PRACTICE EXAM 12: ALGEBRA II REGENTS SIMULATION

PART I — Multiple Choice (24 questions × 2 credits = 48 credits)

1. The expression $(64x^{12}y^6)^{(1/3)}$ is equivalent to

A. $4x^4y^2$

B. $4x^9y^3$

C. $64x^4y^2$

D. $8x^4y^2$

2. The expression $2i(3 + 4i) - i$, written in $a + bi$ form, equals

A. $-8 + 7i$

B. $-8 + 5i$

C. $8 + 5i$

D. $-7 + 6i$

3. Which of the following is a factor of the polynomial $x^3 + 2x^2 - 5x - 6$?

A. $(x - 1)$

B. $(x + 2)$

C. $(x - 3)$

D. $(x + 1)$

4. When the polynomial $p(x) = 2x^3 - 5x^2 + x + 2$ is divided by $(x - 2)$, the remainder is

A. 4

B. 2

C. 0

D. −2

5. The exact solutions to the equation $3x^2 - 2x - 1 = 0$ are

A. $x = 1$ and $x = -1/3$

B. $x = 1$ and $x = 1/3$

C. $x = -1$ and $x = -1/3$

D. $x = -1$ and $x = 1/3$

6. What is the solution to the equation $3/(x + 1) = 5/(x + 3)$?

A. $x = -1$

B. $x = 0$

C. $x = -2$

D. $x = 2$

7. What is the solution to the equation $\sqrt[3]{(2x - 1)} = 3$?

A. $x = 12$

B. $x = 13$

C. $x = 15$

D. $x = 14$

8. How many real solutions does the system $y = x^2 + 3$ and $y = 2x$ have?

A. 1

B. 0

C. 2

D. 4

9. A town's population P (in thousands) is modeled by $P(t) = 25(1.03)^t$, where t is the number of years after the year 2020. According to this model, the population in the year 2020 is

A. 1.03 thousand

B. 28 thousand

C. 25 thousand

D. 75 thousand

10. The expression $x^4 - 81$, factored completely over the integers, is

A. $(x - 3)(x + 3)(x^2 + 9)$

B. $(x^2 - 9)^2$

C. $(x - 3)^2(x + 3)^2$

D. $(x^2 - 9)(x^2 + 9)$

11. If $f(x) = 2x^3 - x + 5$, then the value of $f(-2)$ is

A. −16

B. −9

C. 7

D. −19

12. A geometric sequence has second term $a_2 = 12$ and fifth term $a_5 = 324$. The common ratio is

A. 4

B. 2

C. 3

D. 6

13. The function $f(x) = x^2$ is reflected across the x-axis and shifted 2 units to the left to produce $g(x)$. The equation of $g(x)$ is

A. $-(x - 2)^2$

B. $(x + 2)^2 - 1$

C. $(x - 2)^2$

D. $-(x + 2)^2$

14. Solve algebraically for x: $5^{(2x - 1)} = 25$.

A. $x = 3/2$

B. $x = 1$

C. $x = 2$

D. $x = 1/2$

15. The exact value of $\sin(11\pi/6)$ is

A. $1/2$

B. $-1/2$

C. $\sqrt{3}/2$

D. $-\sqrt{3}/2$

16. Solve algebraically for x: $\log(x - 5) = 1$.

A. $x = 5$

B. $x = 6$

C. $x = 15$

D. $x = 50$

17. The period of the function $y = -2 \sin(3x) + 5$ is

A. $2\pi/3$

B. 3

C. 2π

D. $\pi/3$

18. The inverse of the function $f(x) = \log_3(x + 1)$ is $f^{-1}(x)$ equal to

A. $3^x - 1$

B. 3^(x − 1)

C. 3^(x + 1)

D. $3^x + 1$

19. An arithmetic sequence has a first term of 7 and a common difference of −3. Which formula gives the nth term of the sequence?

A. $a_n = 7 - 3n$

B. $a_n = -3 + 7n$

C. $a_n = 10 + 3n$

D. $a_n = 10 - 3n$

20. If $\tan \theta = -3/4$ and θ is in Quadrant II, the value of $\sin \theta$ is

A. $-3/5$

B. $3/5$

C. $4/5$

D. $-4/5$

21. A high school principal wants to estimate the average GPA at her school. She randomly selects 50 students from each of the four grade levels (9th, 10th, 11th, 12th). This sampling method is best described as

A. simple random sampling

B. cluster random sampling

C. stratified random sampling

D. convenience sampling design

22. Every value in a data set is transformed by adding 10. Which of the following best describes how the mean and standard deviation change?

A. The mean is unchanged and the standard deviation increases by 10

B. Both the mean and the standard deviation increase by 10

C. The mean increases by 10 and the standard deviation decreases by 10

D. The mean increases by 10 and the standard deviation is unchanged

23. A pizza shop offers 8 different toppings. In how many ways can a customer choose exactly 3 different toppings for a pizza, where the order does not matter?

A. 24

B. 36

C. 56

D. 336

24. A jar contains 3 red, 5 blue, and 2 green marbles. One marble is drawn, the color is recorded, and the marble is replaced. A second marble is then drawn. What is the probability that the first marble is green and the second marble is blue?

A. 7/100

B. 1/10

C. 1/5

D. 7/10

PART II — Short Constructed Response (8 questions × 2 credits = 16 credits)

Show all work. A correct answer with no supporting work will receive only 1 credit.

25. Solve the equation $x^2 + 4x + 7 = 0$ algebraically. Express the solutions in $a + bi$ form.

26. Simplify the rational expression $(x^2 + 6x + 8) / (x^2 + 4x)$ for $x \neq 0$ and $x \neq -4$. Show all factoring steps.

27. Use synthetic division to find the quotient when $x^3 + 4x^2 - 7x - 10$ is divided by $(x + 5)$. State both the quotient and the remainder.

28. Solve algebraically for x: $3^{(x + 1)} = 81^x$. Show all algebraic steps using a common base.

29. A geometric series has first term 4 and common ratio 1/3. Find the sum of the first 5 terms of the series. Express your answer as a fraction in simplest form.

30. Use the sine difference identity $\sin(A - B) = \sin(A)\cos(B) - \cos(A)\sin(B)$ to verify algebraically that $\sin(\pi/2 - \theta) = \cos(\theta)$. Show all algebraic steps.

31. Solve the equation $2\cos^2(x) - \cos(x) - 1 = 0$ for all values of x in the interval $[0, 2\pi)$. Show all algebraic work, including factoring.

32. A box contains 5 red, 4 blue, and 3 yellow chips. Two chips are drawn at random without replacement. Determine the probability that one chip drawn is red and the other is blue, in either order. Express your answer as a fraction in simplest form.

PART III — Extended Constructed Response (3 questions × 4 credits = 12 credits)

Show all work. Partial credit is awarded according to the scoring rubric.

33. The amount of caffeine in a person's bloodstream decreases by 13% every hour after consumption. A person drinks a beverage containing 200 milligrams of caffeine.

(a) Write a function C(t) that models the amount of caffeine, in milligrams, remaining in the person's bloodstream t hours after consumption.

(b) Algebraically determine the time at which the caffeine level falls to 50 milligrams. Round your answer to the nearest tenth of an hour.

34. The polynomial function $p(x) = 2x^4 - 3x^3 - 11x^2 + 6x$ has $x = 0$ as a root (the polynomial has no constant term).

(a) Factor x out of p(x), then use synthetic division on the resulting cubic, knowing that $x = 3$ is a root. Show your synthetic division work.

(b) Factor p(x) completely over the integers.

(c) State all real solutions to the equation $p(x) = 0$.

35. A research team tests a new medication's effectiveness. They randomly select 250 patients and find that 175 of them report improvement after taking the medication.

(a) Determine the sample proportion $\hat{p}$ of patients who reported improvement.

(b) Construct a 95% confidence interval for the true population proportion who improve, using the formula $ME = 1.96 \cdot \sqrt{(\hat{p}(1 - \hat{p})/n)}$. Round the bounds to three decimal places.

(c) The previously approved medication has an improvement rate of 0.60. Based on the interval from part (b), can the team conclude with 95% confidence that the new medication is more effective than the older one? Justify your answer.

PART IV — Long Constructed Response (1 question × 6 credits = 6 credits)

Show all work. This problem integrates concepts from several chapters.

36. A buoy bobs on the water with its height d (in meters) above sea level at time t (in seconds) modeled by $d(t) = 1.5 \cos((\pi/4)\ t) + 3$. A positive value of d indicates the buoy is above sea level; a negative value would indicate below.

(a) Determine the period of the buoy's vertical motion. Justify your reasoning based on the model.

(b) Determine the buoy's height at t = 6 seconds. Round your answer to the nearest tenth of a meter.

(c) Algebraically determine all values of t in the interval [0, 16] seconds at which the buoy is exactly 3.5 meters above sea level. Round each value of t to the nearest hundredth of a second. Show all algebraic work.

PRACTICE EXAM 12 – ANSWER KEY AND EXPLANATIONS

1. A — Apply the rational exponent rule to each factor: $64^{(1/3)} = 4$ (since $4^3 = 64$), $x^{(12/3)} = x^4$, and $y^{(6/3)} = y^2$. The result is $4x^4y^2$. Each exponent inside the parentheses is multiplied by 1/3.

2. B — Distribute the first term: $2i(3 + 4i) = 6i + 8i^2 = 6i - 8$ (since $i^2 = -1$). Subtract i: $6i - 8 - i = -8 + 5i$ in standard a + bi form.

3. D — Test each candidate using the Factor Theorem: for (x + 1), evaluate $p(-1) = (-1)^3 + 2(-1)^2 - 5(-1) - 6 = -1 + 2 + 5 - 6 = 0$. Since $p(-1) = 0$, (x + 1) is a factor. The other choices yield nonzero values when tested.

4. C — Apply the Remainder Theorem: dividing by (x − 2) means evaluating at x = 2. Compute $p(2) = 2(8) - 5(4) + 2 + 2 = 16 - 20 + 2 + 2 = 0$. The remainder is 0, so (x − 2) is also a factor.

5. A — Factor the quadratic: $3x^2 - 2x - 1 = (3x + 1)(x - 1)$. Setting each factor to zero gives $3x + 1 = 0 \rightarrow x = -1/3$, and $x - 1 = 0 \rightarrow x = 1$. The two solutions are x = 1 and x = −1/3.

6. D — Cross-multiply (with restrictions $x \neq -1, -3$): $3(x + 3) = 5(x + 1)$, giving $3x + 9 = 5x + 5$ and $4 = 2x$, so $x = 2$. The solution satisfies the domain restrictions.

7. D — Cube both sides to eliminate the cube root: $2x - 1 = 27$, so $2x = 28$ and $x = 14$. Cubing is a one-to-one operation, so no extraneous solutions arise.

8. B — Set equal: $x^2 + 3 = 2x$, giving $x^2 - 2x + 3 = 0$. Discriminant: $(-2)^2 - 4(1)(3) = 4 - 12 = -8$, which is negative. No real solutions exist — the line never intersects the parabola.

9. C — At $t = 0$ (the year 2020), $P(0) = 25(1.03)^0 = 25 \cdot 1 = 25$ thousand. The initial value in an exponential model is the value of P when $t = 0$.

10. A — Apply the difference of squares twice: $x^4 - 81 = (x^2 - 9)(x^2 + 9)$. The first factor splits further: $x^2 - 9 = (x - 3)(x + 3)$. The factor $(x^2 + 9)$ cannot be factored over the integers (no real roots), so the complete factorization is $(x - 3)(x + 3)(x^2 + 9)$.

11. B — Substitute $x = -2$: $f(-2) = 2(-2)^3 - (-2) + 5 = 2(-8) + 2 + 5 = -16 + 7 = -9$. Be careful with the sign of $-x$ when x is negative.

12. C — Apply $a_n = a_1 \cdot r\text{\^{}}(n - 1)$, giving $a_5 = a_2 \cdot r^3$. Substitute: $324 = 12r^3$, so $r^3 = 27$ and $r = 3$. The ratio between terms three positions apart involves r^3.

13. D — Reflecting $f(x) = x^2$ across the x-axis negates the output: $-x^2$. Shifting 2 units to the left replaces x with $(x + 2)$: the composite $g(x) = -(x + 2)^2$. The horizontal shift is encoded inside the function argument.

14. A — Rewrite 25 as 5^2: $5\text{\^{}}(2x - 1) = 5^2$. Equate exponents: $2x - 1 = 2$, giving $2x = 3$ and $x = 3/2$. Matching bases reduces an exponential equation to a linear one.

15. B — The angle $11\pi/6$ lies in Quadrant IV with reference angle $2\pi - 11\pi/6 = \pi/6$. Sine is negative in Quadrant IV, and $\sin(\pi/6) = 1/2$, so $\sin(11\pi/6) = -1/2$.

16. C — Convert to exponential form (base 10 implied): $\log(x - 5) = 1$ means $10^1 = x - 5$, so $10 = x - 5$ and $x = 15$. Verification: $\log(10) = 1$ ✓.

17. A — For $y = A \sin(Bx) + D$, the period is $2\pi/B$. With $B = 3$: period $= 2\pi/3$. The amplitude, vertical shift, and reflection do not affect the period.

18. A — Swap x and y in $y = \log_3(x + 1)$: $x = \log_3(y + 1)$. Convert to exponential form: $3^x = y + 1$, so $y = 3^x - 1$. The inverse undoes both the log and the +1.

19. D — Apply $a_n = a_1 + (n - 1)d$ with $a_1 = 7$ and $d = -3$: $a_n = 7 + (n - 1)(-3) = 7 - 3n + 3 = 10 - 3n$. Verification: $a_1 = 10 - 3 = 7$ ✓, $a_2 = 10 - 6 = 4$ $(= 7 - 3)$ ✓.

20. B — Use the identity $\sec^2\theta = 1 + \tan^2\theta = 1 + 9/16 = 25/16$, so $|\sec \theta| = 5/4$. In Quadrant II, cosine is negative, so $\sec \theta = -5/4$ and $\cos \theta = -4/5$. Then $\sin \theta = \tan \theta \cdot \cos \theta = (-3/4)(-4/5) = 3/5$ (positive, consistent with Q II).

21. C — Stratified random sampling partitions the population into homogeneous subgroups (strata) and then randomly samples within each stratum. Sampling 50 students from each grade level matches this definition. Simple random sampling would treat the entire student body as one pool; cluster sampling would use whole clusters; convenience sampling lacks randomization.

22. D — Adding a constant to every value translates the distribution: every value (and thus the mean) increases by 10. Since the spread of values does not change, the standard deviation is unchanged. Translation affects center but not variability.

23. C — Order does not matter, so use combinations: $C(8, 3) = 8!/(3! \cdot 5!) = (8 \cdot 7 \cdot 6)/(3 \cdot 2 \cdot 1) = 336/6 = 56$. There are 56 different sets of 3 toppings.

24. B — Drawing with replacement makes the events independent. P(green first) = 2/10 = 1/5. P(blue second) = 5/10 = 1/2. Multiply: (1/5)(1/2) = 1/10. The first draw does not change the second's sample space because the marble is replaced.

PART II — Short Constructed-Response Explanations

25. [2 credits] Apply the quadratic formula with $a = 1$, $b = 4$, $c = 7$: $x = [-4 \pm \sqrt{(16 - 28)}]/2 = [-4 \pm \sqrt{(-12)}]/2 = [-4 \pm 2i\sqrt{3}]/2 = \mathbf{-2 \pm i\sqrt{3}}$. Solutions in a + bi form: $x = -2 + i\sqrt{3}$ and $x = -2 - i\sqrt{3}$.

26. [2 credits] Factor numerator and denominator: $x^2 + 6x + 8 = (x + 2)(x + 4)$, and $x^2 + 4x = x(x + 4)$. Cancel the common factor $(x + 4)$: $\mathbf{(x + 2)/x}$ for $x \neq 0$ and $x \neq -4$.

27. [2 credits] Synthetic division with $k = -5$ on coefficients 1, 4, −7, −10:

```
-5 |  1     4    -7    -10
   |       -5     5     10
   |____________________________
      1    -1    -2      0
```

Quotient: $x^2 - x - 2$, **Remainder: 0**. Since the remainder is zero, $(x + 5)$ is a factor.

28. [2 credits] Rewrite 81 as 3^4, so $81^x = (3^4)^x = 3^{(4x)}$. The equation becomes $3^{(x + 1)} = 3^{(4x)}$. Equate exponents: $x + 1 = 4x$, giving $1 = 3x$ and $\mathbf{x = 1/3}$.

29. [2 credits] Apply $S_n = a_1(1 - r^n)/(1 - r)$ with $a_1 = 4$, $r = 1/3$, $n = 5$: $S_5 = 4(1 - (1/3)^5)/(1 - 1/3) = 4(1 - 1/243)/(2/3) = 4 \cdot (242/243) \cdot (3/2) = (4 \cdot 242 \cdot 3)/(243 \cdot 2) = 2{,}904/486 = \mathbf{484/81}$ (in simplest form).

30. [2 credits] Apply the sine difference identity with $A = \pi/2$ and $B = \theta$: $\sin(\pi/2 - \theta) = \sin(\pi/2)\cos(\theta) - \cos(\pi/2)\sin(\theta)$. Substitute the exact values $\sin(\pi/2) = 1$ and $\cos(\pi/2) = 0$: $\sin(\pi/2 - \theta) = (1)\cos(\theta) - (0)\sin(\theta) = \mathbf{\cos(\theta)}$ ✓. The cofunction identity is verified.

31. [2 credits] Let $u = \cos(x)$. The equation becomes $2u^2 - u - 1 = 0$, which factors as $(2u + 1)(u - 1) = 0$, giving $u = -1/2$ or $u = 1$. Solve $\cos(x) = 1$: $\mathbf{x = 0}$. Solve $\cos(x) = -1/2$ (reference angle $\pi/3$, negative in Quadrants II and III): $\mathbf{x = 2\pi/3}$ and $\mathbf{x = 4\pi/3}$.

32. [2 credits] Without replacement, calculate each order separately. P(red then blue) = (5/12)(4/11) = 20/132. P(blue then red) = (4/12)(5/11) = 20/132. Add: 40/132 = **10/33** (in simplest form, since GCD(40, 132) = 4).

PART III — Extended Constructed-Response Explanations

33.

(a) [1 credit] A 13% decrease per hour means 87% remains, so the hourly multiplier is 0.87. The exponential decay model is **$C(t) = 200(0.87)^t$**.

(b) [3 credits] Set C(t) = 50: $200(0.87)^t = 50$, so $(0.87)^t = 0.25$. Take the natural logarithm: t · ln(0.87) = ln(0.25), giving t = ln(0.25)/ln(0.87) ≈ −1.3863/−0.1393 ≈ 9.95. Rounded to the nearest tenth: **t ≈ 10.0 hours**.

34.

(a) [2 credits] Factor x out of p(x): $p(x) = x(2x^3 - 3x^2 - 11x + 6)$. Apply synthetic division with k = 3 to the cubic coefficients 2, −3, −11, 6:

```
3 |  2    -3    -11     6
  |        6      9    -6
  |______________________________
     2     3     -2     0
```

Quotient: **$2x^2 + 3x - 2$**.

(b) [1 credit] Factor $2x^2 + 3x - 2$ by splitting the middle term: $2x^2 + 4x - x - 2 = 2x(x + 2) - 1(x + 2) = (2x - 1)(x + 2)$. Complete factorization: **p(x) = x(x − 3)(2x − 1)(x + 2)**.

(c) [1 credit] Setting each factor to zero: **x = 0, x = 3, x = 1/2, and x = −2**.

35.

(a) [1 credit] Sample proportion: $\hat{p}$ = 175/250 = **0.7** (or 70%).

(b) [2 credits] Compute ME = 1.96 · √(0.7 · 0.3/250) = 1.96 · √(0.21/250) = 1.96 · √0.00084 ≈ 1.96 · 0.02898 ≈ 0.0568. Confidence interval: 0.7 ± 0.0568, giving lower bound 0.7 − 0.0568 ≈ **0.643** and upper bound 0.7 + 0.0568 ≈ **0.757**. CI ≈ **(0.643, 0.757)**.

(c) [1 credit] **Yes** — the entire interval (0.643, 0.757) lies above 0.60, and the lower bound 0.643 is greater than 0.60. With 95% confidence, the true improvement proportion for the new medication exceeds 0.60, so the team can conclude that the new medication is more effective than the older one.

PART IV — Long Constructed-Response Explanation

36.

(a) [2 credits] In the model $d(t) = 1.5 \cos((\pi/4) t) + 3$, the coefficient of t inside cosine is $B = \pi/4$. The period is $2\pi/B = 2\pi/(\pi/4) =$ **8 seconds**. This means the buoy completes one full vertical oscillation every 8 seconds.

(b) [1 credit] Substitute $t = 6$: $d(6) = 1.5 \cos((\pi/4)(6)) + 3 = 1.5 \cos(3\pi/2) + 3$. Since $\cos(3\pi/2) = 0$: $d(6) = 1.5(0) + 3 =$ **3.0 meters**. The buoy is at the midline at this moment.

(c) [3 credits] Set $d(t) = 3.5$: $1.5 \cos((\pi/4) t) + 3 = 3.5$, so $1.5 \cos((\pi/4) t) = 0.5$ and $\cos((\pi/4) t) = 1/3$. Let $u = (\pi/4) t$; as t ranges over [0, 16], u ranges over $[0, 4\pi]$. In $[0, 2\pi]$, $\cos(u) = 1/3$ has solutions $u_1 = \arccos(1/3) \approx 1.2310$ and $u_2 = 2\pi - 1.2310 \approx 5.0522$. Repeating in $[2\pi, 4\pi]$: $u_3 \approx 2\pi + 1.2310 \approx 7.5142$ and $u_4 \approx 4\pi - 1.2310 \approx 11.3354$. Convert via $t = (4/\pi) u$: $t_1 \approx (4/\pi)(1.2310) \approx$ **1.57 seconds**, $t_2 \approx (4/\pi)(5.0522) \approx$ **6.43 seconds**, $t_3 \approx (4/\pi)(7.5142) \approx$ **9.57 seconds**, $t_4 \approx (4/\pi)(11.3354) \approx$ **14.43 seconds**. The buoy reaches exactly 3.5 m above sea level four times during the 16-second interval, with each pair symmetric about the midpoint of each 8-second period.