PRACTICE EXAM 14: ALGEBRA II REGENTS SIMULATION

PART I — Multiple Choice (24 questions × 2 credits = 48 credits)

1. The expression $(x^{(1/2)} \cdot x^{(2/3)})^6$ is equivalent to

A. x^4

B. x^6

C. x^7

D. x^{12}

2. The expression $(1 + 2i)^2$, written in $a + bi$ form, equals

A. $-3 + 4i$

B. $1 - 4i$

C. $5 + 4i$

D. $-3 - 4i$

3. According to the Rational Root Theorem, which of the following values could possibly be a rational root of the polynomial $3x^3 + 2x^2 - 7x + 2$?

A. 1/2

B. 3

C. 1/4

D. 2/3

4. When the polynomial $p(x) = x^3 + 3x^2 - 4$ is divided by $(x - 1)$, the quotient is

A. $x^2 + 3x + 4$

B. $x^2 + 4x + 4$

C. $x^2 + 4$

D. $x^2 - 4x + 4$

5. A quadratic equation has the complex solutions $x = 2 + i$ and $x = 2 - i$. Which equation could this be?

A. $x^2 + 4x + 5 = 0$

B. $x^2 + 4x - 5 = 0$

C. $x^2 - 4x - 5 = 0$

D. $x^2 - 4x + 5 = 0$

6. What is the solution to the equation $2/(x - 1) + 3/(x + 1) = 4/(x^2 - 1)$?

A. $x = 1$

B. $x = -1$

C. $x = 0$

D. no solution

7. What is the solution to the equation $\sqrt[3]{(x + 2)} = -2$?

A. $x = -10$

B. $x = -6$

C. $x = -4$

D. $x = 6$

8. At how many points does the parabola $y = x^2 + 2x$ intersect the parabola $y = -x^2 + 4$?

A. 0

B. 1

C. 2

D. 3

9. A water lily population doubles every 4 days. If the initial population is 50 lilies, which function models the population P after t days?

A. $P(t) = 50 \cdot 2^t$

B. P(t) = 50 · 2^(t/4)

C. $P(t) = 50 \cdot 4^t$

D. $P(t) = 50 + 4t$

10. The expression $x^6 - 64$, factored completely over the integers, is

A. $(x - 2)(x + 2)(x^2 + 2x + 4)(x^2 - 2x + 4)$

B. $(x - 2)^2(x + 2)^2$

C. $(x^3 - 8)(x^3 + 8)$

D. $(x^2 - 4)(x^4 + 4x^2 + 16)$

11. If $f(x) = 2x + 3$ and $h \neq 0$, then the expression $[f(x + h) - f(x)] / h$ is equal to

A. 0

B. 3

C. 2h

D. 2

12. The sum of the first 12 terms of the arithmetic sequence 5, 9, 13, 17, … is

A. 240

B. 288

C. 324

D. 360

13. The inverse of the function $f(x) = (x + 5) / 3$ is $f^{-1}(x)$ equal to

A. $3x + 5$

B. $3x - 5$

C. $(x - 5) / 3$

D. $-(x + 5) / 3$

14. Solve algebraically for x: $5 \cdot 2^x = 80$.

A. $x = 4$

B. $x = 16$

C. $x = 5$

D. $x = 80$

15. The exact value of $\sin(-\pi/3)$ is

A. $\sqrt{3} / 2$

B. $1 / 2$

C. $-\sqrt{3} / 2$

D. $-1 / 2$

16. Solve algebraically for x: $\log_5(x^2) = 2$.

A. $x = 5$ only

B. $x = -5$ only

C. $x = 5$ and $x = -5$

D. $x = 25$

17. The inverse of the function $f(x) = \ln(x - 3)$ is $f^{-1}(x)$ equal to

A. e^(x + 3)

B. $e^x + 3$

C. $\ln(x + 3)$

D. $\ln(x) - 3$

18. A function has the form $y = 4 \sin(B(x - \pi/6)) + 2$ with period π. The value of B is

A. $B = \pi$

B. $B = 1/2$

C. $B = 1$

D. $B = 2$

19. If $\tan \theta = 3$ and θ is in Quadrant I, then the value of $\sin \theta$ is

A. $3\sqrt{10} / 10$

B. $\sqrt{10} / 10$

C. $3 / 10$

D. $1 / 3$

20. The expression $\log(x) - 2\log(y) + 3\log(z)$, written as a single logarithm, is

A. $\log(x - 2y + 3z)$

B. $\log(xz^3 / y^2)$

C. $\log(2x / (yz))$

D. $\log(x^2y / z^3)$

21. A 95% confidence interval for the mean weight of newborn babies at a hospital is (6.2 lbs, 7.4 lbs). The point estimate (sample mean) used to construct this interval is

A. 6.2 lbs

B. 7.4 lbs

C. 6.8 lbs

D. 1.2 lbs

22. A teacher draws names from a hat to select 10 students out of the 30 students in her class. This sampling procedure is best described as

A. simple random sampling

B. stratified random sampling

C. cluster sampling design

D. convenience sampling method

23. A standard deck of 52 cards is shuffled, and one card is drawn at random. Given that the card drawn is a face card (Jack, Queen, or King), the probability that it is a Queen is

A. 1/13

B. 4/13

C. 4/52

D. 1/3

24. A committee of 5 people is to be selected from a group of 12 people. In how many ways can this committee be formed if order does not matter?

A. 60

B. 792

C. 95,040

D. 12

PART II — Short Constructed Response (8 questions × 2 credits = 16 credits)

Show all work. A correct answer with no supporting work will receive only 1 credit.

25. Solve the equation $4x^2 - 6x + 5 = 0$ algebraically. Express the solutions in $a + bi$ form.

26. Solve algebraically: $3 / (x + 2) = 4 / (x - 1)$. State any restrictions on x and identify whether any extraneous solutions arise.

27. A polynomial f(x) has linear factors $(x - 2)$, $(x + 1)$, and $(x - 4)$, with leading coefficient 3. Write f(x) as a polynomial in expanded standard form.

28. Solve algebraically: $\log_3(x + 1) + \log_3(x - 5) = 3$. Show all algebraic work and verify your solution against the domain.

29. A geometric sequence has first term 8 and common ratio 3. Find the 5th term of the sequence using the explicit formula. Show all work.

30. Use a sum or difference identity for sine to find the exact value of $\sin(75°)$. Show all algebraic work, expressing your answer in simplest radical form.

31. Solve the equation $\cos(2x) = -1/2$ for all values of x in the interval $[0, 2\pi)$. Show how you determined each solution.

32. A box contains 10 numbered tickets, numbered 1 through 10. Two tickets are drawn at random without replacement. Determine the probability that both numbers drawn are even. Express your answer as a fraction in simplest form.

PART III — Extended Constructed Response (3 questions × 4 credits = 12 credits)

Show all work. Partial credit is awarded according to the scoring rubric.

33. A car is purchased for $32,000 and depreciates at a rate of 15% per year.

(a) Write a function V(t) that models the value of the car (in dollars) after t years.

(b) Algebraically determine the number of years until the car is worth $15,000. Round your answer to the nearest tenth of a year.

34. The polynomial function $f(x) = x^3 - x^2 - 8x + 12$ has $x = 2$ as a known real root.

(a) Use synthetic division to find the quotient when $f(x)$ is divided by $(x - 2)$. Show your synthetic division work.

(b) Factor $f(x)$ completely over the integers, and identify any repeated roots.

(c) State all real solutions to the equation $f(x) = 0$.

35. A medical researcher tests whether a new drug reduces cholesterol. In a randomized clinical trial of 144 patients, the sample mean reduction is 25 mg/dL, and the sample standard deviation is 12 mg/dL.

(a) Calculate the standard error of the sample mean using $SE = s / \sqrt{n}$.

(b) Construct a 95% confidence interval for the population mean reduction using $\bar{x} \pm 1.96 \cdot SE$. Round bounds to two decimal places.

(c) The researcher requires strong evidence that the drug reduces cholesterol by more than 20 mg/dL. Based on the confidence interval, can the researcher conclude this with 95% confidence? Justify your answer.

PART IV — Long Constructed Response (1 question × 6 credits = 6 credits)

Show all work. This problem integrates concepts from several chapters.

36. A radio tower's signal strength S (in decibels) at a distance d (in kilometers) from the tower is modeled by $S(d) = 100 - 20 \log(d)$, where $d > 0$ and log denotes the common (base-10) logarithm.

(a) Determine the signal strength at d = 1 km, d = 10 km, and d = 100 km. Explain the pattern you observe relating signal strength to distance.

(b) Algebraically determine the distance d at which the signal strength drops to exactly 40 decibels. Show all algebraic work.

(c) Describe the long-term behavior of S(d) as d increases without bound. Algebraically determine whether and where S(d) reaches zero decibels.

PRACTICE EXAM 14 — ANSWER KEY AND EXPLANATIONS

1. C — Apply the product rule inside the parentheses: $x^{(1/2)} \cdot x^{(2/3)} = x^{(1/2 + 2/3)} = x^{(7/6)}$. Then apply the power rule: $(x^{(7/6)})^6 = x^{(7/6 \cdot 6)} = x^7$. The exponent simplification is the key step.

2. A — Apply the square of a binomial: $(1 + 2i)^2 = 1 + 4i + 4i^2$. Since $i^2 = -1$, this becomes $1 + 4i - 4 = -3 + 4i$ in standard a + bi form.

3. D — The Rational Root Theorem states that any rational root p/q (in lowest terms) must have p dividing the constant term 2 and q dividing the leading coefficient 3. Valid candidates: ±1, ±2, ±1/3, ±2/3. Only 2/3 appears among the choices and matches the theorem's constraints.

4. B — Apply synthetic division with k = 1 on coefficients 1, 3, 0, −4 (note the implicit zero coefficient for the missing x term): bring down 1; multiply and add to get 4; continue to get 4 and 0. The quotient is $x^2 + 4x + 4$, with remainder 0.

5. D — Apply Vieta's formulas for a quadratic with roots r_1 and r_2: $x^2 - (\text{sum})x + (\text{product}) = 0$. Sum: $(2 + i) + (2 - i) = 4$. Product: $(2 + i)(2 - i) = 4 - i^2 = 5$. The equation is $x^2 - 4x + 5 = 0$.

6. D — Multiply both sides by $(x - 1)(x + 1) = x^2 - 1$: $2(x + 1) + 3(x - 1) = 4$, giving $5x - 1 = 4$ and $x = 1$. Since $x = 1$ makes the denominator $(x - 1)$ zero, the value is extraneous, and the equation has no solution.

7. A — Cube both sides to eliminate the cube root: $x + 2 = (-2)^3 = -8$, so $x = -10$. Cubing is a one-to-one operation that never produces extraneous solutions.

8. C — Set the parabolas equal: $x^2 + 2x = -x^2 + 4$, giving $2x^2 + 2x - 4 = 0$ or $x^2 + x - 2 = 0$, which factors as $(x + 2)(x - 1) = 0$. The two distinct real solutions $x = -2$ and $x = 1$ correspond to two intersection points.

9. B — Doubling every 4 days means the exponent on 2 must equal 1 when t = 4. The form 2^(t/4) achieves this: at t = 4, $P = 50 \cdot 2^1 = 100$. The structure base^(t/period) is standard for doubling-time models.

10. A — Recognize $x^6 - 64 = (x^3)^2 - 8^2$ and apply the difference of squares: $(x^3 - 8)(x^3 + 8)$. Each factor is a difference/sum of cubes: $x^3 - 8 = (x - 2)(x^2 + 2x + 4)$ and $x^3 + 8 = (x + 2)(x^2 - 2x + 4)$. The quadratic factors have negative discriminants and don't factor further.

11. D — Compute $f(x + h) = 2(x + h) + 3 = 2x + 2h + 3$. Then $f(x + h) - f(x) = (2x + 2h + 3) - (2x + 3) = 2h$. Divided by h: $2h/h = 2$. This is the slope of the linear function, computed from the difference quotient.

12. C — Apply $S_n = (n/2)(2a_1 + (n - 1)d)$ with $a_1 = 5$, $d = 4$, $n = 12$: $S_{12} = 6(10 + 44) = 6 \cdot 54 = 324$. The arithmetic series sum can also be computed as $(n/2)(a_1 + a_n)$.

13. B — Swap x and y in $y = (x + 5)/3$ to get $x = (y + 5)/3$. Solve for y: $3x = y + 5$, so $y = 3x - 5$. Verification: $f(f^{-1}(x)) = (3x - 5 + 5)/3 = 3x/3 = x$ ✓.

14. A — Divide both sides by 5: $2^x = 16$. Since $16 = 2^4$, equate exponents: $x = 4$. Isolating the exponential before applying the equivalence of exponents is the key step.

15. C — Sine is an odd function, so $\sin(-\theta) = -\sin(\theta)$. Therefore $\sin(-\pi/3) = -\sin(\pi/3) = -\sqrt{3}/2$. The negative input flips the sign of the output for odd functions.

16. C — Convert to exponential form: $5^2 = x^2$, so $x^2 = 25$. Take the square root: $x = \pm 5$. Both solutions are valid because the argument x^2 is positive (nonzero), satisfying the domain of the logarithm. The absolute value of x — not its sign — matters here.

17. B — Swap x and y in $y = \ln(x - 3)$: $x = \ln(y - 3)$. Convert to exponential form: $e^x = y - 3$, so $y = e^x + 3$. The constant 3 stays outside the exponential after the inverse swap.

18. D — For $y = A \sin(B(x - C)) + D$, the period is $2\pi/B$. Setting $2\pi/B = \pi$ gives $B = 2\pi/\pi = 2$. The phase shift $\pi/6$ and amplitude 4 do not affect the period.

19. A — In Quadrant I both sine and cosine are positive. Apply $\sec^2\theta = 1 + \tan^2\theta = 1 + 9 = 10$, so $|\sec \theta| = \sqrt{10}$ and $\cos \theta = 1/\sqrt{10}$. Then $\sin \theta = \tan \theta \cdot \cos \theta = 3 \cdot 1/\sqrt{10} = 3/\sqrt{10} = 3\sqrt{10}/10$ after rationalizing.

20. B — Apply the power rule: $-2 \log(y) = -\log(y^2)$, and $3 \log(z) = \log(z^3)$. Combine using product/quotient rules: $\log(x) + \log(z^3) - \log(y^2) = \log(xz^3/y^2)$.

21. C — A confidence interval is symmetric about the point estimate (sample mean): the midpoint of (6.2, 7.4) is $(6.2 + 7.4)/2 = 6.8$ lbs. The interval width 1.2 lbs represents twice the margin of error.

22. A — Drawing names from a hat gives every student an equal and independent chance of being selected, satisfying the definition of simple random sampling. Stratified, cluster, and convenience methods involve grouping or non-random selection.

23. D — Conditional probability restricts attention to the given event. There are 12 face cards (J, Q, K in each of 4 suits), of which 4 are Queens. P(Queen | Face) = 4/12 = 1/3.

24. B — Since order does not matter, use combinations: $C(12, 5) = 12! / (5! \cdot 7!) = (12 \cdot 11 \cdot 10 \cdot 9 \cdot 8) / (5 \cdot 4 \cdot 3 \cdot 2 \cdot 1) = 95{,}040 / 120 = 792$.

PART II — Short Constructed-Response Explanations

25. [2 credits] Apply the quadratic formula with a = 4, b = −6, c = 5: $x = [6 \pm \sqrt{(36 - 80)}]/8 = [6 \pm \sqrt{(-44)}]/8 = [6 \pm 2i\sqrt{11}]/8 = (3 \pm i\sqrt{11})/4$. Solutions in a + bi form: $\mathbf{x = 3/4 + (\sqrt{11}/4)i}$ **and** $\mathbf{x = 3/4 - (\sqrt{11}/4)i}$.

26. [2 credits] Restrictions: $\mathbf{x \neq -2}$ **and** $\mathbf{x \neq 1}$. Cross-multiply: $3(x - 1) = 4(x + 2)$, giving $3x - 3 = 4x + 8$, so $\mathbf{x = -11}$. Since −11 violates neither restriction, the solution is valid and **no extraneous solutions arise**.

27. [2 credits] Multiply two factors first: $(x - 2)(x + 1) = x^2 - x - 2$. Multiply by the third: $(x^2 - x - 2)(x - 4) = x^3 - 4x^2 - x^2 + 4x - 2x + 8 = x^3 - 5x^2 + 2x + 8$. Multiply by leading coefficient 3: $\mathbf{f(x) = 3x^3 - 15x^2 + 6x + 24}$.

28. [2 credits] Apply the product rule: $\log_3((x + 1)(x - 5)) = 3$. Convert to exponential form: $(x + 1)(x - 5) = 27$, giving $x^2 - 4x - 32 = 0$ and $(x - 8)(x + 4) = 0$. Candidate solutions x = 8 and x = −4. Domain requires x > 5 (both log arguments positive), so x = −4 is rejected and **x = 8**. Verification: $\log_3(9) + \log_3(3) = 2 + 1 = 3$ ✓.

29. [2 credits] Apply $a_n = a_1 \cdot r^{(n - 1)}$ with $a_1 = 8$, r = 3, n = 5: $a_5 = 8 \cdot 3^4 = 8 \cdot 81 = \mathbf{648}$.

30. [2 credits] Recognize 75° = 45° + 30°. Apply sin(A + B) = sin(A) cos(B) + cos(A) sin(B): $\sin(75°) = \sin(45°)\cos(30°) + \cos(45°)\sin(30°) = (\sqrt{2}/2)(\sqrt{3}/2) + (\sqrt{2}/2)(1/2) = \sqrt{6}/4 + \sqrt{2}/4 = \mathbf{(\sqrt{6} + \sqrt{2})/4}$.

31. [2 credits] Let u = 2x; as x ranges over [0, 2π), u ranges over [0, 4π). cos(u) = −1/2 has reference angle π/3, and cosine is negative in Quadrants II and III. In [0, 2π), u = 2π/3 and u = 4π/3. Adding 2π gives u = 8π/3 and u = 10π/3 in [2π, 4π). Convert via x = u/2: **x = π/3, 2π/3, 4π/3, and 5π/3**.

32. [2 credits] There are 5 even numbers (2, 4, 6, 8, 10) among the 10 tickets. Without replacement: P(first even) = 5/10 = 1/2. P(second even | first even) = 4/9. Multiply: (1/2)(4/9) = 4/18 = **2/9**.

PART III — Extended Constructed-Response Explanations

33.

(a) [1 credit] A 15% annual depreciation means 85% of the value remains each year, so the multiplier is 0.85. The model is $\mathbf{V(t) = 32{,}000(0.85)^t}$.

(b) [3 credits] Set $V(t) = 15{,}000$: $32{,}000(0.85)^t = 15{,}000$, so $(0.85)^t = 15/32 \approx 0.46875$. Take the natural logarithm: $t \cdot \ln(0.85) = \ln(0.46875)$, giving $t = \ln(0.46875)/\ln(0.85) \approx -0.7577/-0.1625 \approx$ **4.7 years**.

34.

(a) [2 credits] Apply synthetic division with $k = 2$ on coefficients 1, −1, −8, 12:

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  2 |  1    -1    -8    12
    |        2     2   -12
    |______________________________
       1     1    -6     0
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The quotient is $\mathbf{x^2 + x - 6}$ with remainder 0, confirming $x = 2$ as a root.

(b) [1 credit] Factor $x^2 + x - 6 = (x + 3)(x - 2)$. Combined with $(x - 2)$: $f(x) = (x - 2)(x + 3)(x - 2) = \mathbf{(x - 2)^2(x + 3)}$. The factor $(x - 2)$ appears twice, so **$x = 2$ is a repeated root** of multiplicity 2.

(c) [1 credit] Setting each factor to zero: **$x = 2$ (with multiplicity 2) and $x = -3$**.

35.

(a) [1 credit] $\mathbf{SE = 12/\sqrt{144} = 12/12 = 1}$ **mg/dL**. The standard error measures the typical variation of the sample mean from the population mean.

(b) [2 credits] $CI = \bar{x} \pm 1.96 \cdot SE = 25 \pm 1.96(1) = 25 \pm 1.96$. Lower bound: $25 - 1.96 =$ **23.04 mg/dL**. Upper bound: $25 + 1.96 =$ **26.96 mg/dL**. CI ≈ **(23.04, 26.96) mg/dL**.

(c) [1 credit] **Yes** — the entire interval (23.04, 26.96) lies above 20 mg/dL, and the lower bound 23.04 is greater than 20. The researcher can conclude with 95% confidence that the drug reduces cholesterol by more than 20 mg/dL, since the data are inconsistent with reductions of 20 mg/dL or less.

PART IV — Long Constructed-Response Explanation

36.

(a) [2 credits] Compute the signal at each distance: $\mathbf{S(1) = 100 - 20\log(1) = 100 - 0 = 100}$ **dB**, $\mathbf{S(10) = 100 - 20(1) = 80}$ **dB**, and $\mathbf{S(100) = 100 - 20(2) = 60}$ **dB**. **Pattern**: each tenfold increase in distance decreases the signal strength by 20 decibels. This reflects the logarithmic relationship between distance and signal attenuation — signal loss follows a fixed rate per decade of distance.

(b) [2 credits] Set $S(d) = 40$: $100 - 20\log(d) = 40$, so $-20\log(d) = -60$ and $\log(d) = 3$. Convert to exponential form: $\mathbf{d = 10^3 = 1{,}000}$ **km**.

(c) [2 credits] As $d \to \infty$, $\log(d) \to \infty$ (slowly), so $S(d) = 100 - 20\log(d) \to -\infty$. The signal strength **decreases without bound** as the distance grows, although at a slow logarithmic rate. To find where $S(d) = 0$: $0 = 100 - 20\log(d)$, so $\log(d) = 5$ and $\mathbf{d = 10^5 = 100{,}000}$ **km**. The signal strength reaches zero decibels at this distance and becomes negative for larger distances — representing no measurable signal in practical terms.