PRACTICE EXAM 18: ALGEBRA II REGENTS SIMULATION

PART I — Multiple Choice (24 questions × 2 credits = 48 credits)

1. What is the multiplicative inverse of $(2 + 3i)$?

A. $2 + 3i$

B. $-2 + 3i$

C. $2/5 - 3/5\ i$

D. $2/13 - 3/13\ i$

2. If $P(x)$ is a polynomial of degree 5 with real coefficients, what is the minimum possible number of real zeros?

A. 0

B. 5

C. 1

D. 3

3. For what values of k does the equation $x^2 + kx + 9 = 0$ have two distinct real solutions?

A. $k = 6$ or $k = -6$

B. $-6 < k < 6$

C. $k > 6$ or $k < -6$

D. all real values of k

4. For the function $y = -3\cos(2x) + 4$, the amplitude is

A. 3

B. −3

C. 2

D. 4

5. In a geometric sequence, the second term is 12 and the fifth term is 324. What is the common ratio?

A. 4

B. 3

C. 27

D. 9

6. Which expression is equivalent to $\log_3(50)$?

A. log(3) / log(50)

B. 50 / 3

C. 3 / 50

D. log(50) / log(3)

7. What is the value of arctan(1) in radians, using the principal range of the arctangent function?

A. π/2

B. π/4

C. π/3

D. π/6

8. What is the y-intercept of the function $f(x) = (x - 2)(x + 3)(x - 5)$?

A. 30

B. −30

C. 0

D. −10

9. A function $f(x)$ is reflected over the x-axis and then translated up 7 units to produce a new function $g(x)$. Which expression represents $g(x)$?

A. $g(x) = f(-x) + 7$

B. $g(x) = f(x) - 7$

C. $g(x) = -f(x) - 7$

D. $g(x) = -f(x) + 7$

10. Solve algebraically for x: $e^{(2x)} = 7$.

A. ln(3.5)

B. ln(14)

C. 2 ln(7)

D. (1/2) ln(7)

11. Which function has domain all real numbers and range $y \geq 3$?

A. $y = (x - 3)^2$

B. $y = \sqrt{(x - 3)}$

C. $y = x^2 + 3$

D. $y = |x - 3|$

12. If $\log_b(M) = 4$ and $\log_b(N) = 6$, what is the value of $\log_b(MN^2)$?

A. 24

B. 16

C. 10

D. 14

13. A bank account earns 6 percent annual interest compounded continuously. The balance $B(t)$ is modeled by $B(t) = 1000 e^{0.06t}$. How long, to the nearest tenth of a year, will it take for the balance to double?

A. 11.6 years

B. 16.7 years

C. 33.3 years

D. 12.0 years

14. Which expression is equivalent to $(\sin\theta + \cos\theta)^2$?

A. $1 - \sin(2\theta)$

B. $2 + \sin(2\theta)$

C. $1 + \sin(2\theta)$

D. $\sin^2(\theta) + \cos^2(\theta)$

15. Which of the following could NOT be a rational root of $f(x) = 2x^3 - 5x^2 + x - 6$?

A. 3

B. $-1/2$

C. 6

D. 4

16. In a survey of 200 voters, 110 reported that they support candidate X. What is the sample proportion?

A. 0.55

B. 110

C. 200

D. 0.45

17. A spinner is divided into 4 equal sectors numbered 1, 2, 3, and 4. If the spinner is spun twice, what is the probability that the sum of the two results is 5?

A. 1/8

B. 1/4

C. 1/2

D. 1/16

18. What is the value of the summation Σ (from k = 1 to 4) of k^2?

A. 14

B. 10

C. 16

D. 30

19. What is the period of the function y = tan(2x)?

A. π/2

B. π

C. 2π

D. π/4

20. A circle is defined by the equation $x^2 + y^2 + 6x - 4y - 12 = 0$. What are the coordinates of its center and the length of its radius?

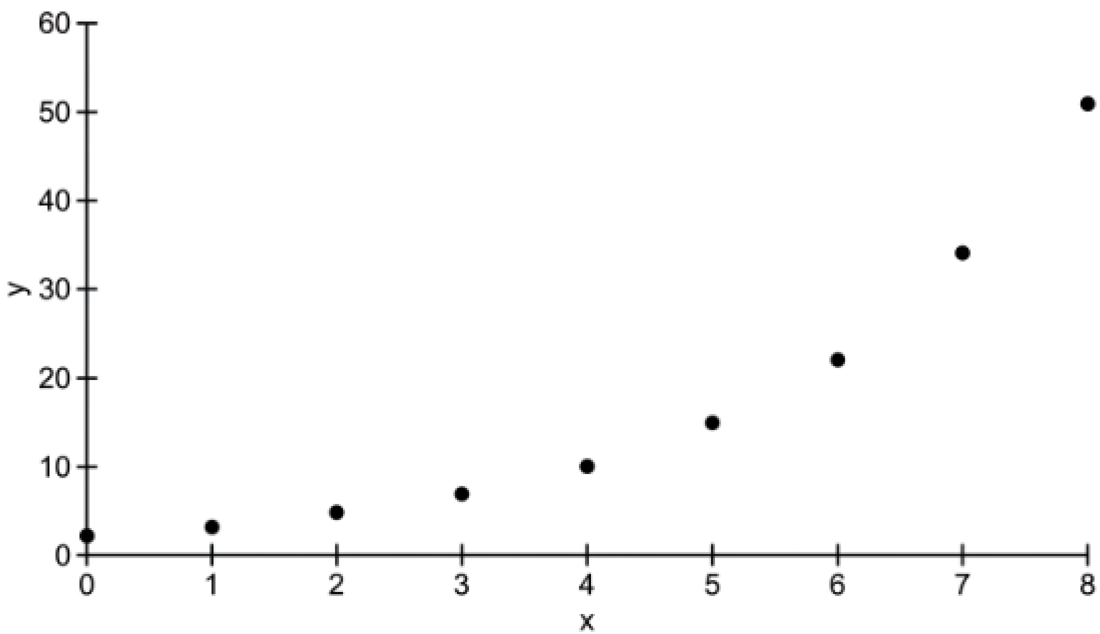
A. Center $(3, -2)$, radius 5

B. Center $(-3, 2)$, radius $\sqrt{12}$

C. Center $(-3, 2)$, radius 5

D. Center $(6, -4)$, radius 12

21. The scatter plot below shows the relationship between two variables x and y. Which type of regression model would best fit the data?



A. Linear

B. Logarithmic

C. Exponential

D. Constant

22. In a standard normal distribution, approximately what percentage of values fall between $z = -1$ and $z = 1$?

A. 50%

B. 68%

C. 95%

D. 99.7%

23. Add and simplify: $x / (x + 2) + 2 / (x - 1)$.

A. $(x^2 + x + 4) / [(x + 2)(x - 1)]$

B. $(x^2 - x + 4) / [(x + 2)(x - 1)]$

C. $(x + 2) / [(x + 2)(x - 1)]$

D. $(x^2 + 4) / [(x + 2)(x - 1)]$

24. What is the coefficient of x^2 in the expansion of $(3x + 1)^4$?

A. 12

B. 54

C. 36

D. 6

PART II — Short Constructed Response (8 questions × 2 credits = 16 credits)

Show all work. A correct answer with no supporting work will receive only 1 credit.

25. Solve the equation $x^2 + 8x + 25 = 0$ algebraically. Express your solutions in $a + bi$ form.

26. Solve algebraically for x: $3^{(x + 2)} = 7$. Round your answer to the nearest hundredth.

27. Use the binomial theorem to expand $(x - 2)^4$. Write the result as a polynomial in standard form.

28. Find the inverse function $g^{-1}(x)$ for $g(x) = 4x - 7$. Then evaluate $g(g^{-1}(11))$ and explain why this result confirms the inverse relationship.

29. Solve algebraically for x: $\log_2(x + 3) - \log_2(x - 1) = 2$. State any extraneous solutions and indicate which (if any) are rejected.

30. A sample of student heights has a mean of 165 cm and a standard deviation of 8 cm. Calculate the z-score for a student whose height is 174 cm. Round your answer to the nearest hundredth, and interpret what the z-score indicates about this student's height relative to the sample.

31. The sum of an infinite geometric series is 36 and the first term is 24. Find the common ratio r. Show the algebraic work that justifies your answer.

32. Verify the trigonometric identity $\sin(\theta) \cdot \sec(\theta) = \tan(\theta)$. Show all algebraic steps.

PART III — Extended Constructed Response (3 questions × 4 credits = 12 credits)

Show all work. Partial credit is awarded according to the scoring rubric.

33. The population of a town is modeled by the function $P(t) = 5000\ e^{(0.04t)}$, where t represents the number of years after the year 2000.

(a) State the initial population in the year 2000 and the continuous annual growth rate of the population. Explain what the exponent 0.04 represents.

(b) Algebraically determine the year in which the population will first reach 8,000 residents. Round your answer to the nearest whole year.

34. The table below shows the number of users N(t), in thousands, of a social media application after t years of operation.

t (years)	1	2	3	4	5
N(t) (thousands)	50	85	105	119	130

(a) Using logarithmic regression, write a function of the form $N(t) = a + b \ln(t)$ that best models the data. Round each coefficient to the nearest tenth.

(b) Use the regression model from part (a) to predict the number of users, in thousands, after 10 years of operation. Round your answer to the nearest tenth.

35. Consider the function $f(x) = 2x^2 - 4x + 5$, with the domain restricted to $x \geq 1$.

(a) Algebraically find the equation for $f^{-1}(x)$. Show all steps, including completing the square as part of your method.

(b) State the domain and range of $f^{-1}(x)$, and explain how they relate to the domain and range of the original function $f(x)$.

PART IV — Long Constructed Response (1 question × 6 credits = 6 credits)

Show all work. This problem requires multiple steps and integrates concepts from several chapters.

36. A high school chess club begins the school year with 30 members. The number of club members $M(t)$ at the end of week t is modeled by a piecewise function:

$M(t) = 30 + 4t$ for $0 \leq t \leq 10$ (linear growth from new student recruitment)

$M(t) = 70(0.95)^{(t - 10)}$ for $t > 10$ (gradual exponential decline as members leave for other activities)

(a) Determine the maximum number of members the club has during the school year, and identify the week at which this maximum occurs. Justify your answer using the structure of the piecewise function.

(b) Algebraically determine the week in which the membership count first drops below 50 members. Round your answer to the nearest tenth of a week.

(c) Calculate the average rate of change of M(t) on the interval from t = 0 to t = 10, and the average rate of change on the interval from t = 10 to t = 20. Compare the two rates in context, explaining what each value represents about the club's membership trends. Show all algebraic work.

PRACTICE EXAM 18 – ANSWER KEY WITH EXPLANATIONS

1. D — The multiplicative inverse of a + bi is 1/(a + bi). Multiplying numerator and denominator by the conjugate (2 − 3i) gives (2 − 3i)/[(2 + 3i)(2 − 3i)] = (2 − 3i)/(4 + 9) = (2 − 3i)/13 = 2/13 − (3/13)i. Multiplying by the conjugate eliminates i from the denominator using the difference-of-squares pattern.

2. C — Complex roots of a polynomial with real coefficients always occur in conjugate pairs, so the count of non-real roots must be even. A degree-5 polynomial has exactly 5 total roots, leaving at least 1 real root (5 minus an even number is at least 1). Every odd-degree polynomial therefore must cross the x-axis at least once.

3. C — Two distinct real solutions require a strictly positive discriminant. For $x^2 + kx + 9 = 0$, the discriminant is $k^2 - 36$, which must satisfy $k^2 > 36$, equivalent to $|k| > 6$. This produces two disjoint intervals: k > 6 or k < −6.

4. A — In the sinusoidal form y = a cos(bx) + d, the amplitude equals |a|. With a = −3, the amplitude is |−3| = 3. The negative sign produces a vertical reflection of the cosine curve but does not change the amplitude itself.

5. B — Using $a_n = a_1 \cdot r^{n-1}$, the ratio $a_5/a_2 = r^3$ equals 324/12 = 27, so $r = \sqrt[3]{27} = 3$. Dividing two distant terms isolates a power of r because the first-term factor cancels.

6. D — The change-of-base formula states log_b(N) = log(N)/log(b) for any consistent base used on the right side. Substituting b = 3 and N = 50 yields log(50)/log(3). This formula allows logarithms in any base to be evaluated using common or natural logarithms.

7. B — The arctangent function returns the angle whose tangent equals the input value, with principal range (−π/2, π/2). Since tan(π/4) = 1 and π/4 lies within that range, arctan(1) = π/4. The principal-range restriction guarantees a unique output for each input.

8. A — The y-intercept is the value of f(0). Substituting x = 0: f(0) = (0 − 2)(0 + 3)(0 − 5) = (−2)(3)(−5) = 30. An even number of negative factors multiplies to a positive result.

9. D — Reflection over the x-axis multiplies the output by -1, producing $-f(x)$. A vertical translation up 7 units adds 7 to the result, giving $g(x) = -f(x) + 7$. The reflection is performed before the translation because each point is first flipped, then shifted upward.

10. D — Taking the natural logarithm of both sides of $e^{2x} = 7$ gives $2x = \ln(7)$, so $x = (1/2) \ln(7)$. The natural logarithm is the inverse of the base-e exponential function, allowing direct isolation of the exponent.

11. C — The function $y = x^2 + 3$ is defined for every real x and has minimum value 3 at $x = 0$ since $x^2 \geq 0$ always. Its range is therefore $[3, \infty)$, matching $y \geq 3$. The +3 vertical shift raises the parabola's vertex from (0, 0) to (0, 3).

12. B — Apply the product rule $\log_b(MN^2) = \log_b(M) + \log_b(N^2)$ and the power rule $\log_b(N^2) = 2 \log_b(N)$. Substituting: $\log_b(MN^2) = 4 + 2(6) = 4 + 12 = 16$. Logarithm properties convert multiplication and exponentiation into addition.

13. A — Setting $B(t) = 2000$ gives $e^{0.06t} = 2$, so $0.06t = \ln(2) \approx 0.6931$. Solving: $t \approx 0.6931/0.06 \approx 11.55$, rounded to 11.6 years. Continuous compounding doubles any principal in time $t = \ln(2)/r$, independent of the starting amount.

14. C — Expand the square: $(\sin \theta + \cos \theta)^2 = \sin^2\theta + 2 \sin \theta \cos \theta + \cos^2\theta$. Apply the Pythagorean identity $\sin^2\theta + \cos^2\theta = 1$ and the double-angle identity $2 \sin \theta \cos \theta = \sin(2\theta)$, giving $1 + \sin(2\theta)$. Two identities collapse the expanded form into a clean expression.

15. D — The Rational Root Theorem states that any rational root must have the form $\pm p/q$, where p divides the constant term (6) and q divides the leading coefficient (2). The complete list of possibilities is $\pm\{1, 2, 3, 6, 1/2, 3/2\}$. The value 4 does not appear because 4 is not a factor of 6.

16. A — Sample proportion equals the count of successes divided by the sample size: $\hat{p} = 110/200 = 0.55$. Sample proportions are decimal values between 0 and 1 that serve as point estimates of the underlying population proportion.

17. B — Total equally likely outcomes for two spins equal $4 \times 4 = 16$. Favorable outcomes summing to 5 are (1, 4), (2, 3), (3, 2), and (4, 1), totaling 4 pairs. Therefore $P(\text{sum} = 5) = 4/16 = 1/4$.

18. D — Substitute $k = 1, 2, 3, 4$ into k^2 and add: $1 + 4 + 9 + 16 = 30$. Sigma notation compactly represents a finite sum that must be evaluated term-by-term and totaled.

19. A — The base function $y = \tan(x)$ has period π. For $y = \tan(bx)$, the period is $\pi/|b|$. With $b = 2$, the period equals $\pi/2$. Tangent's shorter natural period (compared to sine and cosine) means horizontal compression by a factor of 2 yields $\pi/2$.

20. C — Group x and y terms and complete the square on each: $(x^2 + 6x + 9) + (y^2 - 4y + 4) = 12 + 9 + 4$. This yields $(x + 3)^2 + (y - 2)^2 = 25$, matching the standard form $(x - h)^2 + (y - k)^2 = r^2$. Reading off: center $(-3, 2)$, radius $\sqrt{25} = 5$.

21. C — The plotted points show very slow initial growth that accelerates dramatically — the signature of exponential growth, in which successive y-values multiply by a roughly constant factor. Linear regression produces constant slope, and logarithmic regression decelerates, so neither matches the upward-curving acceleration. An exponential model of the form $y = a \cdot b^x$ fits the pattern best.

22. B — The empirical rule states that approximately 68 percent of values in a normal distribution lie within one standard deviation of the mean. The interval from $z = -1$ to $z = 1$ corresponds exactly to this one-SD band. The 68-95-99.7 rule provides quick estimates for whole-number SD boundaries in any normal distribution.

23. A — Use common denominator $(x + 2)(x - 1)$: rewrite as $x(x - 1)/[(x + 2)(x - 1)] + 2(x + 2)/[(x + 2)(x - 1)]$. The numerators combine as $x^2 - x + 2x + 4 = x^2 + x + 4$, producing $(x^2 + x + 4)/[(x + 2)(x - 1)]$. The combined numerator does not factor over the rationals, so further simplification is impossible.

24. B — In the binomial expansion of $(a + b)^n$, the general term is $C(n, k) \cdot a^{n-k} \cdot b^k$. For $(3x + 1)^4$, the x^2 term requires $(3x)$ to appear with exponent 2, so $k = 2$: $C(4, 2) \cdot (3x)^2 \cdot (1)^2 = 6 \cdot 9x^2 \cdot 1 = 54x^2$. The coefficient of x^2 is therefore 54.

PART II — Short Constructed Response Solutions

25. Apply the quadratic formula with $a = 1$, $b = 8$, $c = 25$: $x = (-8 \pm \sqrt{(64 - 100)})/2 = (-8 \pm \sqrt{(-36)})/2 = (-8 \pm 6i)/2 = \mathbf{-4 \pm 3i}$. Solutions: $x = -4 + 3i$ and $x = -4 - 3i$.

26. Take the natural logarithm of both sides: $3^{(x + 2)} = 7$ $(x + 2) \cdot \ln(3) = \ln(7)$ $x + 2 = \ln(7)/\ln(3) \approx 1.9459/1.0986 \approx 1.7712$ $x \approx 1.7712 - 2 = \mathbf{-0.23}$.

27. Apply the binomial theorem to $(x + (-2))^4$: $(x - 2)^4 = C(4,0)x^4 + C(4,1)x^3(-2) + C(4,2)x^2(-2)^2 + C(4,3)x(-2)^3 + C(4,4)(-2)^4 = x^4 + 4x^3(-2) + 6x^2(4) + 4x(-8) + 16 = \mathbf{x^4 - 8x^3 + 24x^2 - 32x + 16}$.

28. Solve $y = 4x - 7$ for x: $y + 7 = 4x \rightarrow x = (y + 7)/4$ Swap variables: $\mathbf{g^{-1}(x) = (x + 7)/4}$. Evaluate $g^{-1}(11) = (11 + 7)/4 = 18/4 = 9/2$. Evaluate $g(9/2) = 4(9/2) - 7 = 18 - 7 = \mathbf{11}$. The result returns the original input 11, confirming that $g(g^{-1}(x)) = x$ for any valid x.

29. Apply the quotient rule of logarithms: $\log_2((x + 3)/(x - 1)) = 2$ $(x + 3)/(x - 1) = 2^2 = 4$ $x + 3 = 4(x - 1) = 4x - 4$ $7 = 3x \rightarrow \mathbf{x = 7/3}$. Restrictions: $x + 3 > 0$ $(x > -3)$ AND $x - 1 > 0$ $(x > 1)$, so $x > 1$. Since $7/3 > 1$, the solution is valid. **No extraneous solutions** to reject.

30. Apply the z-score formula $z = (x - \mu)/\sigma$: $z = (174 - 165)/8 = 9/8 = \mathbf{1.125 \approx 1.13}$. Interpretation: This student's height is approximately 1.13 standard deviations above the sample mean, indicating that the student is somewhat taller than typical members of the sample (taller than about 87 percent of students under a normal distribution assumption).

31. The sum of an infinite geometric series with $|r| < 1$ is $S = a/(1 - r)$: $36 = 24/(1 - r)$ $36(1 - r) = 24$ $36 - 36r = 24$ $36r = 12 \rightarrow \mathbf{r = 1/3}$.

32. Rewrite $\sec(\theta)$ using its reciprocal definition: $\sin(\theta) \cdot \sec(\theta) = \sin(\theta) \cdot (1/\cos(\theta)) = \sin(\theta)/\cos(\theta) =$ **$\tan(\theta)$** = RHS ✓. The identity is verified.

PART III — Extended Constructed Response Solutions

33. (a) Initial population: $P(0) = 5000 \cdot e^0 =$ **5,000 residents**. Continuous annual growth rate: **0.04, or 4 percent per year**. The exponent 0.04 represents the instantaneous (continuous) rate at which the population is increasing — the proportional rate of growth at every instant of time, rather than a discrete annual percentage increase.

(b) Set $P(t) = 8000$: $8000 = 5000\, e^{(0.04t)}$ $1.6 = e^{(0.04t)}$ $\ln(1.6) = 0.04t$ $t = \ln(1.6)/0.04 \approx 0.4700/0.04 \approx 11.75$. Rounded to the nearest whole year, $t = 12$, meaning the population first reaches 8,000 in the year 2000 + 12 = **2012**.

34. (a) For a logarithmic model $N(t) = a + b \ln(t)$, at $t = 1$: $N(1) = a + b \ln(1) = a + 0 = a = 50$. Use $t = 2$: $85 = 50 + b \ln(2) \rightarrow b = 35/\ln(2) \approx 35/0.6931 \approx 50.5$. Cross-checking with the remaining data points (t = 3, 4, 5) confirms $b \approx 50$.

Rounded to the nearest tenth: **$N(t) = 50.0 + 50.0 \ln(t)$**.

(b) Predict for $t = 10$: $N(10) = 50 + 50 \cdot \ln(10) = 50 + 50(2.3026) \approx 50 + 115.13 \approx 165.13$. Rounded to the nearest tenth: **$N(10) \approx 165.1$ thousand users**.

35. (a) Begin with $f(x) = 2x^2 - 4x + 5$. Complete the square: $f(x) = 2(x^2 - 2x) + 5 = 2(x^2 - 2x + 1) - 2 + 5 = 2(x - 1)^2 + 3$.

Set $y = 2(x - 1)^2 + 3$ and solve for x: $y - 3 = 2(x - 1)^2$ $(y - 3)/2 = (x - 1)^2$ $x - 1 = \pm\sqrt{(y - 3)/2}$.

Since the domain is restricted to $x \geq 1$, $x - 1 \geq 0$, requiring the positive square root: $x = 1 + \sqrt{(y - 3)/2}$. Swapping variables: **$f^{-1}(x) = 1 + \sqrt{(x - 3)/2}$**.

(b) On the restricted domain $x \geq 1$, f is increasing with minimum $f(1) = 3$, so the range of f is $[3, \infty)$. **Domain of f^{-1}: $x \geq 3$** (which equals the range of f). **Range of f^{-1}: $y \geq 1$** (which equals the domain of f). The inverse relationship swaps domain and range because f and f^{-1} reflect each other across the line $y = x$.

PART IV — Long Constructed Response Solution

36. (a) During the linear phase ($0 \leq t \leq 10$), $M(t) = 30 + 4t$ increases steadily from 30 to 30 + 40 = 70. At $t = 10$, the exponential phase begins with $M(10) = 70(0.95)^0 = 70$, matching the linear endpoint. For $t > 10$, the factor $(0.95)^{(t - 10)} < 1$ and decreases as t grows, so M(t) decreases throughout the exponential phase. The maximum membership of **70 students occurs at t = 10 weeks**, the moment the recruitment phase ends and exponential decay begins.

(b) During the linear phase, M(t) increases from 30 toward 70 and stays above 50 from t = 5 onward; it never drops below 50 in this phase. After t = 10, M(t) decays from 70. Set M(t) = 50 and solve: $50 = 70(0.95)^{(t-10)}$ $50/70 = 5/7 = (0.95)^{(t-10)}$ $\ln(5/7) = (t-10) \cdot \ln(0.95)$ $t - 10 = \ln(5/7)/\ln(0.95) \approx (-0.3365)/(-0.0513) \approx 6.56$ $t \approx 10 + 6.56 =$ **16.6 weeks**.

(c) **Linear phase, t = 0 to t = 10:** Average rate of change = [M(10) − M(0)] / (10 − 0) = (70 − 30)/10 = 40/10 = **+4 members per week**. This represents the steady weekly gain in membership during the recruitment phase.

Decay phase, t = 10 to t = 20: $M(20) = 70(0.95)^{10} \approx 70(0.5987) \approx 41.91$. Average rate of change = [M(20) − M(10)] / (20 − 10) = (41.91 − 70)/10 ≈ −28.09/10 ≈ **−2.81 members per week**. This represents the weekly loss in membership during the decline phase.

Comparison: The club's growth rate during recruitment (+4 members/week) is larger in magnitude than its average loss rate during decline (−2.81 members/week), indicating that members joined faster than they later departed. The net effect over weeks 0–20 is positive growth from 30 to approximately 42 members.