PRACTICE EXAM 16: ALGEBRA II REGENTS SIMULATION

PART I — Multiple Choice (24 questions × 2 credits = 48 credits)

1. The expression $\log_2(8x^3) - \log_2(x)$ is equivalent to

A. $3\log_2(x)$ B. $3 + 2\log_2(x)$ C. $2 + 3\log_2(x)$ D. $\log_2(8x^4)$

2. If $f(x) = \ln(x + 2)$, then $f^{-1}(x)$ is equal to

A. $\ln(x) - 2$ B. $\ln(x) + 2$ C. e^x + 2 D. e^x − 2

3. What is the sum of the roots of the polynomial equation $2x^3 - 6x^2 + 4x - 5 = 0$?

A. −3 B. 5/2 C. 3 D. −5/2

4. Which value of c makes the trinomial $x^2 - 10x + c$ a perfect square trinomial?

A. 25 B. 100 C. 20 D. 5

5. What is the exact value of $\tan(5\pi/4)$?

A. 1 B. −1 C. √2 D. −√2

6. What is the sum of the first 20 terms of the arithmetic sequence 3, 7, 11, 15, ...?

A. 760 B. 1640 C. 200 D. 820

7. If $f(x) = 2x + 1$ and $g(x) = x^2 - 3$, then the value of $(f \circ g)(2)$ is

A. 11 B. 3 C. 7 D. 6

8. The function $f(x) = -2(x - 3)^2 + 5$ has its vertex at

A. (−3, 5) B. (3, −5) C. (3, 5) D. (−3, −5)

9. What is the solution to the equation 5^(x + 2) = 125?

A. x = 3 B. x = 25 C. x = 5 D. x = 1

10. A bag contains 4 red, 3 blue, and 5 green marbles. What is the probability of drawing two red marbles in a row without replacement?

A. 1/11 B. 1/9 C. 1/16 D. 1/6

11. The expression $(x^2 - 9) / (x - 3)$, where $x \neq 3$, is equivalent to

A. $x - 3$ B. $x + 3$ C. $x^2 + 3$ D. 1

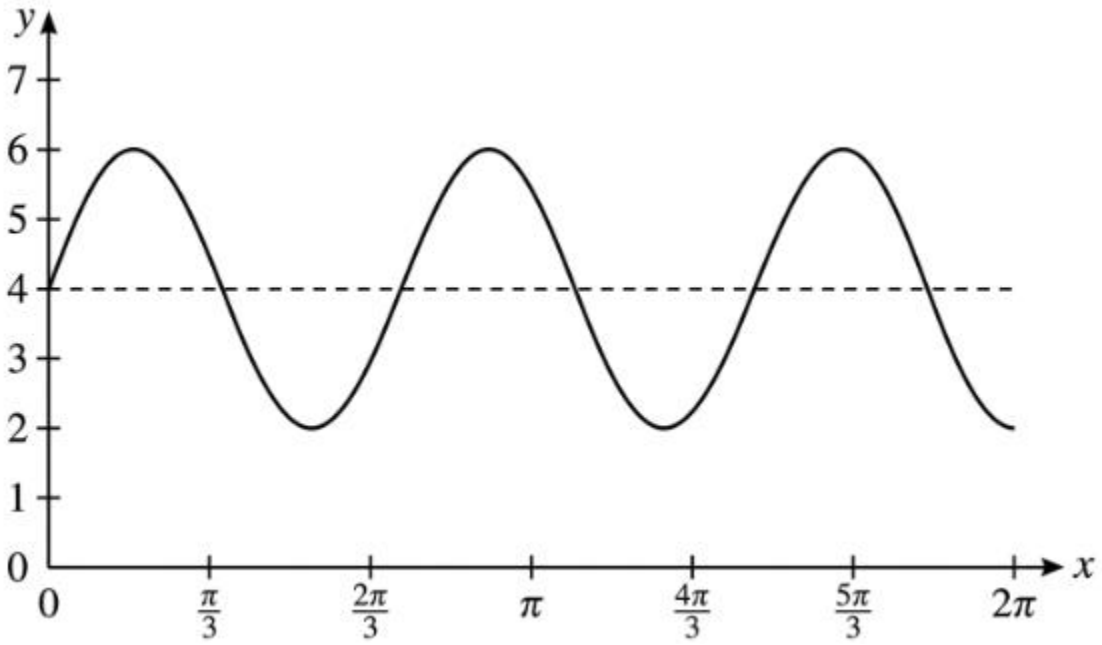
12. What is the domain of $f(x) = \sqrt{(x - 4)}$?

A. $x > 4$ B. $x \geq 4$ C. $x \leq 4$ D. all real numbers

13. As $x \to +\infty$, the function $f(x) = -3x^4 + 2x^2 - 5$ approaches

A. $-\infty$ B. $+\infty$ C. 0 D. 3

14. The sinusoidal function graphed below has the form $y = a \sin(bx) + c$. What is the amplitude of the function?



A. 4 B. 6 C. 2 D. 8

15. What is the solution to the equation $\log(x) + \log(x - 3) = 1$?

A. $x = -2$ B. $x = 2$ C. $x = 10$ D. $x = 5$

16. The third term of the binomial expansion of $(x + 2)^5$ is

A. $10x^3$ B. $20x^3$ C. $40x^3$ D. $80x^2$

17. The box-and-whisker plot below summarizes test scores for a high school class. What is the interquartile range of the data?

A. 30 B. 20 C. 45 D. 10

18. Which expression is equivalent to $(1 - \sin^2\theta) / \cos\theta$, where $\cos\theta \neq 0$?

A. $\cos\theta$ B. $\sin\theta$ C. $\tan\theta$ D. $1 / \cos\theta$

19. The solutions of the system $y = x^2 - 1$ and $y = 3x + 3$ include which point?

A. (1, 6) B. (2, 9) C. (4, 15) D. (−2, −3)

20. $5,000 is invested at an annual interest rate of 4%, compounded quarterly. What is the value of the investment after 3 years, to the nearest cent?

A. $5,600.00 B. $5,612.45 C. $5,624.32 D. $5,634.13

21. Which sampling method is most likely to produce biased results in a survey about a city's transportation needs?

A. Survey people waiting at downtown bus stops during morning rush hour B. Survey random households selected from the city directory C. Survey people at multiple shopping centers across the city D. Survey households randomly selected from voter registration lists

22. The expression $(3 + i) / (2 - i)$, expressed in $a + bi$ form, is

A. 6/5 + 7/5 i B. 1 − i C. 5/3 + i D. 1 + i

23. What is the sum of the first 6 terms of the geometric sequence 4, 12, 36, 108, ...?

A. 1460 B. 972 C. 1456 D. 1820

24. In a normal distribution with mean 100 and standard deviation 15, approximately what percent of values fall above 130?

A. 5% B. 2.5% C. 16% D. 32%

PART II — Short Constructed Response (8 questions × 2 credits = 16 credits)

Show all work. A correct answer with no supporting work will receive only 1 credit.

25. Solve the equation $2x^2 - 4x + 7 = 0$ algebraically. Express your solutions in a + bi form.

26. Solve algebraically for x: $\log_4(x) + \log_4(x - 6) = 2$. State any extraneous solutions and indicate which (if any) are rejected.

27. Factor the expression $3x^3 - 24$ completely over the real numbers. Show each step of the factoring process.

28. An arithmetic sequence has first term $a_1 = 7$ and fifteenth term $a_{15} = 49$. Find the common difference d and the value of the 25th term a_{25}.

29. Verify the trigonometric identity $(\sec^2\theta - 1) / \sec^2\theta = \sin^2\theta$. Show all algebraic steps.

30. Determine the inverse function $f^{-1}(x)$ for $f(x) = (3x - 5) / 2$ and verify algebraically that $f(f^{-1}(x)) = x$.

31. A jar contains 5 red marbles and 4 blue marbles. Two marbles are drawn at random without replacement. Determine the probability that both marbles drawn are blue. Express your answer as a fraction in simplest form.

32. A biased coin has probability P(heads) = 0.6 on each toss. If the coin is tossed 3 times, find the probability of obtaining exactly 2 heads. Round your answer to the nearest thousandth.

PART III — Extended Constructed Response (3 questions × 4 credits = 12 credits)

Show all work. Partial credit is awarded according to the scoring rubric.

33. A ball is thrown upward from a platform 5 feet above the ground with an initial velocity of 60 feet per second. The height $h(t)$, in feet, of the ball above the ground t seconds after release is modeled by $h(t) = -16t^2 + 60t + 5$.

(a) Algebraically determine the maximum height reached by the ball and the time at which the maximum occurs. Round each answer to the nearest hundredth.

(b) Algebraically determine the time at which the ball hits the ground. Round your answer to the nearest tenth of a second.

34. The table below shows the temperature $T(t)$, in degrees Fahrenheit, recorded at a weather station every 3 hours over a 24-hour period.

t (hours)	0	3	6	9	12	15	18	21
T(t) (°F)	45	50	58	65	70	65	55	48

(a) Using sinusoidal regression, write a function of the form $T(t) = A\sin(B(t - C)) + D$ that best models the data. Round each coefficient to the nearest hundredth.

(b) Use the regression model from part (a) to predict the temperature at $t = 30$ hours. Round your answer to the nearest tenth of a degree Fahrenheit.

35. Given the polynomial function $f(x) = x^3 - 2x^2 - 5x + 6$.

(a) Using the Rational Root Theorem, list all possible rational roots of $f(x)$.

(b) Algebraically find all real zeros of $f(x)$, and express $f(x)$ as a product of linear factors. Show all algebraic work, including the polynomial division.

PART IV — Long Constructed Response (1 question × 6 credits = 6 credits)

Show all work. This problem requires multiple steps and integrates concepts from several chapters.

36. A psychologist studying memory retention found that the percent P of information retained from a single lecture decreases over time according to the model $P(d) = 90(0.85)^d$, where d is the number of days after the lecture and $P(d)$ is the percent of information retained.

(a) State the initial percent of information retained on the day of the lecture ($d = 0$) and explain what the value 0.85 represents in the context of the model.

(b) Algebraically determine the number of days it takes for retention to drop to 30 percent. Round your answer to the nearest tenth of a day.

(c) The psychologist hypothesizes that a follow-up review session conducted on day 5 will boost retention, with retention thereafter modeled by R(d) = 95(0.92)^(d − 5) for d ≥ 5. Algebraically determine the number of days after the original lecture at which retention under this new model first drops below 50 percent. Round your answer to the nearest tenth of a day. Show all algebraic work.

PRACTICE EXAM 16 — ANSWER KEY WITH EXPLANATIONS

1. B — Apply the quotient rule for logarithms: $\log_2(8x^3) - \log_2(x) = \log_2(8x^3/x) = \log_2(8x^2)$. Splitting using the product and power rules gives $\log_2(8) + \log_2(x^2) = 3 + 2\log_2(x)$. Logarithmic expressions collapse cleanly when the quotient, product, and power rules are applied in sequence.

2. D — To invert f(x) = ln(x + 2), swap x and y to get x = ln(y + 2), then exponentiate both sides: $e^x = y + 2$, giving $y = e^x - 2$. The exponential function e^x is the natural inverse of ln(x), and the inverse always reverses every operation performed by the original function.

3. C — By Vieta's formulas, the sum of the roots of a polynomial $ax^n + bx^{n-1} + \ldots = 0$ equals $-b/a$. For $2x^3 - 6x^2 + 4x - 5 = 0$, this gives $-(-6)/2 = 3$. Vieta's formulas extract symmetric functions of the roots directly from the coefficients without ever solving for the individual roots.

4. A — A trinomial $x^2 + bx + c$ is a perfect square when $c = (b/2)^2$. With $b = -10$, $c = (-5)^2 = 25$. The result factors as $(x - 5)^2$, which is the canonical perfect-square form needed for completing the square.

5. A — The angle $5\pi/4$ lies in Quadrant III, where tangent is positive because both sine and cosine are negative and their ratio is therefore positive. The reference angle is $\pi/4$, and $\tan(\pi/4) = 1$, so $\tan(5\pi/4) = 1$. Quadrant sign analysis combined with reference-angle values recovers every unit-circle tangent value.

6. D — For an arithmetic sequence, $S_n = (n/2)(2a_1 + (n - 1)d)$. With $a_1 = 3$, $d = 4$, $n = 20$: $S_{20} = 10(6 + 76) = 10(82) = 820$. The closed-form sum formula replaces tedious term-by-term addition for any finite arithmetic series.

7. B — Function composition is evaluated from the inside out: first compute $g(2) = (2)^2 - 3 = 1$, then apply f to that result, giving f(1) = 2(1) + 1 = 3. The order in composition cannot be reversed because $f \circ g$ and $g \circ f$ generally produce different outputs.

8. C — In the vertex form $f(x) = a(x - h)^2 + k$, the vertex sits at (h, k). For $f(x) = -2(x - 3)^2 + 5$, h = 3 and k = 5, so the vertex is (3, 5). The sign convention places the horizontal coordinate at the value that makes $(x - h)^2$ equal zero.

9. D — Express both sides with the same base: $125 = 5^3$, so $5^{(x + 2)} = 5^3$. Equating exponents gives x + 2 = 3, so x = 1. The one-to-one property of exponential functions guarantees a unique solution whenever both sides share a common base.

10. A — Without replacement, successive probabilities multiply using updated counts. P(R then R) = (4/12)(3/11) = 12/132 = 1/11. After the first red is drawn, both the favorable and total counts decrease by one, reflecting the lost outcome from the sample space.

11. B — Factor the numerator as a difference of squares: $x^2 - 9 = (x + 3)(x - 3)$. Canceling the common factor $(x - 3)$ leaves $x + 3$. The cancellation is valid only because the stated restriction $x \neq 3$ excludes the value at which the original denominator would be zero.

12. B — The square root function is real-valued only when the radicand is nonnegative. Setting $x - 4 \geq 0$ gives $x \geq 4$. The inequality is inclusive because $\sqrt{0} = 0$ is a defined real value, while negative radicands produce no real output.

13. A — End behavior of a polynomial is determined entirely by the leading term. The leading term $-3x^4$ has even degree with a negative coefficient, so $f(x) \to -\infty$ as $x \to +\infty$ (and as $x \to -\infty$). Lower-order terms become negligible relative to the leading term at large $|x|$.

14. C — Amplitude equals half the vertical distance between the maximum and minimum values of a sinusoidal curve. From the graph, max = 6 and min = 2, so amplitude = $(6 - 2)/2 = 2$. Amplitude measures vertical displacement from the midline and is independent of period or phase shift.

15. D — Combine the logarithms using the product rule: $\log(x(x - 3)) = 1$, which converts to $x(x - 3) = 10^1 = 10$. Solving $x^2 - 3x - 10 = 0$ by factoring yields $(x - 5)(x + 2) = 0$, so $x = 5$ or $x = -2$. The value $x = -2$ is rejected because $\log(-2)$ and $\log(-5)$ are undefined; only $x = 5$ satisfies both original logarithmic terms.

16. C — In the binomial expansion of $(a + b)^n$, the $(k + 1)^{th}$ term is $C(n, k) \cdot a^{n-k} \cdot b^k$. For the third term of $(x + 2)^5$, $k = 2$: $C(5, 2) \cdot x^3 \cdot 2^2 = 10 \cdot x^3 \cdot 4 = 40x^3$. The binomial coefficient $C(5, 2) = 10$ counts the arrangements that produce $x^3 \cdot 2^2$ as a product.

17. B — The interquartile range equals Q3 − Q1, the boundaries marked by the box in a box-and-whisker plot. Reading from the figure, Q3 = 85 and Q1 = 65, giving IQR = 85 − 65 = 20. The IQR captures the spread of the middle 50% of the data and is resistant to outliers in the whiskers.

18. A — Apply the Pythagorean identity $\sin^2\theta + \cos^2\theta = 1$ to rewrite the numerator: $1 - \sin^2\theta = \cos^2\theta$. The expression becomes $\cos^2\theta / \cos\theta = \cos\theta$. Simplification of trigonometric ratios typically begins by recognizing and substituting a standard identity.

19. C — Set the two expressions equal: $x^2 - 1 = 3x + 3$, which simplifies to $x^2 - 3x - 4 = 0$. Factoring gives $(x - 4)(x + 1) = 0$, and substituting $x = 4$ yields $y = 15$, so $(4, 15)$ is a solution. The other intersection $(-1, 0)$ does not appear among the choices, leaving C as the only valid option.

20. D — Use the compound interest formula $A = P(1 + r/n)^{nt}$ with $P = 5000$, $r = 0.04$, $n = 4$, $t = 3$. Substituting gives $A = 5000(1.01)^{12} \approx 5000(1.126825) \approx \$5{,}634.13$. Quarterly compounding produces slightly more interest than annual compounding because earnings are reinvested four times per year.

21. A — Surveying only people already waiting at downtown bus stops during rush hour preselects current transit users who depend on the existing system. Their preferences cannot represent the broader population — including drivers, cyclists, and people who avoid public transit — producing strong selection bias. Random sampling from a comprehensive list of all residents is required to obtain a representative sample.

22. D — Multiply numerator and denominator by the conjugate of the denominator: $(3 + i)(2 + i) / [(2 - i)(2 + i)]$. The numerator expands to $6 + 3i + 2i + i^2 = 5 + 5i$, and the denominator becomes $4 - i^2 = 5$, giving $(5 + 5i)/5 = 1 + i$. Multiplying by the conjugate eliminates i from the denominator through the difference-of-squares pattern.

23. C — The sum of a finite geometric series is $S_n = a(1 - r^n)/(1 - r)$. With $a = 4$, $r = 3$, $n = 6$: $S_6 = 4(1 - 729)/(1 - 3) = 4(-728)/(-2) = 1456$. The formula works for any $r \neq 1$ and is derived from telescoping the partial sums.

24. B — Under the empirical rule, approximately 95% of values in a normal distribution fall within 2 standard deviations of the mean, leaving 5% in the two tails combined. The value 130 is exactly 2 SD above the mean ($100 + 2 \cdot 15$), so the area above it is half of 5%, or 2.5%. The 68-95-99.7 rule provides quick percentage estimates whenever boundaries align with whole-number standard deviations.

PART II — Short Constructed Response Solutions

25. Apply the quadratic formula with $a = 2$, $b = -4$, $c = 7$: $x = (4 \pm \sqrt{(16 - 56)})/4 = (4 \pm \sqrt{(-40)})/4 = (4 \pm 2i\sqrt{10})/4 =$ **$1 \pm (\sqrt{10}/2)i$**. Solutions: $x = 1 + (\sqrt{10}/2)i$ and $x = 1 - (\sqrt{10}/2)i$.

26. Combine logarithms using the product rule: $\log_4(x(x - 6)) = 2$ $x(x - 6) = 4^2 = 16$ $x^2 - 6x - 16 = 0$ $(x - 8)(x + 2) = 0 \rightarrow x = 8$ or $x = -2$. Domain restriction: $x > 0$ AND $x - 6 > 0$, so $x > 6$. The value **$x = -2$ is rejected as extraneous**, and the only valid solution is **$x = 8$**.

27. Factor out the GCF: $3x^3 - 24 = 3(x^3 - 8)$ Apply the difference of cubes formula $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$ with $a = x$, $b = 2$: $x^3 - 8 = (x - 2)(x^2 + 2x + 4)$ Final factorization: **$3(x - 2)(x^2 + 2x + 4)$**. The quadratic factor has discriminant $4 - 16 = -12 < 0$, so it is irreducible over the real numbers.

28. Use the arithmetic sequence formula $a_n = a_1 + (n - 1)d$: $49 = 7 + (15 - 1)d$ $42 = 14d \rightarrow$ **$d = 3$**. Apply the formula again for the 25th term: $a_{25} = 7 + 24(3) = 7 + 72 =$ **79**.

29. Begin with the left-hand side: $(\sec^2\theta - 1) / \sec^2\theta = \sec^2\theta/\sec^2\theta - 1/\sec^2\theta = 1 - \cos^2\theta$ (since $1/\sec^2\theta = \cos^2\theta$) = **$\sin^2\theta$** (by the Pythagorean identity $\sin^2\theta + \cos^2\theta = 1$) = RHS ✓ The identity is verified.

30. Solve for the inverse: $y = (3x - 5)/2 \rightarrow 2y = 3x - 5 \rightarrow x = (2y + 5)/3$ Swapping x and y: **$f^{-1}(x) = (2x + 5)/3$**. Verify $f(f^{-1}(x))$: $f((2x + 5)/3) = [3 \cdot (2x + 5)/3 - 5] / 2 = [(2x + 5) - 5] / 2 = 2x/2 =$ **x** ✓.

31. Total marbles = 5 + 4 = 9. Apply the multiplication rule without replacement: $P(B_1 \text{ and } B_2) = P(B_1) \cdot P(B_2 \mid B_1) = (4/9)(3/8) = 12/72 =$ **1/6**.

32. Apply the binomial probability formula: $P(X = 2) = C(3, 2) \cdot (0.6)^2 \cdot (0.4)^1 = 3 \cdot 0.36 \cdot 0.4 =$ **0.432**.

PART III — Extended Constructed Response Solutions

33. (a) The vertex of $h(t) = -16t^2 + 60t + 5$ occurs at $t = -b/(2a)$: $t = -60 / (2 \cdot (-16)) = -60/-32 = 1.875 \approx$ **1.88 seconds**. Maximum height: $h(1.875) = -16(1.875)^2 + 60(1.875) + 5 = -56.25 + 112.5 + 5 =$ **61.25 feet**.

(b) Set $h(t) = 0$: $-16t^2 + 60t + 5 = 0$ Apply the quadratic formula: $t = (-60 \pm \sqrt{(3600 + 320)}) / (-32) = (-60 \pm \sqrt{3920}) / (-32)$ $\sqrt{3920} \approx 62.610$ Positive root: $t = (-60 - 62.610) / (-32) = 122.610/32 \approx 3.83$. The ball hits the ground at **$t \approx 3.8$ seconds** (the negative root is rejected).

34. (a) Identify model parameters from the data: • Amplitude $A = (\max - \min)/2 = (70 - 45)/2 =$ **12.50** • Midline $D = (\max + \min)/2 = (70 + 45)/2 =$ **57.50** • Period ≈ 24 hours → $B = 2\pi/24 = \pi/12 \approx$ **0.26** • Maximum at $t = 12 \rightarrow B(t - C) = \pi/2$ when $t = 12 \rightarrow C = 12 - 6 =$ **6.00**

Sinusoidal model: **$T(t) = 12.50 \sin(0.26(t - 6.00)) + 57.50$**.

(b) Evaluate $T(30)$: $T(30) = 12.50 \sin((\pi/12)(30 - 6)) + 57.50 = 12.50 \sin(2\pi) + 57.50 = 12.50(0) + 57.50 =$ **57.5°F**.

35. (a) By the Rational Root Theorem, possible rational roots are ±(factors of constant term) / ±(factors of leading coefficient). Constant = 6, leading coefficient = 1. **Possible rational roots: ±1, ±2, ±3, ±6**.

(b) Test $f(1)$: $f(1) = 1 - 2 - 5 + 6 = 0$ ✓, so $(x - 1)$ is a factor. Synthetic division of $x^3 - 2x^2 - 5x + 6$ by $(x - 1)$:

1 | 1 −2 −5 6 | 1 −1 −6 | 1 −1 −6 0

Quotient: $x^2 - x - 6 = (x - 3)(x + 2)$. Complete factorization: **$f(x) = (x - 1)(x - 3)(x + 2)$**. Real zeros: **$x = 1$, $x = 3$, $x = -2$**.

PART IV — Long Constructed Response Solution

36. (a) Initial retention: $P(0) = 90(0.85)^0 =$ **90 percent**. The value **0.85 represents the daily retention factor** — each day, 85 percent of the previous day's retained information is kept, equivalent to a 15 percent daily loss of retained material.

(b) Set $P(d) = 30$: $30 = 90(0.85)^d$ $1/3 = (0.85)^d$ $\ln(1/3) = d \cdot \ln(0.85)$ $d = \ln(1/3) / \ln(0.85) \approx (-1.0986) / (-0.1625) \approx 6.76$. Retention drops to 30 percent after approximately **6.8 days**.

(c) Set $R(d) = 50$: $50 = 95(0.92)^{(d - 5)}$ $50/95 = (0.92)^{(d - 5)}$ $10/19 = (0.92)^{(d - 5)}$ $\ln(10/19) = (d - 5) \cdot \ln(0.92)$ $d - 5 = \ln(10/19) / \ln(0.92) \approx (-0.6419) / (-0.0834) \approx 7.70$. Solving: $d \approx 5 + 7.70 = 12.70$. Under the new model, retention first drops below 50 percent at approximately **12.7 days** after the original lecture.