, $c = 5$: $x = (6 \pm \sqrt{36 - 40})/4 = (6 \pm \sqrt{-4})/4 = (6 \pm 2i)/4$ = **$3/2 \pm (1/2)i$**. Solutions: $x = 3/2 + (1/2)i$ and $x = 3/2 - (1/2)i$.

26. Convert $\log_3(x + 5) = 2$ to exponential form: $x + 5 = 3^2 = 9$ $x = 9 - 5 =$ **4**. Check restriction: $x + 5 = 9 > 0$ ✓.

27. Apply the Rational Root Theorem; test $x = 2$: $p(2) = 8 - 16 - 22 + 30 = 0$ ✓, so $(x - 2)$ is a factor. Synthetic division by $(x - 2)$:

2 | 1 −4 −11 30 | 2 −4 −30 | 1 −2 −15 0

Quotient: $x^2 - 2x - 15 = (x - 5)(x + 3)$. Complete factorization: $p(x) = (x - 2)(x - 5)(x + 3)$. **Real zeros: $x = -3$, $x = 2$, $x = 5$**.

28. Complete the square: $f(x) = 2x^2 - 12x + 5 = 2(x^2 - 6x) + 5 = 2(x^2 - 6x + 9) - 18 + 5 =$ **$2(x - 3)^2 - 13$**. **Vertex: $(3, -13)$**.

29. Apply the arithmetic series formula $S_n = (n/2)(2a_1 + (n - 1)d)$ with $a_1 = 7$, $d = 4$, $n = 25$: $S_{25} = (25/2)(2(7) + 24(4)) = (25/2)(14 + 96) = (25/2)(110) = 25 \cdot 55 =$ **1,375**.

30. Total balls = 10. Calculate the two ordered ways one of each color can be drawn: P(W then B) = $(6/10)(4/9) = 24/90$ P(B then W) = $(4/10)(6/9) = 24/90$ Total: P(one white, one black) = $24/90 + 24/90 = 48/90 =$ **8/15**.

31. Solve $2 \sin(\theta) = 1$: $\sin(\theta) = 1/2$ Sine equals 1/2 at reference angle $\pi/6$ in Quadrants I and II. In $[0, 2\pi)$: **$\theta = \pi/6$ and $\theta = 5\pi/6$**.

32. Multiply $(2 - 3i)(1 + 4i)$ using the distributive property: $= 2(1) + 2(4i) - 3i(1) - 3i(4i) = 2 + 8i - 3i - 12i^2 = 2 + 5i - 12(-1) = 2 + 5i + 12 =$ **$14 + 5i$**.

PART III — Extended Constructed Response Solutions

33. (a) **Initial temperature:** $T(0) = 70 + 110 \cdot e^0 = 70 + 110(1) =$ **180°F**. **Room temperature:** As $t \to \infty$, the term $e^{-0.05t}$ approaches 0, so $T(t) \to 70 + 110(0) =$ **70°F**. The constant 70 in the model represents the ambient room temperature that the coffee asymptotically approaches.

(b) Set $T(t) = 100$ and solve: $100 = 70 + 110 e^{-0.05t}$ $30 = 110 e^{-0.05t}$ $3/11 = e^{-0.05t}$ $\ln(3/11) = -0.05t$ $t = \ln(3/11)/(-0.05)$ $\ln(3/11) \approx -1.2993$ $t \approx -1.2993/(-0.05) \approx 25.99$. Rounded to the nearest tenth: **$t \approx 26.0$ minutes**.

34. (a) The values 400 and 600 are each exactly 1 standard deviation from the mean (since $(600 - 500)/100 = 1$ and $(400 - 500)/100 = -1$). By the empirical rule, approximately **68 percent** of normally distributed data fall within 1 standard deviation of the mean, so about 68 percent of test takers scored between 400 and 600.

(b) $z = (x - \mu)/\sigma = (720 - 500)/100 = 220/100 =$ **2.2**. **Interpretation:** This student's score is 2.2 standard deviations above the mean, indicating very strong performance well above the typical range. Approximately 1.4 percent of test takers scored at or above this level under a normal distribution, placing the student near the top of the score distribution.

35. (a) Factor numerator and denominator: Numerator: $x^2 - 4 = (x - 2)(x + 2)$ Denominator: $x^2 - x - 6 = (x - 3)(x + 2)$ Canceling the common factor $(x + 2)$: Simplified form: $\mathbf{(x - 2)/(x - 3)}$. **Restrictions:** $x \neq 3$ (denominator zero in simplified form) AND $x \neq -2$ (denominator zero in the original expression). Both restrictions must be stated because the canceled factor $(x + 2)$ hides one of them.

(b) At $x = 5$ in the simplified form: $(5 - 2)/(5 - 3) = 3/2$. Verification with the original expression: $(25 - 4)/(25 - 5 - 6) = 21/14 = 3/2$ ✓. Both forms yield the same value, **3/2**, confirming the simplification is correct.

PART IV — Long Constructed Response Solution

36. (a) **Alex's balance:** $A(t) = 2000(1.04)^t$. **Bailey's balance:** $B(t) = 1500(1.06)^t$.

(b) After 10 years:

$\mathbf{A(10) = 2000(1.04)^{10}}$ $(1.04)^2 = 1.0816$ $(1.04)^4 = (1.0816)^2 \approx 1.16986$ $(1.04)^5 \approx 1.21665$ $(1.04)^{10} = (1.21665)^2 \approx 1.48024$ $A(10) \approx 2000 \times 1.48024 \approx$ **$2,960.49**.

$\mathbf{B(10) = 1500(1.06)^{10}}$ $(1.06)^2 = 1.1236$ $(1.06)^5 \approx 1.33823$ $(1.06)^{10} \approx (1.33823)^2 \approx 1.79085$ $B(10) \approx 1500 \times 1.79085 \approx$ **$2,686.27**.

After 10 years, Alex's balance exceeds Bailey's by approximately $274.22.

(c) Set $B(t) = A(t)$: $1500(1.06)^t = 2000(1.04)^t$ Divide both sides by $1500(1.04)^t$: $(1.06/1.04)^t = 2000/1500 = 4/3$.

Take natural logarithms of both sides: $t \cdot \ln(1.06/1.04) = \ln(4/3)$ $t = \ln(4/3) / \ln(1.06/1.04)$.

Evaluating: $\ln(4/3) \approx 0.28768$ $\ln(1.06/1.04) = \ln(1.06) - \ln(1.04) \approx 0.05827 - 0.03922 \approx 0.01905$ $t \approx 0.28768 / 0.01905 \approx 15.10$.

Rounded to the nearest tenth: $\mathbf{t \approx 15.1}$ **years**. Bailey's higher interest rate eventually overtakes Alex's larger initial principal at approximately 15.1 years after the deposits.