

PRACTICE EXAM 17: ALGEBRA II

REGENTS SIMULATION

PART I — Multiple Choice (24 questions \times 2 credits = 48 credits)

1. Which expression is equivalent to $(x^3 - 1) / (x - 1)$ for $x \neq 1$?

- A. $x^2 + x + 1$
- B. $x^2 - x + 1$
- C. $x^2 + 1$
- D. $x^2 - 1$

2. What are the roots of the equation $4x^2 - 12x + 9 = 0$?

- A. $x = 3/2$ with multiplicity 2
- B. $x = 3/2$ and $x = -3/2$
- C. $x = 3$ and $x = -3/4$
- D. $x = 9/4$ with multiplicity 2

3. What is the solution to the equation $8^x = 32$?

- A. $x = 2/5$
- B. $x = 3/5$
- C. $x = 4$
- D. $x = 5/3$

4. The value of $\log_5(125)$ is

- A. 5
- B. 25
- C. 3
- D. $1/3$

5. Express $(4 + i)^2$ in $a + bi$ form.

- A. $17 + 8i$
- B. $15 + 8i$
- C. $16 + i$
- D. $15 - 8i$

6. The 10th term of the arithmetic sequence with $a_1 = -4$ and common difference $d = 5$ is

- A. 50
- B. 46
- C. 36
- D. 41

7. If $f(x) = x^2 + 1$ and $g(x) = \sqrt{x}$, what is the value of $f(g(9))$?

- A. 82
- B. 10
- C. 81
- D. 100

8. As $x \rightarrow -\infty$, the function $f(x) = 5x^5 - 2x^3 + 7$ approaches

- A. $+\infty$
- B. 7
- C. $-\infty$
- D. 0

9. What is the horizontal asymptote of $f(x) = (3x^2 + 2) / (x^2 - 5)$?

- A. $y = 3$
- B. $y = -5$
- C. $x = \sqrt{5}$
- D. $y = 0$

10. How many real solutions exist for the system $y = x^2 + 3$ and $y = 2x$?

- A. 1
- B. 2
- C. 3
- D. 0

11. What is the average rate of change of $f(x) = 2x^2 + 3x$ on the interval $[0, 4]$?

- A. 8
- B. 14
- C. 11
- D. 44

12. A radioactive isotope has a half-life of 8 years. Starting with 100 grams, the amount remaining after t years is given by $A(t) = 100(1/2)^{(t/8)}$. How many grams remain after 24 years?

- A. 50
- B. 25
- C. 75
- D. 12.5

13. The value of $\sin(-\pi/6)$ is

- A. $1/2$
- B. $-1/2$
- C. $\sqrt{3}/2$
- D. $-\sqrt{3}/2$

14. What is the solution to the equation $\log(x + 4) = 2$?

- A. $x = 96$
- B. $x = 6$
- C. $x = 4$
- D. $x = 100$

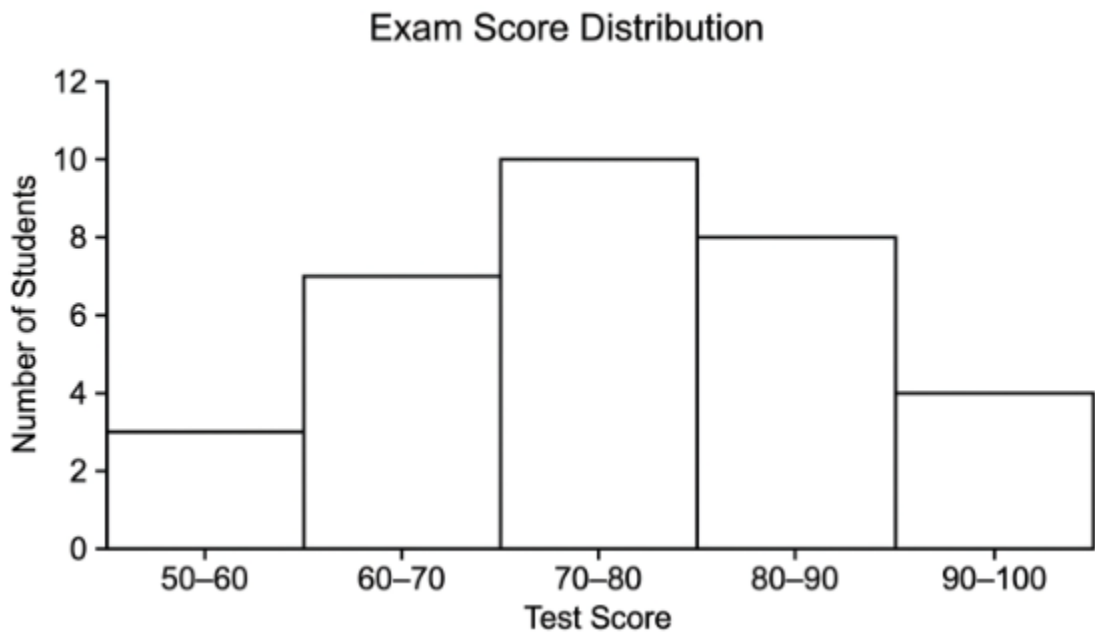
15. For a linear regression line of best fit, what does a residual value represent?

- A. The slope of the line of best fit
- B. The y-intercept of the line of best fit
- C. The vertical distance from a data point to the line
- D. The horizontal distance from a data point to the line

16. What is the inverse of $f(x) = 2x^3 + 1$?

- A. $f^{-1}(x) = (x - 1) / 2$
- B. $f^{-1}(x) = \sqrt[3]{2x + 1}$
- C. $f^{-1}(x) = (\sqrt[3]{x} - 1) / 2$
- D. $f^{-1}(x) = \sqrt[3]{(x - 1) / 2}$

17. The histogram below shows the distribution of scores on a recent exam. Approximately how many students scored at least 80 but less than 90?



- A. 4
- B. 8
- C. 10
- D. 12

18. A sequence is defined recursively by $a_1 = 2$ and $a_n = (a_{n-1})^2 - 1$ for $n \geq 2$. What is the value of a_4 ?

- A. 63

- B. 35
- C. 48
- D. 80

19. If $\cos \theta = -1/2$ and $\pi \leq \theta \leq 2\pi$, then θ equals

- A. $2\pi/3$
- B. $4\pi/3$
- C. $5\pi/3$
- D. $7\pi/6$

20. What value of k makes $(x - 2)$ a factor of $f(x) = x^3 - 3x^2 + kx - 4$?

- A. $k = -2$
- B. $k = 8$
- C. $k = 4$
- D. $k = -4$

21. What is the value of the summation Σ (from $k = 1$ to 5) of $(2k + 1)$?

- A. 35
- B. 30
- C. 25
- D. 21

22. In a class, 60 percent of students passed mathematics and 40 percent of students passed both mathematics and science. What is the probability that a randomly selected student passed science, given that the student passed mathematics?

- A. $1/2$
- B. $2/3$
- C. $3/5$
- D. $2/5$

23. The expression $3 \log(x) - 2 \log(y)$ can be written as a single logarithm as

- A. $\log(x^3 - y^2)$
- B. $\log(3x / 2y)$
- C. $\log(x^3 \cdot y^2)$
- D. $\log(x^3 / y^2)$

24. A circle has center $(3, -2)$ and passes through the point $(7, 1)$. What is the equation of this circle?

- A. $(x + 3)^2 + (y - 2)^2 = 25$
- B. $(x - 3)^2 + (y + 2)^2 = 5$
- C. $(x - 3)^2 + (y + 2)^2 = 25$
- D. $(x - 7)^2 + (y - 1)^2 = 25$

PART II — Short Constructed Response (8 questions \times 2 credits = 16 credits)

Show all work. A correct answer with no supporting work will receive only 1 credit.

25. Solve the equation $3x^2 + 4x + 2 = 0$ algebraically. Express your solutions in a + bi form.

26. Solve algebraically for x : $\sqrt{3x + 1} = x - 3$. Identify and reject any extraneous solutions.

27. Use synthetic division or long division to divide $(2x^3 - 7x^2 + 4x + 1)$ by $(x - 3)$. State the quotient and the remainder, and write the result in the form $\text{dividend} = (\text{divisor})(\text{quotient}) + \text{remainder}$.

28. Solve algebraically for x : $4^{(2x - 1)} = 64$.

29. Express the function $f(x) = x^2 + 8x + 18$ in vertex form by completing the square. State the coordinates of the vertex.

30. A fair coin is tossed 4 times. Find the probability of obtaining exactly 3 heads. Express your answer as a fraction in simplest form.

31. Find the sum of the first seven terms of the geometric sequence 8, 4, 2, 1, ... Express your answer as a fraction in simplest form.

32. Given that θ terminates in Quadrant II and $\sin \theta = 5/13$, find the exact value of $\sin(2\theta)$. Show the algebraic work that justifies your answer.

PART III — Extended Constructed Response (3 questions \times 4 credits = 12 credits)

Show all work. Partial credit is awarded according to the scoring rubric.

33. The pH level of a fluid is defined by the equation $\text{pH} = -\log[\text{H}^+]$, where $[\text{H}^+]$ is the concentration of hydrogen ions in moles per liter.

(a) A fruit juice sample has a hydrogen ion concentration of 5×10^{-4} moles per liter. Algebraically determine its pH. Round your answer to the nearest tenth.

(b) Pure water has a pH of 7. Algebraically determine the hydrogen ion concentration $[\text{H}^+]$ of pure water, and express your answer in scientific notation.

34. The table below shows the profit $P(x)$, in thousands of dollars, generated by a small business for various levels of monthly advertising spending x , in thousands of dollars.

| x (thousand \$) | 1 | 2 | 3 | 4 | 5 |

|---|---|---|---|---|---|

| P(x) (thousand \$) | 5 | 18 | 27 | 32 | 33 |

(a) Using regression, write a quadratic function $P(x) = ax^2 + bx + c$ that best models the data. Round each coefficient to the nearest tenth.

(b) Use the regression model from part (a) to determine the level of advertising spending that maximizes profit and the maximum profit attained. Round each answer to the nearest hundredth.

35. Given the polynomial function $f(x) = 2x^3 + x^2 - 8x - 4$.

(a) Algebraically factor $f(x)$ completely over the real numbers using the method of factoring by grouping. Show all steps.

(b) State all real zeros of $f(x)$ and the multiplicity of each zero.

PART IV — Long Constructed Response (1 question × 6 credits = 6 credits)

Show all work. This problem requires multiple steps and integrates concepts from several chapters.

36. A new car was purchased for \$25,000 and depreciates at an annual rate of 12 percent. Its value t years after purchase is modeled by the function $V(t) = 25000(0.88)^t$.

(a) State the initial value of the car and the annual decay factor, and explain what the decay factor 0.88 represents in the context of this depreciation model.

(b) Algebraically determine the value of the car 5 years after purchase. Round your answer to the nearest dollar.

(c) A second car was purchased on the same day for \$30,000 and depreciates at an annual rate of 15 percent. Its value t years after purchase is modeled by $W(t) = 30000(0.85)^t$. Algebraically determine the

number of years after purchase at which both cars will have the same value. Round your answer to the nearest tenth of a year. Show all algebraic work.

PRACTICE EXAM 17 – ANSWER KEY WITH EXPLANATIONS

1. A — Factor the numerator using the difference of cubes pattern: $x^3 - 1 = (x - 1)(x^2 + x + 1)$. Canceling the common factor $(x - 1)$ leaves $x^2 + x + 1$. The restriction $x \neq 1$ is preserved because the original denominator equals zero there.

2. A — The discriminant of $4x^2 - 12x + 9$ is $b^2 - 4ac = 144 - 144 = 0$, indicating a repeated real root. Applying $x = -b/(2a)$ gives $x = 12/8 = 3/2$ with multiplicity 2. A zero discriminant always corresponds to a parabola touching the x-axis at exactly one point.

3. D — Rewrite both sides with the common base 2: $8 = 2^3$ and $32 = 2^5$, so $2^{3x} = 2^5$. Equating exponents yields $3x = 5$, giving $x = 5/3$. The one-to-one property of exponential functions guarantees a unique solution whenever both sides share the same base.

4. C — The logarithm $\log_b(N)$ asks "to what power must b be raised to produce N ?" Since $5^3 = 125$, $\log_5(125) = 3$. Logarithmic evaluation hinges on recognizing N as a known power of the base.

5. B — Apply the binomial pattern $(a + b)^2 = a^2 + 2ab + b^2$: $(4 + i)^2 = 16 + 8i + i^2 = 16 + 8i + (-1) = 15 + 8i$. Substituting $i^2 = -1$ is the critical move that converts the imaginary square back into a real contribution.

6. D — For an arithmetic sequence, $a_n = a_1 + (n - 1)d$. Substituting $a_1 = -4$, $d = 5$, and $n = 10$ gives $a_{10} = -4 + 9(5) = -4 + 45 = 41$. The explicit formula reaches any term directly without iterating through earlier ones.

7. B — Function composition evaluates from the inside out: first compute $g(9) = \sqrt{9} = 3$, then $f(3) = 3^2 + 1 = 10$. The order in composition cannot be reversed because $f \circ g$ and $g \circ f$ generally produce different outputs.

8. C — End behavior is governed entirely by the leading term. The leading term $5x^5$ has odd degree and a positive coefficient, so $f(x) \rightarrow -\infty$ as $x \rightarrow -\infty$. Odd-degree polynomials with positive leading coefficients always fall on the left and rise on the right.

9. A — When the numerator and denominator have equal degrees, the horizontal asymptote equals the ratio of the leading coefficients. Here both degrees are 2, so $y = 3/1 = 3$. Lower-order terms become negligible as $|x|$ grows large, so the function approaches this ratio.

10. D — Setting $x^2 + 3 = 2x$ gives $x^2 - 2x + 3 = 0$, with discriminant $b^2 - 4ac = 4 - 12 = -8$. A negative discriminant means no real solutions, so the line and parabola do not intersect on the real plane. Geometrically, the parabola sits entirely above the line.

11. C — Average rate of change equals $(f(b) - f(a))/(b - a)$. With $f(0) = 0$ and $f(4) = 32 + 12 = 44$, the rate is $(44 - 0)/(4 - 0) = 11$. This value represents the slope of the secant line connecting the two endpoints on the curve.

12. D — After 24 years, the number of half-lives elapsed is $24/8 = 3$. The remaining amount is $100 \cdot (1/2)^3 = 100/8 = 12.5$ grams. Each half-life cuts the remaining quantity in half, and three successive halvings reduce 100 to 12.5.

13. B — Sine is an odd function, so $\sin(-x) = -\sin(x)$. Since $\sin(\pi/6) = 1/2$, $\sin(-\pi/6) = -1/2$. The reflection across the x-axis on the unit circle inverts the y-coordinate, which is exactly the sine value.

14. A — Convert the logarithmic equation to exponential form, using base 10 (the implied base when no base is shown): $x + 4 = 10^2 = 100$. Solving gives $x = 96$. The defining relationship $\log_b(y) = c \Leftrightarrow b^c = y$ underlies every log-equation solution.

15. C — A residual is defined as the difference between an observed y-value and the y-value predicted by the regression line at the same x. This difference is measured vertically because least-squares regression minimizes vertical distances. Residuals indicate how well individual data points are captured by the model.

16. D — To invert $y = 2x^3 + 1$, swap variables and solve: $x = 2y^3 + 1$, so $y^3 = (x - 1)/2$, giving $y = \sqrt[3]{((x - 1)/2)}$. The inverse reverses the original operations in reverse order — cube root undoes cubing, division by 2 undoes the multiplication, and subtraction of 1 undoes the addition.

17. B — The histogram bar covering the bin 80 to 90 has a height of 8, meaning 8 students fell within that score range. Histogram bar heights directly represent the frequency of values within each bin interval. Interpreting frequency requires reading the y-axis at each bar's top edge.

18. A — Apply the recursion step by step: $a_1 = 2$, then $a_2 = 2^2 - 1 = 3$, $a_3 = 3^2 - 1 = 8$, and $a_4 = 8^2 - 1 = 63$. Recursive sequences must be evaluated term by term, with each new term computed from the previous one using the defining rule.

19. B — Cosine equals $-1/2$ at reference angle $\pi/3$ in Quadrants II and III. The constraint $\pi \leq \theta \leq 2\pi$ restricts θ to Quadrants III and IV, leaving only the Quadrant III solution $\theta = \pi + \pi/3 = 4\pi/3$. Combining sign analysis with reference-angle work locates unit-circle solutions efficiently.

20. C — By the Factor Theorem, $(x - 2)$ is a factor of $f(x)$ if and only if $f(2) = 0$. Substituting gives $8 - 12 + 2k - 4 = 0$, so $2k - 8 = 0$ and $k = 4$. The Factor Theorem links polynomial divisibility to evaluation at a single point.

21. A — Expand the summation by substituting $k = 1$ through $k = 5$: $3 + 5 + 7 + 9 + 11 = 35$. The expression $2k + 1$ generates consecutive odd numbers starting at 3. Sigma notation compactly represents finite sums that can be evaluated term-by-term.

22. B — Apply the conditional probability formula $P(S | M) = P(S \cap M) / P(M)$. Substituting the given values: $P(S | M) = 0.4 / 0.6 = 2/3$. Conditional probability rescales the joint probability by the size of the conditioning event, effectively restricting the sample space to that subset.

23. D — Apply the power rule of logarithms to each term: $3 \log(x) = \log(x^3)$ and $2 \log(y) = \log(y^2)$. The difference becomes $\log(x^3) - \log(y^2) = \log(x^3 / y^2)$ by the quotient rule. Power and quotient rules together compress sums and differences of logs into a single logarithm.

24. C — The center of the circle is $(h, k) = (3, -2)$. The radius equals the distance from the center to the given point on the circle: $r = \sqrt{((7 - 3)^2 + (1 - (-2))^2)} = \sqrt{(16 + 9)} = 5$. Substituting into the standard form $(x - h)^2 + (y - k)^2 = r^2$ gives $(x - 3)^2 + (y + 2)^2 = 25$.

PART II — Short Constructed Response Solutions

25. Apply the quadratic formula with $a = 3$, $b = 4$, $c = 2$: $x = (-4 \pm \sqrt{(16 - 24)})/6 = (-4 \pm \sqrt{(-8)})/6 = (-4 \pm 2i\sqrt{2})/6 = -2/3 \pm (\sqrt{2}/3)i$. Solutions: $x = -2/3 + (\sqrt{2}/3)i$ and $x = -2/3 - (\sqrt{2}/3)i$.

26. Domain requirement: $x - 3 \geq 0$ (the principal square root is nonnegative), so $x \geq 3$. Squaring both sides: $3x + 1 = x^2 - 6x + 9$ $x^2 - 9x + 8 = 0$ $(x - 1)(x - 8) = 0 \rightarrow x = 1$ or $x = 8$. Check $x = 1$: $\sqrt{(3(1) + 1)} = \sqrt{4} = 2$, but $1 - 3 = -2 \rightarrow \mathbf{x = 1}$ rejected as extraneous. Check $x = 8$: $\sqrt{(3(8) + 1)} = \sqrt{25} = 5$, and $8 - 3 = 5$ ✓ Only valid solution: $\mathbf{x = 8}$.

27. Synthetic division of $2x^3 - 7x^2 + 4x + 1$ by $(x - 3)$ using $c = 3$:

$$3 \mid 2 \ -7 \ 4 \ 1 \mid 6 \ -3 \ 3 \mid 2 \ -1 \ 1 \ 4$$

Quotient: $2x^2 - x + 1$. Remainder: 4 . Result: $2x^3 - 7x^2 + 4x + 1 = (x - 3)(2x^2 - x + 1) + 4$.

28. Rewrite 64 as a power of 4: $64 = 4^3$. $4^{(2x - 1)} = 4^3$ $2x - 1 = 3$ $2x = 4 \rightarrow \mathbf{x = 2}$.

29. Take half of the linear coefficient 8 (which is 4), square it (16), and add and subtract: $f(x) = x^2 + 8x + 16 - 16 + 18 = (x + 4)^2 + 2$. Vertex form: $\mathbf{f(x) = (x + 4)^2 + 2}$. Vertex: $\mathbf{(-4, 2)}$.

30. Apply the binomial probability formula with $n = 4$, $k = 3$, $p = 1/2$: $P(X = 3) = C(4, 3) \cdot (1/2)^3 \cdot (1/2)^1 = 4 \cdot (1/8) \cdot (1/2) = 4/16 = \mathbf{1/4}$.

31. Use the geometric series formula $S_n = a(1 - r^n)/(1 - r)$ with $a = 8$, $r = 1/2$, $n = 7$: $S_7 = 8(1 - (1/2)^7)/(1 - 1/2) = 8(1 - 1/128)/(1/2) = 8 \cdot (127/128) \cdot 2 = 16 \cdot 127/128 = \mathbf{127/8}$.

32. Given $\sin \theta = 5/13$ with θ in Quadrant II, cosine is negative. $\cos^2 \theta = 1 - \sin^2 \theta = 1 - 25/169 = 144/169 \rightarrow \cos \theta = -12/13$. Apply the double-angle identity $\sin(2\theta) = 2 \sin \theta \cos \theta$: $\sin(2\theta) = 2 \cdot (5/13) \cdot (-12/13) = -120/169$.

PART III — Extended Constructed Response Solutions

33. (a) Apply the definition $\text{pH} = -\log[\text{H}^+]$ with $[\text{H}^+] = 5 \times 10^{-4}$: $\text{pH} = -\log(5 \times 10^{-4}) = -[\log(5) + \log(10^{-4})] = -[\log(5) - 4] = 4 - \log(5) \approx 4 - 0.6990 \approx 3.3010$. Rounded to the nearest tenth: **pH \approx 3.3**.

(b) Substitute $\text{pH} = 7$ and solve for $[\text{H}^+]$: $7 = -\log[\text{H}^+] \log[\text{H}^+] = -7 [\text{H}^+] = 10^{-7} = \mathbf{1 \times 10^{-7} \text{ moles per liter}}$.

34. (a) Compute first differences of $P(x)$: 13, 9, 5, 1. Compute second differences: -4, -4, -4 (constant), confirming a quadratic model with $2a = -4$, so $a = -2$. Using $P(1) = 5$: $-2 + b + c = 5 \rightarrow b + c = 7$. Using $P(2) = 18$: $-8 + 2b + c = 18 \rightarrow 2b + c = 26$. Subtracting: $b = 19$, $c = -12$.

Regression model rounded to the nearest tenth: **$P(x) = -2.0x^2 + 19.0x - 12.0$** .

(b) Maximum profit occurs at the vertex: $x = -b/(2a) = -19/(2 \cdot (-2)) = 4.75$. Maximum value: $P(4.75) = -2(4.75)^2 + 19(4.75) - 12 = -45.125 + 90.25 - 12 = 33.125$.

Rounded to the nearest hundredth: **Advertising spending: $x = 4.75$ thousand dollars (\$4,750) Maximum profit: $P = 33.13$ thousand dollars (\$33,130)**.

35. (a) Group the four terms in pairs: $f(x) = 2x^3 + x^2 - 8x - 4 = (2x^3 + x^2) + (-8x - 4) = x^2(2x + 1) - 4(2x + 1) = (2x + 1)(x^2 - 4)$ Factor $x^2 - 4$ as a difference of squares: **$f(x) = (2x + 1)(x - 2)(x + 2)$** .

(b) Setting each factor equal to zero: $2x + 1 = 0 \rightarrow x = -1/2$ (multiplicity 1) $x - 2 = 0 \rightarrow x = 2$ (multiplicity 1) $x + 2 = 0 \rightarrow x = -2$ (multiplicity 1) All three zeros are simple roots, each with multiplicity 1.

PART IV — Long Constructed Response Solution

36. (a) Initial value: $V(0) = 25000(0.88)^0 = \mathbf{\$25,000}$. Annual decay factor: **0.88**. The factor **0.88 represents the proportion of value retained each year** — the car keeps 88 percent of its previous year's value, equivalent to a 12 percent annual decrease in value.

(b) Compute $V(5) = 25000(0.88)^5$: $(0.88)^2 = 0.7744$ $(0.88)^4 = (0.7744)^2 = 0.59969536$ $(0.88)^5 = 0.59969536 \times 0.88 \approx 0.527732$ $V(5) \approx 25000 \times 0.527732 \approx 13193.32$. Rounded to the nearest dollar: **$V(5) \approx \$13,193$** .

(c) Set $V(t) = W(t)$ and solve for t : $25000(0.88)^t = 30000(0.85)^t$ Divide both sides by $25000(0.85)^t$: $(0.88/0.85)^t = 30000/25000 = 6/5$ Take natural logarithms: $t \cdot \ln(0.88/0.85) = \ln(6/5)$ $t = \ln(6/5) / \ln(0.88/0.85)$ $t = \ln(1.2) / \ln(1.03529)$ $t \approx 0.18232 / 0.03469$ $t \approx 5.255$. Rounded to the nearest tenth: **$t \approx 5.3$ years** after purchase.