

PRACTICE EXAM 13: ALGEBRA II

REGENTS SIMULATION

PART I — Multiple Choice (24 questions \times 2 credits = 48 credits)

1. The expression $i^{13} + i^{18}$, written in $a + bi$ form, is

- A. $1 + i$
- B. $-1 - i$
- C. $1 - i$
- D. $-1 + i$

2. For $x > 0$, the expression $(x^{-2})^{3/4} \cdot x^{1/2}$ is equivalent to

- A. x^2
- B. x^{-2}
- C. $1/x$
- D. x

3. The exact solutions to the equation $2x^2 + 8x = -3$ are

- A. $(-4 \pm \sqrt{10}) / 4$

B. $(-4 \pm \sqrt{10}) / 2$

C. $-4 \pm \sqrt{10}$

D. $(4 \pm \sqrt{10}) / 2$

4. When the polynomial $3x^3 - 2x^2 + x - 5$ is divided by $(x + 1)$, the remainder is

A. -11

B. -3

C. -7

D. 5

5. The polynomial $x^4 - 13x^2 + 36$, factored completely over the integers, is

A. $(x^2 - 4)(x^2 - 9)$

B. $(x - 2)(x - 3)(x + 6)$

C. $(x - 2)(x + 2)(x - 3)(x + 3)$

D. $(x^2 - 4)^2$

6. What is the solution to the equation $5/(x - 2) - 3/(x - 2) = 4/(x^2 - 4)$?

A. $x = 2$

B. $x = 0$

C. $x = -2$

D. no solution

7. The solution set of the equation $\sqrt{2x - 3} = 1 - x$ is

A. $\{2\}$

B. $\{ \}$

C. $\{-1, 2\}$

D. $\{-1\}$

8. The system $y = x^2 - 4$ and $y = 3x$ intersects at the points

A. $(-1, -3)$ and $(4, 8)$

B. $(1, 3)$ and $(-4, -12)$

C. $(4, 0)$ and $(-1, 0)$

D. $(-1, -3)$ and $(4, 12)$

9. A company's profit P (in dollars) is modeled by $P(x) = -2x^2 + 100x - 400$, where x is the number of units sold. The number of units that must be sold to maximize the profit is

A. 25 units

B. 50 units

C. 100 units

D. 200 units

10. The expression $8x^3 + 125$, factored completely over the integers, is

A. $(2x - 5)(4x^2 + 10x + 25)$

B. $(2x + 5)(4x^2 + 10x + 25)$

C. $(2x + 5)(4x^2 - 10x + 25)$

D. $(2x - 5)(4x^2 - 10x + 25)$

11. If $f(x) = \sqrt{x}$ and $g(x) = 3x + 1$, then the value of $f(g(5))$ is

A. 4

B. 16

C. 8

D. 12

12. The inverse of the function $f(x) = \sqrt{x + 4} - 1$, defined for $x \geq -4$, is $f^{-1}(x)$ equal to

A. $(x - 1)^2 + 4$

B. $(x + 1)^2 - 4$

C. $\sqrt{x + 4} + 1$

D. $\sqrt{x - 4} + 1$

13. The n th term of a geometric sequence is given by $a_n = 1,024 \cdot (1/2)^{(n - 1)}$. The 6th term of this sequence is

- A. 64
- B. 16
- C. 128
- D. 32

14. An investment doubles in value after 12 years under continuous compounding. The equation $2 = e^{(12r)}$ gives the annual interest rate r . Solving for r yields approximately

- A. $r = 6\%$
- B. $r = 4.5\%$
- C. $r = 5.78\%$
- D. $r = 8\%$

15. The exact value of $\cos(3\pi/4)$ is

- A. $-\sqrt{2}/2$
- B. $\sqrt{2}/2$
- C. $1/2$
- D. $-1/2$

16. Solve algebraically for x : $\log_2(x) - \log_2(3) = 2$.

- A. $x = 7$
- B. $x = 9$

C. $x = 12$

D. $x = 6$

17. The inverse of the function $f(x) = 4^x + 2$ is $f^{-1}(x)$ equal to

A. $\log_4(x + 2)$

B. $4^{(x - 2)}$

C. $\log_4(x - 2)$

D. $4^x - 2$

18. A sinusoidal function has amplitude 5, period 4π , midline $y = -2$, and a maximum value at $x = 0$. Which equation describes this function?

A. $y = 5 \sin(x/2) - 2$

B. $y = -2 \cos(x/2) + 5$

C. $y = 5 \cos(2x) - 2$

D. $y = 5 \cos(x/2) - 2$

19. If $\cos \theta = -1/3$ and θ is in Quadrant II, then the value of $\sin \theta$ is

A. $-2\sqrt{2} / 3$

B. $2\sqrt{2} / 3$

C. $\sqrt{8} / 9$

D. $-1/3$

20. The expression $2 \ln(x) - 3 \ln(y) + (1/2) \ln(z)$, written as a single logarithm, is

A. $\ln(x^2 \sqrt{z} / y^3)$

B. $\ln((2x - 3y + z) / 2)$

C. $\ln(x^2 - y^3 + \sqrt{z})$

D. $\ln(x^2 y^3 \sqrt{z})$

21. A researcher wants to compare the effectiveness of two new headache medicines. Which experimental design is most likely to produce valid, unbiased results?

A. Give Medicine A to women and Medicine B to men only

B. Allow each patient to choose which medicine to take

C. Give Medicine A first, observe results, then give Medicine B

D. Randomly assign each patient to receive Medicine A or Medicine B

22. A normally distributed data set has a mean of 60 and a standard deviation of 8. A data value of 76 corresponds to a z-score of

A. 2

B. 1.5

C. 2.5

D. 1

23. A box contains 4 red, 5 blue, and 6 green marbles. Two marbles are drawn at random without replacement. The probability that the second marble drawn is red, given that the first marble drawn was red, is

A. $\frac{4}{14}$

B. $\frac{3}{14}$

C. $\frac{4}{15}$

D. $\frac{3}{15}$

24. In how many different orders can 5 distinct students be arranged in a line for a photograph?

A. 25

B. 24

C. 60

D. 120

PART II — Short Constructed Response (8 questions \times 2 credits = 16 credits)

Show all work. A correct answer with no supporting work will receive only 1 credit.

25. Solve the equation $3x^2 - 2x + 4 = 0$ algebraically. Express the solutions in $a + bi$ form.

26. Simplify the rational expression $(x^2 - 7x + 12) / (x^2 - 16)$ for $x \neq \pm 4$. Show all factoring steps.

27. A polynomial $p(x) = x^3 + bx^2 + cx + d$ has roots at $x = 1$, $x = -2$, and $x = 3$. Determine the values of b , c , and d , then write $p(x)$ in expanded standard form.

28. Solve algebraically for x : $2 \cdot 3^{(2x)} = 162$. Show all algebraic steps.

29. A geometric series has first term 9 and common ratio $1/3$. Find the sum of the infinite series and explain why the series converges.

30. Verify algebraically the identity $(1 + \tan^2\theta) \cos^2\theta = 1$ for $\cos \theta \neq 0$. Show all algebraic steps using Pythagorean and reciprocal identities.

31. Solve the equation $\tan^2(x) = 3$ for all values of x in the interval $[0, 2\pi)$. Show how you determined each solution.

32. A bag contains 6 red, 4 blue, and 2 white marbles. Two marbles are drawn at random without replacement. Determine the probability that both marbles drawn are the same color. Express your answer as a fraction in simplest form.

PART III — Extended Constructed Response (3 questions \times 4 credits = 12 credits)

Show all work. Partial credit is awarded according to the scoring rubric.

33. The population of a city is modeled by $P(t) = 100,000 \cdot (1.025)^t$, where t is measured in years since the start of 2020.

(a) State the population at the start of 2020 and the annual percent growth rate, and interpret both values in context.

(b) Algebraically determine the year in which the population will first reach 150,000. Round t to the nearest tenth of a year, then state the corresponding calendar year.

34. The polynomial function $p(x) = x^3 - 4x^2 + x + 6$ has $x = 2$ as a known real root.

(a) Use synthetic division to find the quotient when $p(x)$ is divided by $(x - 2)$. Show your synthetic division work.

(b) Factor $p(x)$ completely over the integers.

(c) State all real solutions to the equation $p(x) = 0$.

35. A national education survey samples 800 randomly selected high school seniors and asks whether they plan to attend college. 528 respond "yes."

(a) Determine the sample proportion \hat{p} of seniors planning to attend college.

(b) Construct a 95% confidence interval for the true population proportion using the formula $ME = 1.96 \cdot \sqrt{(\hat{p}(1 - \hat{p}))/n}$. Round the bounds to three decimal places.

(c) A previous national study estimated that 65% of seniors planned to attend college. Are the current data consistent with that estimate at the 95% confidence level? Justify your answer using the interval.

PART IV — Long Constructed Response (1 question \times 6 credits = 6 credits)

Show all work. This problem integrates concepts from several chapters.

36. The temperature of a refrigerated drink as it warms in a room is modeled by $T(t) = 70 - 36e^{-0.04t}$, where T is in degrees Fahrenheit and t is the time in minutes since the drink was placed on the counter.

(a) Determine the initial temperature of the drink at $t = 0$ and the long-term temperature the drink approaches as t becomes large. Justify each value using the model.

(b) Determine the temperature of the drink after 25 minutes. Round your answer to the nearest tenth of a degree Fahrenheit.

(c) Algebraically determine the time at which the drink reaches a temperature of exactly 60°F . Round your answer to the nearest tenth of a minute. Show all algebraic work.

PRACTICE EXAM 13 – ANSWER KEY AND EXPLANATIONS

1. D — Powers of i cycle with period 4. Since $13 = 4(3) + 1$, $i^{13} = i$; since $18 = 4(4) + 2$, $i^{18} = i^2 = -1$. Sum: $i + (-1) = -1 + i$ in standard $a + bi$ form.

2. C — Apply exponent rules: $(x^{-2})^{3/4} = x^{-3/2}$, and combining with $x^{1/2}$ gives $x^{-3/2 + 1/2} = x^{-1} = 1/x$. Exponents add when bases match.

3. B — Rewrite as $2x^2 + 8x + 3 = 0$. Apply the quadratic formula: $x = \frac{-8 \pm \sqrt{(64 - 24)}}{4} = \frac{-8 \pm \sqrt{40}}{4} = \frac{-8 \pm 2\sqrt{10}}{4} = \frac{-4 \pm \sqrt{10}}{2}$. Simplifying $\sqrt{40}$ to $2\sqrt{10}$ and reducing the fraction by 2 produces the simplest radical form.

4. A — Apply the Remainder Theorem: dividing by $(x + 1)$ means evaluating at $x = -1$. Compute $p(-1) = 3(-1)^3 - 2(-1)^2 + (-1) - 5 = -3 - 2 - 1 - 5 = -11$.

5. C — Substitute $u = x^2$: $u^2 - 13u + 36 = (u - 4)(u - 9)$. Back-substitute: $(x^2 - 4)(x^2 - 9)$. Each factor is a difference of squares: $x^2 - 4 = (x - 2)(x + 2)$ and $x^2 - 9 = (x - 3)(x + 3)$, giving the complete factorization.

6. B — Multiply both sides by $(x - 2)(x + 2)$ (with restrictions $x \neq \pm 2$): $5(x + 2) - 3(x + 2) = 4$, so $2(x + 2) = 4$, giving $x + 2 = 2$ and $x = 0$. Since $0 \neq \pm 2$, the solution is valid.

- 7. B** — Square both sides: $2x - 3 = (1 - x)^2 = x^2 - 2x + 1$, giving $x^2 - 4x + 4 = 0$ and $(x - 2)^2 = 0$, so $x = 2$. Check: $\sqrt{1} = 1$ but $1 - 2 = -1$, which the principal square root cannot equal. The candidate is extraneous, so the solution set is empty.
- 8. D** — Substitute to eliminate y : $x^2 - 4 = 3x$, giving $x^2 - 3x - 4 = 0$ and $(x - 4)(x + 1) = 0$, so $x = 4$ or $x = -1$. Evaluate y : at $x = 4$, $y = 12$; at $x = -1$, $y = -3$. Intersection points: $(4, 12)$ and $(-1, -3)$.
- 9. A** — For a parabola $P(x) = ax^2 + bx + c$ with $a < 0$, the maximum occurs at the vertex $x = -b/(2a) = -100/(-4) = 25$. The leading coefficient -2 ensures the parabola opens downward, so the vertex is the maximum.
- 10. C** — Apply the sum of cubes identity $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$ with $a = 2x$ and $b = 5$: $(2x + 5)((2x)^2 - (2x)(5) + 5^2) = (2x + 5)(4x^2 - 10x + 25)$. The middle term carries a negative sign for sums of cubes.
- 11. A** — Composition is evaluated inside-out: $g(5) = 3(5) + 1 = 16$, then $f(16) = \sqrt{16} = 4$. The output of g becomes the input of f .
- 12. B** — Swap x and y in $y = \sqrt{x + 4} - 1$: $x = \sqrt{y + 4} - 1$, giving $(x + 1)^2 = y + 4$ after isolating the radical and squaring. Therefore $f^{-1}(x) = (x + 1)^2 - 4$. Since the original domain requires $x \geq -4$, the range of the inverse confirms $(x + 1)^2 \geq 0$.
- 13. D** — Substitute $n = 6$: $a_6 = 1,024 \cdot (1/2)^5 = 1,024 \cdot 1/32 = 32$. The exponent is $n - 1$, not n , in the geometric formula $a_n = a_1 \cdot r^{(n - 1)}$.
- 14. C** — Take the natural logarithm: $12r = \ln(2)$, so $r = \ln(2)/12 \approx 0.6931/12 \approx 0.0578$, or about 5.78%. The Rule of 72 gives a quick estimate ($72/12 = 6\%$), but the continuous-compounding formula refines this slightly downward.
- 15. A** — The angle $3\pi/4$ lies in Quadrant II with reference angle $\pi - 3\pi/4 = \pi/4$. Cosine is negative in Quadrant II, and $\cos(\pi/4) = \sqrt{2}/2$, so $\cos(3\pi/4) = -\sqrt{2}/2$.
- 16. C** — Apply the quotient rule of logarithms: $\log_2(x/3) = 2$. Convert to exponential form: $x/3 = 2^2 = 4$, so $x = 12$. Verification: $\log_2(12) - \log_2(3) = \log_2(12/3) = \log_2(4) = 2 \checkmark$.
- 17. C** — Swap x and y in $y = 4^x + 2$: $x = 4^y + 2$. Isolate the exponential: $x - 2 = 4^y$, then convert to logarithmic form: $y = \log_4(x - 2)$. The inverse undoes both the exponentiation and the $+2$.
- 18. D** — Amplitude 5 and midline -2 give the form $y = 5 \text{ trig}(Bx) - 2$. Period 4π means $B = 2\pi/(4\pi) = 1/2$. A maximum at $x = 0$ calls for cosine ($\cos(0) = 1$, the maximum), so $y = 5 \cos(x/2) - 2$.
- 19. B** — Apply the Pythagorean identity: $\sin^2\theta = 1 - \cos^2\theta = 1 - 1/9 = 8/9$, so $|\sin \theta| = \sqrt{8/9} = (2\sqrt{2})/3$. In Quadrant II sine is positive: $\sin \theta = 2\sqrt{2}/3$.
- 20. A** — Apply the power rule first: $2 \ln(x) = \ln(x^2)$, $3 \ln(y) = \ln(y^3)$, and $(1/2) \ln(z) = \ln(\sqrt{z})$. Combine using product/quotient rules: $\ln(x^2) - \ln(y^3) + \ln(\sqrt{z}) = \ln(x^2 \cdot \sqrt{z} / y^3)$.

21. D — Random assignment is the gold standard of experimental design for comparing treatments. It eliminates selection bias and balances both known and unknown confounders across the treatment groups. The other options introduce gender confounding, self-selection bias, or order effects that compromise causal inference.

22. A — The z-score formula is $z = (\text{value} - \text{mean})/\text{standard deviation} = (76 - 60)/8 = 16/8 = 2$. A z-score of 2 means the value lies 2 standard deviations above the mean.

23. B — After the first red marble is drawn, 14 marbles remain in the bag: 3 red, 5 blue, and 6 green. The conditional probability $P(\text{2nd red} \mid \text{1st red}) = 3/14$. Without-replacement sampling reduces both the favorable count and the total count by 1.

24. D — Arrangements of n distinct objects use factorials: $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$. Each position in the line is filled by one of the remaining unplaced students.

PART II — Short Constructed-Response Explanations

25. [2 credits] Apply the quadratic formula with $a = 3$, $b = -2$, $c = 4$: $x = [2 \pm \sqrt{(4 - 48)}]/6 = [2 \pm \sqrt{(-44)}]/6 = [2 \pm 2i\sqrt{11}]/6 = (1 \pm i\sqrt{11})/3$. Solutions in $a + bi$ form: $\mathbf{x = 1/3 + (\sqrt{11}/3)i}$ and $\mathbf{x = 1/3 - (\sqrt{11}/3)i}$.

26. [2 credits] Factor: $x^2 - 7x + 12 = (x - 3)(x - 4)$, and $x^2 - 16 = (x - 4)(x + 4)$. Cancel the common factor $(x - 4)$, leaving $\mathbf{(x - 3)/(x + 4)}$ for $x \neq \pm 4$.

27. [2 credits] Write $p(x) = (x - 1)(x + 2)(x - 3)$. Multiply step by step: $(x - 1)(x + 2) = x^2 + x - 2$, then $(x^2 + x - 2)(x - 3) = x^3 - 3x^2 + x^2 - 3x - 2x + 6 = x^3 - 2x^2 - 5x + 6$. Therefore $\mathbf{b = -2}$, $\mathbf{c = -5}$, $\mathbf{d = 6}$, and $\mathbf{p(x) = x^3 - 2x^2 - 5x + 6}$.

28. [2 credits] Divide both sides by 2: $3^{(2x)} = 81 = 3^4$. Equate exponents: $2x = 4$, so $\mathbf{x = 2}$.

29. [2 credits] Since $|r| = 1/3 < 1$, the series **converges**. Apply the infinite geometric sum formula $S = a_1/(1 - r) = 9/(1 - 1/3) = 9/(2/3) = \mathbf{27/2}$. The series converges because successive terms shrink toward zero quickly enough.

30. [2 credits] Apply the Pythagorean identity $1 + \tan^2\theta = \sec^2\theta$ to the left side: $(1 + \tan^2\theta) \cos^2\theta = \sec^2\theta \cdot \cos^2\theta$. By the reciprocal identity $\sec^2\theta = 1/\cos^2\theta$: $\sec^2\theta \cdot \cos^2\theta = (1/\cos^2\theta) \cdot \cos^2\theta = \mathbf{1}$ ✓. The identity is verified.

31. [2 credits] Take square roots: $\tan(x) = \pm\sqrt{3}$. Reference angle for both is $\pi/3$ (since $\tan(\pi/3) = \sqrt{3}$). Tangent is positive in Quadrants I and III: $\mathbf{x = \pi/3}$ and $\mathbf{x = 4\pi/3}$. Tangent is negative in Quadrants II and IV: $\mathbf{x = 2\pi/3}$ and $\mathbf{x = 5\pi/3}$. Four solutions in $[0, 2\pi)$.

32. [2 credits] Total marbles = 12. Compute each same-color probability without replacement: $P(\text{both red}) = (6/12)(5/11) = 30/132$, $P(\text{both blue}) = (4/12)(3/11) = 12/132$, $P(\text{both white}) = (2/12)(1/11) = 2/132$. Sum: $30 + 12 + 2 = 44$ favorable, so $P = 44/132 = \mathbf{1/3}$.

PART III — Extended Constructed-Response Explanations

33.

(a) [2 credits] **Population at start of 2020:** $P(0) = 100,000(1.025)^0 = 100,000$ — the city's population in 2020 is 100,000. **Annual growth rate:** $1.025 = 1 + 0.025$, so the population grows by **2.5% per year**.

(b) [2 credits] Set $P(t) = 150,000$: $100,000(1.025)^t = 150,000$, so $(1.025)^t = 1.5$. Take the natural logarithm: $t \cdot \ln(1.025) = \ln(1.5)$, giving $t = \ln(1.5)/\ln(1.025) \approx 0.4055/0.02469 \approx \mathbf{16.4}$ years. Calendar year: $2020 + 16 \approx \mathbf{2036}$ (the population reaches 150,000 partway through 2036).

34.

(a) [2 credits] Apply synthetic division with $k = 2$ on coefficients 1, -4, 1, 6:

$$\begin{array}{r|rrrr} 2 & 1 & -4 & 1 & 6 \\ & & 2 & -4 & -6 \\ \hline & 1 & -2 & -3 & 0 \end{array}$$

The quotient is $x^2 - 2x - 3$ with remainder 0, confirming $x = 2$ as a root.

(b) [1 credit] Factor $x^2 - 2x - 3 = (x - 3)(x + 1)$, so $p(x) = (x - 2)(x - 3)(x + 1)$.

(c) [1 credit] Setting each factor to zero: $x = 2$, $x = 3$, and $x = -1$.

35.

(a) [1 credit] Sample proportion: $\hat{p} = 528/800 = \mathbf{0.66}$ (or 66%).

(b) [2 credits] Compute $ME = 1.96 \cdot \sqrt{(0.66 \cdot 0.34/800)} = 1.96 \cdot \sqrt{(0.2244/800)} = 1.96 \cdot \sqrt{0.0002805} \approx 1.96 \cdot 0.01675 \approx 0.0328$. Confidence interval: 0.66 ± 0.0328 , giving lower bound $0.66 - 0.0328 \approx \mathbf{0.627}$ and upper bound $0.66 + 0.0328 \approx \mathbf{0.693}$. $CI \approx (\mathbf{0.627}, \mathbf{0.693})$.

(c) [1 credit] **Yes** — the value 0.65 lies within the interval (0.627, 0.693). The current data are consistent with the previous estimate of 65% at the 95% confidence level; there is insufficient evidence to claim that the true proportion differs from 0.65.

PART IV — Long Constructed-Response Explanation

36.

(a) [2 credits] **Initial temperature** at $t = 0$: $T(0) = 70 - 36e^0 = 70 - 36 = \mathbf{34^\circ F}$ — the temperature when the drink was placed on the counter. **Long-term temperature:** as $t \rightarrow \infty$, $e^{(-0.04t)} \rightarrow 0$, so $T(t) \rightarrow 70 - 0 = \mathbf{70^\circ F}$ — the drink asymptotically approaches the ambient room temperature, never quite reaching it but coming arbitrarily close.

(b) [1 credit] Substitute $t = 25$: $T(25) = 70 - 36 e^{(-0.04 \cdot 25)} = 70 - 36 e^{-1}$. Since $e^{-1} \approx 0.3679$: $T(25) \approx 70 - 36 \cdot 0.3679 \approx 70 - 13.24 \approx \mathbf{56.8^\circ\text{F}}$.

(c) [3 credits] Set $T(t) = 60$: $70 - 36 e^{(-0.04t)} = 60$, so $-36 e^{(-0.04t)} = -10$ and $e^{(-0.04t)} = 10/36 = 5/18$. Take the natural logarithm: $-0.04t = \ln(5/18) \approx \ln(0.2778) \approx -1.2810$. Solve for t : $t \approx 1.2810/0.04 \approx \mathbf{32.0 \text{ minutes}}$. The drink reaches 60°F approximately 32 minutes after being placed on the counter.