PRACTICE EXAM 15: ALGEBRA II REGENTS SIMULATION

PART I — Multiple Choice (24 questions × 2 credits = 48 credits)

1. Simplify the expression $(x^{(1/2)} \cdot x^{(1/3)})^6$.

A. $x^{(5/6)}$

B. $x^{(1/6)}$

C. x^6

D. x^5

2. Express the product $(5 + 2i)^2$ in a + bi form.

A. 25 + 4i

B. 21 + 20i

C. 29 + 20i

D. 21 − 20i

3. Factored completely, the expression $2x^3 - 8x$ is equivalent to

A. $2x(x + 2)(x - 2)$

B. $2x(x^2 - 4)$

C. $(2x + 4)(x^2 - 2)$

D. $2(x^3 - 4x)$

4. Which equation has two distinct rational solutions?

A. $x^2 + 4x + 5 = 0$

B. $x^2 - 6x + 9 = 0$

C. $3x^2 - 7x + 2 = 0$

D. $x^2 - 2x - 4 = 0$

5. The solutions to the equation $x^2 + 6x + 25 = 0$ are

A. $x = 3 \pm 4i$

B. $x = -6 \pm 8i$

C. $x = -3 \pm 4i$

D. $x = -3 \pm 2i$

6. The expression $(x^2 + x - 12) / (x^2 - 16)$, in fully simplified form with restrictions, is

A. $(x + 3) / (x + 4)$, $x \neq -4$, $x \neq 3$

B. $(x - 3) / (x - 4)$, $x \neq 4$, $x \neq -4$

C. $(x - 3) / (x + 4)$, $x \neq -4$, $x \neq 4$

D. $(x + 4) / (x - 4)$, $x \neq 4$, $x \neq -3$

7. What is the solution set of the equation $\sqrt{(2x - 3)} = x - 3$?

A. {2, 6}

B. {2}

C. { }

D. {6}

8. When $p(x) = 2x^3 + x^2 - 5x + 4$ is divided by $(x + 2)$, the remainder is

A. 2

B. −2

C. 14

D. −10

9. How many points of intersection do the line $y = 2x - 1$ and the parabola $y = x^2$ have?

A. 0

B. 1

C. 2

D. 3

10. The expression $8x^3 - 27$ is equivalent to

A. $(2x + 3)(4x^2 - 6x + 9)$

B. $(2x - 3)(4x^2 - 6x + 9)$

C. $(2x - 3)(4x^2 + 6x + 9)$

D. $(2x - 3)(4x^2 + 9)$

11. If $g(x) = x^2 - 4x + 7$, what is the value of $g(3)$?

A. 4

B. −2

C. 10

D. 22

12. The first three terms of a geometric sequence are 2, 6, 18. What is the 8th term?

A. 1458

B. 13122

C. 486

D. 4374

13. The graph of $f(x) = x^2$ is transformed to produce $g(x) = (x - 2)^2 + 5$. Which best describes the transformation?

A. Shift left 2 and down 5

B. Shift left 2 and up 5

C. Shift right 2 and down 5

D. Shift right 2 and up 5

14. The average rate of change of $f(x) = x^3$ on the interval [1, 3] is

A. 13

B. 9

C. 14

D. 26

15. A population grows according to $P(t) = 800(1.05)^t$, where t is the time in years. The annual growth rate is

A. 0.5%

B. 1.05%

C. 5%

D. 50%

16. What is the solution to the equation $\log_3(2x - 1) = 2$?

A. $x = 4$

B. $x = 5$

C. $x = 8$

D. $x = 10$

17. The value of $\sin(7\pi/6)$ is

A. $-1/2$

B. $1/2$

C. $\sqrt{3}/2$

D. $-\sqrt{3}/2$

18. For the function $y = 4\cos(\pi x/2) - 1$, what is the period?

A. π

B. 2

C. $\pi/2$

D. 4

19. If $f(x) = (x - 4) / 2$, then $f^{-1}(x)$ is equal to

A. $(x + 4) / 2$

B. $2x + 4$

C. $2x - 4$

D. $2 / (x - 4)$

20. Given that tan θ = −5/12 and θ is in Quadrant IV, the value of sin θ is

A. 12/13

B. −12/13

C. −5/13

D. 5/13

21. Test scores on a state exam are normally distributed with a mean of 80 and a standard deviation of 5. Approximately what percent of test scores fall between 70 and 90?

A. 68%

B. 50%

C. 95%

D. 99.7%

22. Which study design is most appropriate for establishing whether a new fertilizer causes increased crop yield?

A. A randomized controlled experiment with treatment and control groups

B. An observational study comparing farms that already use the fertilizer

C. A survey of farmers about their fertilizer preferences and yields

D. A census of all farms in the region currently growing similar crops

23. A scatter plot of data appears to follow a parabolic curve opening upward, with a clear minimum value in the interior of the data set. Which regression model is most appropriate?

A. Linear regression

B. Quadratic regression

C. Exponential regression

D. Logarithmic regression

24. A fair coin is tossed three times. What is the probability of obtaining exactly two heads?

A. 1/2

B. 1/8

C. 1/4

D. 3/8

PART II — Short Constructed Response (8 questions × 2 credits = 16 credits)

Show all work. A correct answer with no supporting work will receive only 1 credit.

25. Solve the equation $x^2 - 4x + 29 = 0$ algebraically. Express your solutions in a + bi form.

26. Solve algebraically for x: $x / (x + 2) + 1 / (x - 1) = 1$. State any extraneous solutions and indicate which (if any) are rejected.

27. Given $f(x) = x^3 - 4x^2 + x + 6$, determine whether $(x - 2)$ is a factor of f(x). Justify your answer using the Factor Theorem.

28. Write an equation, in the form $g(x) = a \cdot f(x - h) + k$, for the function obtained by vertically stretching $f(x) = x^2$ by a factor of 3 and then translating the resulting graph 2 units to the left and 5 units down.

29. Solve algebraically for x: $2^{(3x + 1)} = 32$.

30. Express $3 / (x - 2) - 4 / (x + 5)$ as a single rational expression in simplest form. State any restrictions on the variable.

31. A sequence is defined recursively by $a_1 = 3$ and $a_n = 3a_{n-1} - 2$ for $n \geq 2$. Find the value of a_5.

32. Given that $\cos \theta = 8/17$ and θ terminates in Quadrant IV, find the exact value of $\tan \theta$. Show the algebraic work that justifies your answer.

PART III — Extended Constructed Response (3 questions × 4 credits = 12 credits)

Show all work. Partial credit is awarded according to the scoring rubric.

33. A bacteria culture doubles every 4 hours. The initial population is 150 bacteria, modeled by the function $P(t) = 150 \cdot 2^{(t/4)}$, where $P(t)$ is the population and t is the time elapsed in hours.

(a) State the initial population and the doubling time, and determine the population after 12 hours.

(b) Algebraically determine the number of hours it will take for the population to reach 2,400 bacteria. Round your answer to the nearest tenth of an hour.

34. The table below shows the number of visitors $V(t)$ to a newly launched website, where t is the number of weeks after launch.

t (weeks)	0	1	2	3	4
V(t)	50	80	128	205	328

(a) Using regression, write an exponential function $V(t) = a \cdot b^t$ that best models the data. Round each coefficient to the nearest hundredth.

(b) Use the regression model from part (a) to determine after how many full weeks the number of visitors will first exceed 1,000. Show the algebraic work supporting your answer.

35. A circle is defined by the equation $(x - 2)^2 + (y + 1)^2 = 25$, and a line is defined by the equation $y = x - 4$.

(a) Algebraically determine all points of intersection between the circle and the line. Express coordinates in exact form.

(b) Verify your answer by substituting one of the intersection points back into both original equations. Show the substitution clearly.

PART IV — Long Constructed Response (1 question × 6 credits = 6 credits)

Show all work. This problem requires multiple steps and integrates concepts from several chapters.

36. The water depth $d(t)$, in feet, at a coastal harbor varies sinusoidally with time. At low tide, the depth reaches a minimum of 2 feet; at high tide, the depth reaches a maximum of 14 feet. One complete cycle, from one low tide to the next, takes 12 hours. A low tide occurs at time $t = 0$ hours.

(a) Write a function d(t) that models the water depth in feet as a function of time t in hours.

(b) State the amplitude, period, and midline of the function d(t), and explain what each represents in the context of the harbor's tide cycle.

(c) Algebraically determine all times during the first 24 hours of observation at which the water depth is exactly 11 feet. Round each answer to the nearest hundredth of an hour. Show all algebraic work.

PRACTICE EXAM 15 — ANSWER KEY WITH EXPLANATIONS

1. D — Apply the product rule inside the parentheses: $x^{1/2} \cdot x^{1/3} = x^{1/2 + 1/3} = x^{5/6}$. Raising this to the 6th power gives $(x^{5/6})^6 = x^{5/6 \cdot 6} = x^5$. The power-of-a-power rule clears the fractional exponent whenever the outer integer exponent matches the denominator.

2. B — Apply FOIL: $(5 + 2i)(5 + 2i) = 25 + 10i + 10i + 4i^2 = 25 + 20i + 4(-1) = 21 + 20i$. The critical move is replacing i^2 with -1, which shifts the squared imaginary term into the real component.

3. A — Factor out the GCF 2x to get $2x(x^2 - 4)$, then apply the difference of squares: $x^2 - 4 = (x + 2)(x - 2)$. Complete factoring requires extracting common factors and recognizing standard patterns, yielding $2x(x + 2)(x - 2)$.

4. C — Two distinct rational solutions require a discriminant that is a positive perfect square. For $3x^2 - 7x + 2$, $b^2 - 4ac = 49 - 24 = 25$, which is a perfect square. The other equations yield discriminants of −4 (complex), 0 (repeated), and 20 (irrational).

5. C — By the quadratic formula: $x = (-6 \pm \sqrt{36 - 100})/2 = (-6 \pm \sqrt{-64})/2 = (-6 \pm 8i)/2 = -3 \pm 4i$. A negative discriminant always produces complex conjugate solutions in $a \pm bi$ form.

6. B — Factor numerator and denominator: $x^2 + x - 12 = (x + 4)(x - 3)$ and $x^2 - 16 = (x + 4)(x - 4)$. Canceling the common factor leaves $(x - 3)/(x - 4)$, but both restrictions $x \neq -4$ and $x \neq 4$ from the original denominator must be preserved.

7. D — Squaring both sides yields $2x - 3 = x^2 - 6x + 9$, which rearranges to $x^2 - 8x + 12 = (x - 2)(x - 6) = 0$. Testing $x = 2$: $\sqrt{1} = 1$ but $x - 3 = -1$, so $x = 2$ is extraneous. Only $x = 6$ satisfies the original equation since $\sqrt{9} = 3 = 6 - 3$.

8. A — By the Remainder Theorem, the remainder when p(x) is divided by $(x - c)$ equals p(c). Here $c = -2$, so $p(-2) = 2(-8) + 4 - 5(-2) + 4 = -16 + 4 + 10 + 4 = 2$. The theorem bypasses long division entirely for remainder problems.

9. B — Setting $x^2 = 2x - 1$ gives $x^2 - 2x + 1 = (x - 1)^2 = 0$. The discriminant equals 0, indicating a single repeated root and exactly one point of tangency between the line and the parabola.

10. C — The difference of cubes pattern is $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$. Here $a = 2x$ and $b = 3$, so $8x^3 - 27 = (2x - 3)(4x^2 + 6x + 9)$. The trinomial factor always carries a positive middle term in the difference-of-cubes formula.

11. A — Substitute $x = 3$ directly: $g(3) = (3)^2 - 4(3) + 7 = 9 - 12 + 7 = 4$. Direct substitution is the standard procedure for evaluating polynomial functions at a specified input.

12. D — The common ratio is $r = 18/6 = 3$, and the explicit formula is $a_n = a_1 \cdot r^{(n-1)}$. For $n = 8$: $a_8 = 2 \cdot 3^7 = 2 \cdot 2187 = 4374$. The explicit formula avoids iterative term-by-term computation.

13. D — In the vertex form $(x - h)^2 + k$, h indicates horizontal shift and k indicates vertical shift. Here $h = 2$ (right 2) and $k = 5$ (up 5). The sign of the horizontal shift is opposite the sign that appears inside the parentheses.

14. A — Average rate of change equals $(f(b) - f(a))/(b - a)$. With $f(3) = 27$ and $f(1) = 1$, the rate is $(27 - 1)/(3 - 1) = 26/2 = 13$. This value represents the slope of the secant line connecting the two endpoints on the curve.

15. C — In the exponential model $P(t) = a \cdot b^t$ with $b > 1$, the annual growth rate equals $(b - 1) \cdot 100\%$. Here $b = 1.05$, so the rate is $0.05 = 5\%$. The base 1.05 decomposes as $1 + r$, where r is the per-period growth rate.

16. B — Convert $\log_3(2x - 1) = 2$ to exponential form: $2x - 1 = 3^2 = 9$. Solving gives $2x = 10$, so $x = 5$. The defining relationship $\log_b(y) = c \Leftrightarrow b^c = y$ underpins all logarithmic equation solving.

17. A — The angle $7\pi/6$ lies in Quadrant III, where sine is negative. The reference angle is $7\pi/6 - \pi = \pi/6$, and $\sin(\pi/6) = 1/2$, so $\sin(7\pi/6) = -1/2$. Quadrant signs combined with reference-angle values reproduce every unit-circle value.

18. D — For $y = A \cos(Bx) + D$, the period equals $2\pi/|B|$. Here $B = \pi/2$, so the period is $2\pi \div (\pi/2) = 4$. The amplitude and vertical shift do not affect the period of a sinusoidal function.

19. B — Swap x and y in $y = (x - 4)/2$ to get $x = (y - 4)/2$, then solve: $2x = y - 4$, so $y = 2x + 4$. The inverse function reverses the order of operations — multiplying by 2 and then adding 4 undoes subtracting 4 and dividing by 2.

20. C — In Quadrant IV, sine is negative and cosine is positive. With $\tan \theta = -5/12$, the reference triangle has legs 5 and 12 and hypotenuse $\sqrt{(25 + 144)} = 13$. Therefore $\sin \theta = -5/13$ once the Quadrant IV sign is applied.

21. C — Under the empirical (68-95-99.7) rule, approximately 95% of normally distributed data falls within 2 standard deviations of the mean. Here 70 and 90 are each exactly 2 standard deviations from 80 (since $2 \cdot 5 = 10$), so [70, 90] captures ~95% of test scores.

22. A — Only randomized controlled experiments with random assignment can establish a causal relationship between an intervention and an outcome. Observational studies, surveys, and censuses can demonstrate association but cannot rule out confounding variables. Causation requires controlled manipulation of the independent variable.

23. B — Parabolic data with a clearly visible turning point in the interior of the dataset is best modeled by a quadratic function $y = ax^2 + bx + c$. Linear regression cannot capture curvature, and exponential or logarithmic models have no interior minimum or maximum.

24. D — By the binomial probability formula, P(exactly k successes) = $C(n, k) \cdot p^k \cdot (1 - p)^{(n - k)}$. For $n = 3, k = 2, p = 1/2$: $P = C(3, 2) \cdot (1/2)^2 \cdot (1/2)^1 = 3 \cdot 1/8 = 3/8$. The combination factor accounts for the three arrangements HHT, HTH, and THH.

PART II — Short Constructed Response Solutions

25. Apply the quadratic formula with $a = 1, b = -4, c = 29$: $x = (4 \pm \sqrt{(16 - 116)})/2 = (4 \pm \sqrt{-100})/2 = (4 \pm 10i)/2 =$ **$2 \pm 5i$**. Solutions: $x = 2 + 5i$ and $x = 2 - 5i$.

26. Multiply both sides by the LCD $(x + 2)(x - 1)$: $x(x - 1) + (x + 2) = (x + 2)(x - 1)$ $x^2 - x + x + 2 = x^2 + x - 2$ $x^2 + 2 = x^2 + x - 2$ $2 = x - 2 \rightarrow$ **$x = 4$**. Restrictions: $x \neq -2, x \neq 1$. The solution $x = 4$ violates neither restriction, so there are **no extraneous solutions**.

27. By the Factor Theorem, $(x - c)$ is a factor of $f(x)$ if and only if $f(c) = 0$. $f(2) = (2)^3 - 4(2)^2 + 2 + 6 = 8 - 16 + 2 + 6 =$ **0**. Since $f(2) = 0$, **(x − 2) IS a factor** of f(x).

28. Build the transformation step by step starting from $f(x) = x^2$: • Vertical stretch by 3: $3x^2$ • Translate 2 left: $3(x + 2)^2$ • Translate 5 down: $3(x + 2)^2 - 5$ **$g(x) = 3(x + 2)^2 - 5$** (with $a = 3, h = -2, k = -5$).

29. Rewrite both sides with base 2: $2^{(3x + 1)} = 32 = 2^5$ $3x + 1 = 5$ $3x = 4 \rightarrow$ **$x = 4/3$**.

30. Combine over the common denominator $(x - 2)(x + 5)$: $3/(x - 2) - 4/(x + 5) = [3(x + 5) - 4(x - 2)] / [(x - 2)(x + 5)] = [3x + 15 - 4x + 8] / [(x - 2)(x + 5)] =$ **$(23 - x) / [(x - 2)(x + 5)]$** Restrictions: $x \neq 2, x \neq -5$.

31. Apply the recursion step by step: $a_1 = 3$ $a_2 = 3(3) - 2 = 7$ $a_3 = 3(7) - 2 = 19$ $a_4 = 3(19) - 2 = 55$ $a_5 = 3(55) - 2 =$ **163**.

32. Use the Pythagorean identity $\sin^2\theta + \cos^2\theta = 1$: $\sin^2\theta = 1 - (8/17)^2 = 1 - 64/289 = 225/289$ $\sin \theta = \pm 15/17$. In Quadrant IV, sine is negative, so $\sin \theta = -15/17$. **$\tan \theta = \sin \theta / \cos \theta = (-15/17) \div (8/17) = -15/8$**.

PART III — Extended Constructed Response Solutions

33. (a) **Initial population P(0) = 150 bacteria** (since $150 \cdot 2^0 = 150$). **Doubling time = 4 hours.** After 12 hours: $P(12) = 150 \cdot 2^{(12/4)} = 150 \cdot 2^3 = 150 \cdot 8 =$ **1200 bacteria**.

(b) Set $P(t) = 2400$: $2400 = 150 \cdot 2^{(t/4)}$ $16 = 2^{(t/4)}$ $2^4 = 2^{(t/4)}$ $t/4 = 4 \rightarrow$ **t = 16.0 hours**.

34. (a) Compute successive ratios: $80/50 = 1.60$, $128/80 = 1.60$, $205/128 \approx 1.60$, $328/205 \approx 1.60$. The common ratio is consistent at $b \approx 1.60$, with initial value $a = 50$. **$V(t) = 50(1.60)^t$**.

(b) Solve $V(t) = 1000$: $1000 = 50(1.60)^t$ $20 = (1.60)^t$ $\ln(20) = t \cdot \ln(1.60)$ $t = \ln(20) / \ln(1.60) \approx 2.9957 / 0.4700 \approx 6.37$ Verify with integer weeks: $V(6) = 50(1.60)^6 \approx 838.9$ (below 1000); $V(7) = 50(1.60)^7 \approx 1342.2$ (above 1000). The visitor count first exceeds 1000 during **week 7**.

35. (a) Substitute $y = x - 4$ into the circle equation: $(x - 2)^2 + (x - 4 + 1)^2 = 25$ $(x - 2)^2 + (x - 3)^2 = 25$ $(x^2 - 4x + 4) + (x^2 - 6x + 9) = 25$ $2x^2 - 10x + 13 = 25$ $2x^2 - 10x - 12 = 0$ $x^2 - 5x - 6 = 0$ $(x - 6)(x + 1) = 0$ $\rightarrow x = 6$ or $x = -1$.

For $x = 6$: $y = 6 - 4 = 2 \rightarrow$ **(6, 2)** For $x = -1$: $y = -1 - 4 = -5 \rightarrow$ **(−1, −5)**.

(b) Verify (6, 2) in both original equations: Circle: $(6 - 2)^2 + (2 + 1)^2 = 16 + 9 = 25$ ✓ Line: $y = 6 - 4 = 2$ ✓ Both equations are satisfied, confirming (6, 2) is a valid intersection point.

PART IV — Long Constructed Response Solution

36. (a) Build the sinusoidal model: • Amplitude $A = (\max - \min)/2 = (14 - 2)/2 =$ **6** • Midline $k = (\max + \min)/2 = (14 + 2)/2 =$ **8** • Period $= 12 \rightarrow B = 2\pi/12 =$ **$\pi/6$** • At $t = 0$, $d = 2$ (minimum), so use negative cosine. **$d(t) = -6 \cos(\pi t/6) + 8$**

(b) **Amplitude = 6 feet**: the maximum displacement of water depth above or below the average depth — equal to half the tidal range. **Period = 12 hours**: the time required for one complete tidal cycle from one low tide to the next. **Midline = 8 feet**: the average water depth — the level around which the tide oscillates between its high and low extremes.

(c) Set $d(t) = 11$: $11 = -6 \cos(\pi t/6) + 8$ $3 = -6 \cos(\pi t/6)$ $\cos(\pi t/6) = -1/2$

Within one period ($0 \le \pi t/6 \le 2\pi$), $\cos(\theta) = -1/2$ at $\theta = 2\pi/3$ and $\theta = 4\pi/3$. • $\pi t/6 = 2\pi/3 \rightarrow t = 4$ • $\pi t/6 = 4\pi/3 \rightarrow t = 8$

Add one full period (12 hours) to obtain solutions in the next cycle: • $t = 4 + 12 = 16$ • $t = 8 + 12 = 20$

The water depth equals 11 feet at **t = 4.00, 8.00, 16.00, and 20.00 hours**.